The Actuarial Profession

making financial sense of the future

Projecting a Realistic Balance Sheet

2004 Life Convention, EICC Edinburgh Scotland

Session D12, 9 November

Stephen Keane

Agenda

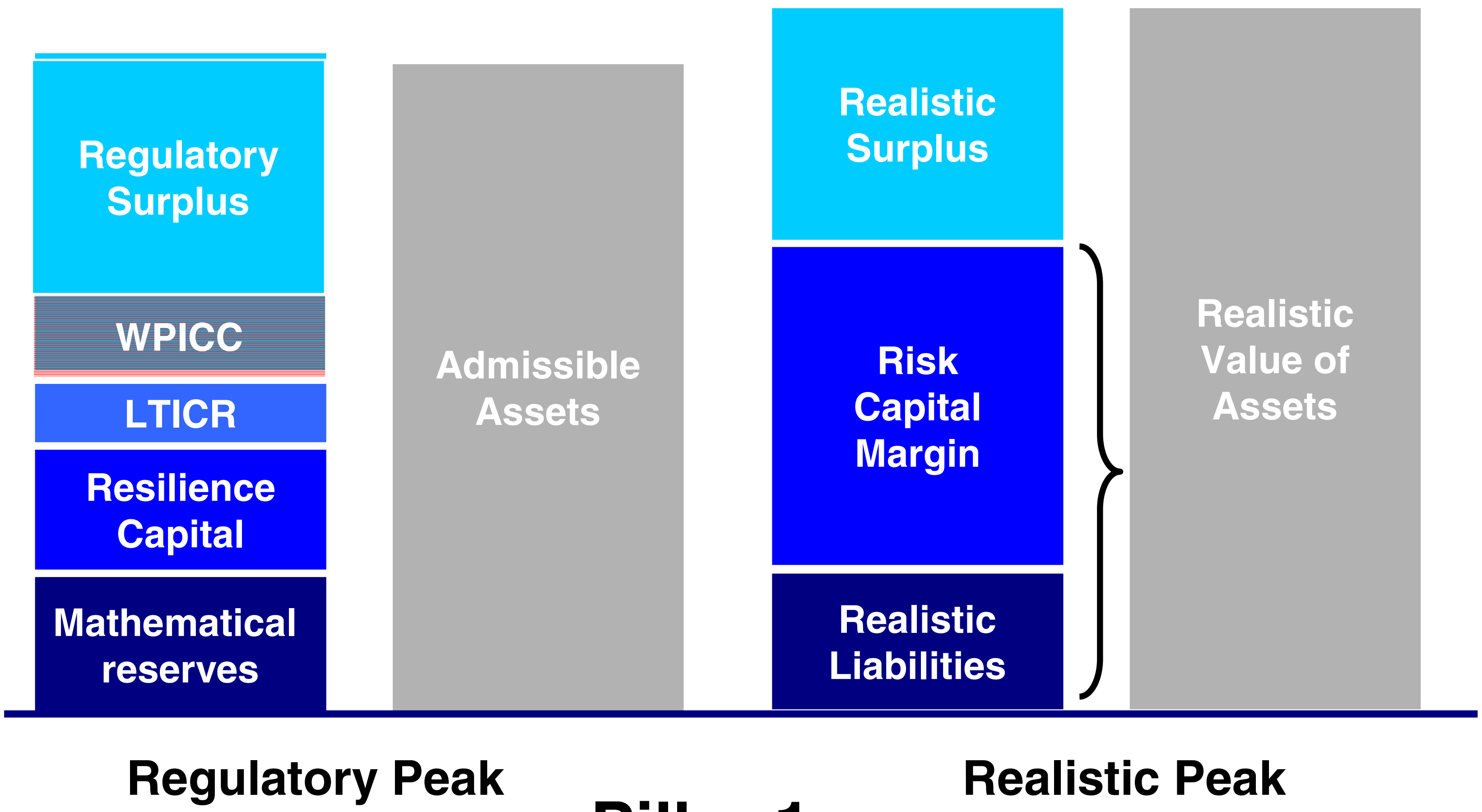
- Why do we need to project the Realistic Balance Sheet ?
- What economic information do we have about the future ?
- Methods used to project the RBS :
  - Deterministic scenario followed by stochastic valuation
  - Closed form approach
  - Simulation within simulation
- Question & Answers

Projecting the RBS – Point zero

- I can calculate my RBS and fulfil my duties to the FSA in meeting their reporting requirements
- I can calculate my ICA via a series of stresses to my RBS with some further calculations outside my model
- I have been preparing business plans and forecasts for my board in the manner they require for years now
- I don't have the time, the energy or the inclination, so …..

WHY BOTHER ?

The intersection between the RBS and the projected RBS



Pillar 1

The intersection between the RBS and the projected RBS

- Management decisions
  - Pillar I is the higher of two peaks - we should take account of which peak is biting when making management decisions in our models

- Management decisions again
  - The RBS model is invaluable at explaining cause and effect to senior management

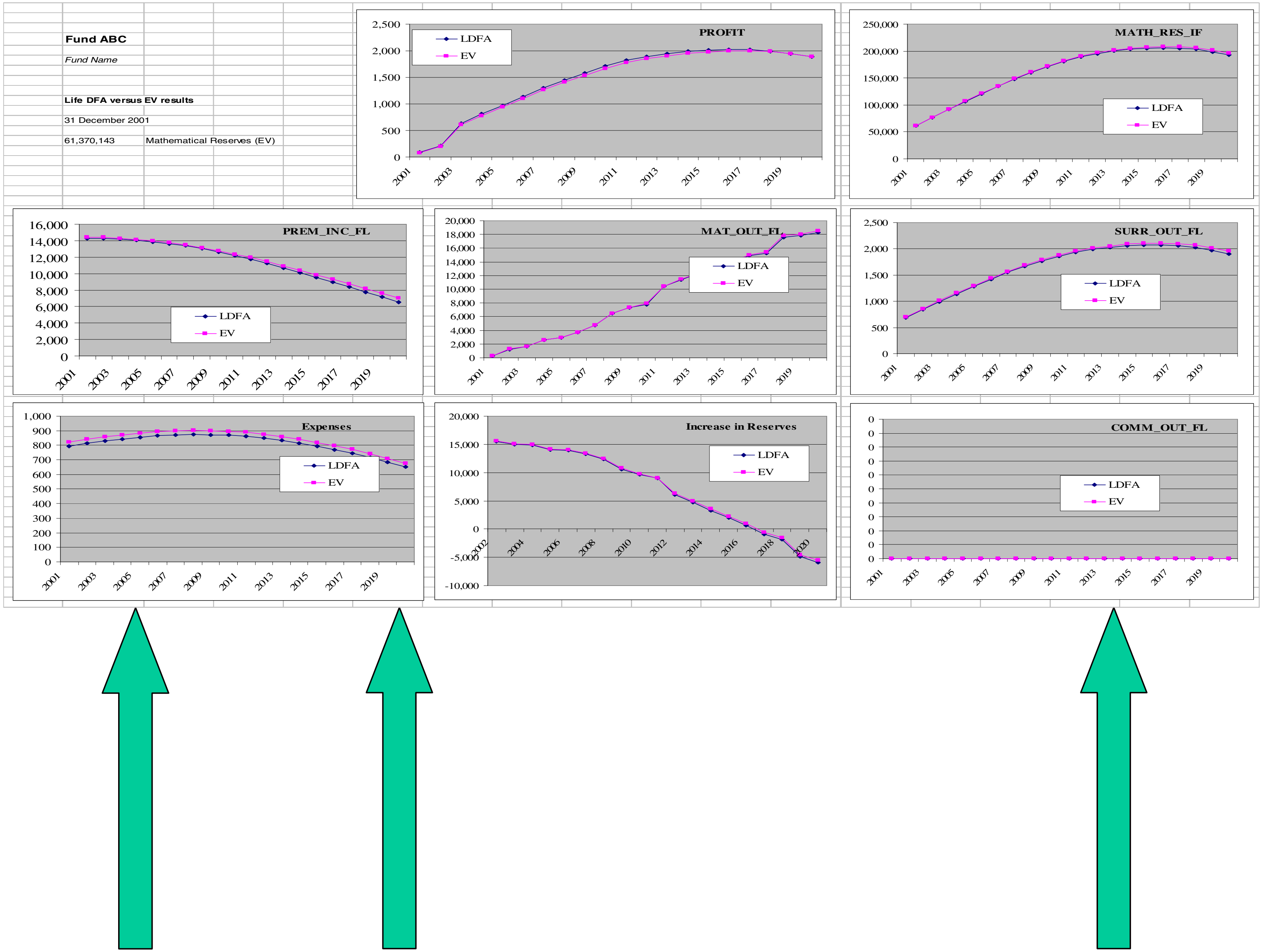
- PPFM
  - In drafting and communicating the PPFM to the policyholders - we will want to have the utmost credibility on that articulation based upon many different views of outcomes to the fund

The intersection between the ICA and the projected RBS

- Individual Capital Guidance
  - ICG will be set taking into consideration capital consistent with a 99.5% confidence level over a one year period or to a lower level of confidence over a longer period

- Regulation (PRU 2.3.14 G)
  - Throughout whatever timeframe is adapted by firms, firms should ensure that their **projected** assets are, and will continue to be sufficient, to enable their **projected** liabilities to be paid, and it would be reasonable for firms to test that this is the case at the end of each year of the timeframe

- Professional guidance (TS3 / GN46, 4.11)
  - This means that the value of assets must exceed the "value" of liabilities at each year end (or at the end of the year for a one-year projection). For this purpose, 'value' of assets and liabilities should be interpreted consistently with PRU 7.4 (ie realistic values calculated on a market consistent basis) for a realistic basis firm



Time 0 Time 1 .... Time n

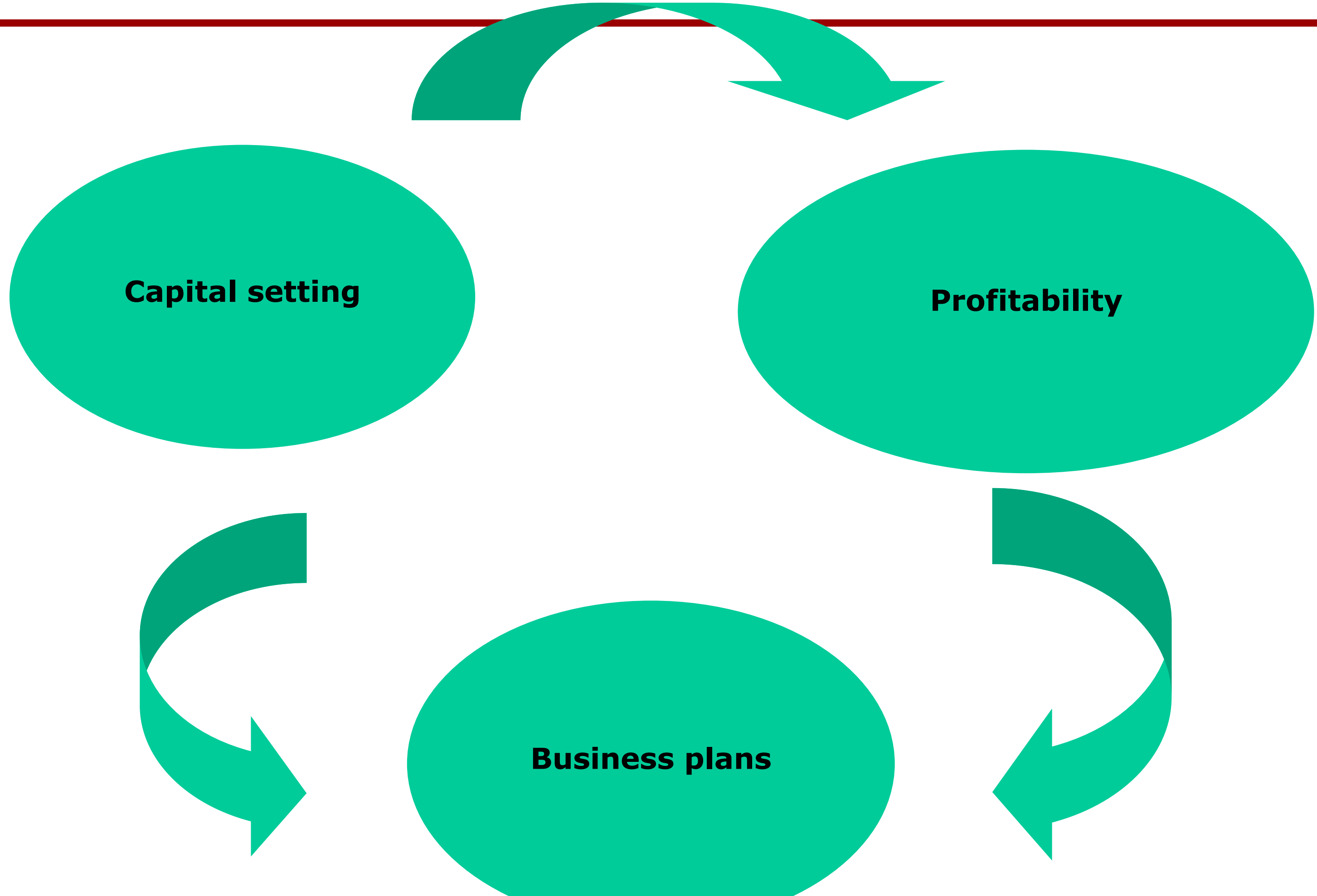
The intersection between the ICA and the projected RBS – the reality for most

- Calculating the ICA against a time zero or a future year balance sheet ?
  - If companies can project the RBS then they will set the ICA against the distribution of the time one or time ‘n’ balance sheet
  - If companies cannot project the RBS then they use the time zero balance sheet as a proxy for achieving this

The intersection between the business plans and the projected RBS

- Business plans are forward looking
- They are the articulation of the managements goals in developing their business over the time horizon
- A business plan only makes sense if it can be delivered against the companies level of available capital
- Companies are now operating in a world where the marked to market balance sheet is the key focus – as is the pure economic view which sits behind their internal capital models
- The nature of business planning needs to change to incorporate a projection of the marked to market balance sheet
  - Range of outcomes rather than a single point estimate
  - Likelihood of an outcomes

The virtuous circle

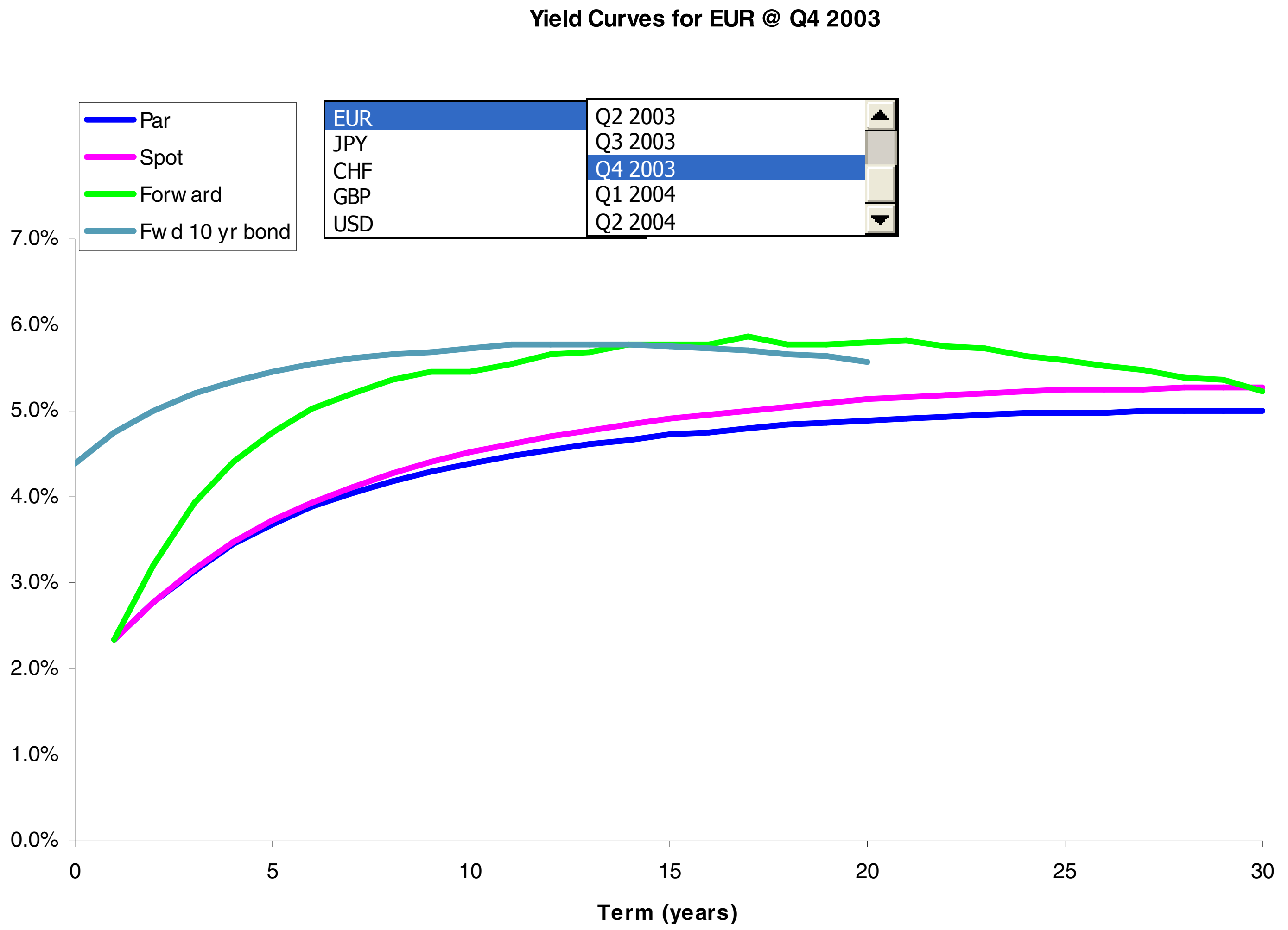


STARTING TO CARE ?

Economic information about the future

- Yield curve
- Economic Scenario Generator

The Yield Curve



The Yield Curve

- As a predictor of what will happen in the future the information in the yield curve will almost certainly not work out to be correct
- What is important though is that these are the rates against which the markets will trade future interest rates
- This is consistent with the option that companies have in hedging their risk to underlying asset classes. Once properly hedged – making forecasts about the future is strengthened

The Economic Scenario Generator

- The ESG captures the historic correlations between different asset classes and projects them coherently bearing in mind their link with interest rate structures
- It is also calibrated to the implied volatility in today's market prices
- Two challenges for using a Monte Carlo approach within a projection :
  - What happens to the correlations ?
  - What happens to volatilities ?

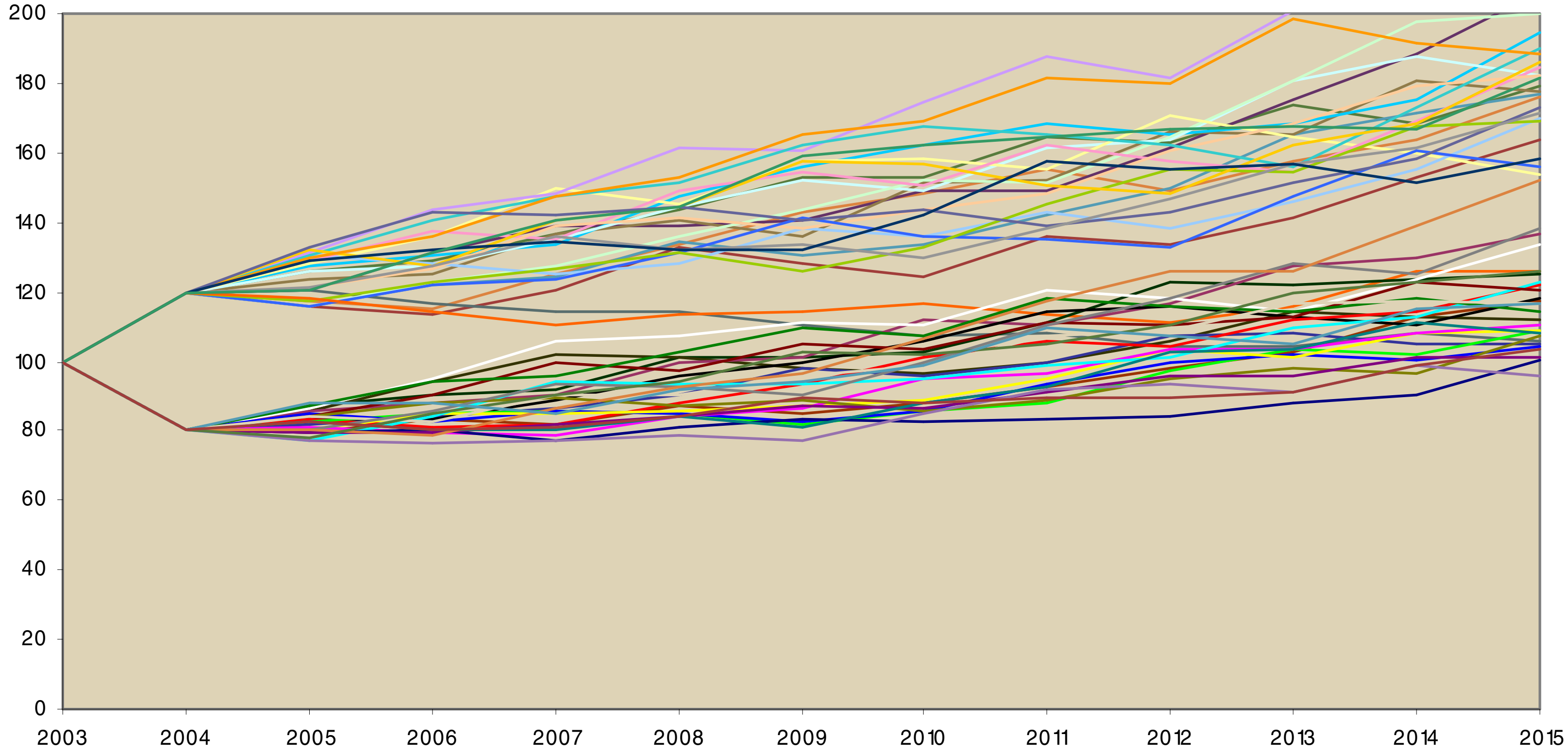
The techniques already used to project the RBS

- Deterministic scenario followed by stochastic valuation – the roll forward
- Closed form approach
- Simulation within simulation – the fan

The roll forward

- This method involves taking a single deterministic step into the projection and then uses a Monte Carlo technique to draw up the marked to market balance sheet after that step

The roll forward



The roll forward

- Things to note about this approach :
- The choice about the step is up to the user, it may have or it may not have any economic justification
- A set of simulations appropriate for the time 1 calculation is required
  - Reflect the shape of the yield curve at that time based on forward rates at time 0
  - The underlying relationships between the asset classes are not reconsidered after the roll forward
- Compliments an optimistic, central, pessimistic approach to planning which is currently widely used by companies
- For each deterministic step the computer runtime required would be approximately the same as a RBS run

Closed form solutions

- If you can find a replicating portfolio for the market consistent valuation of life insurance business you can use this portfolio to consistently project both sides of the balance sheet
- Finding a replicating portfolio is not easy particularly when you want to reflect things like a dynamic reversionary bonus process
- One solution is to decompose the liability cashflows into their component parts and to consider the appropriate portfolios for each of these components
- If we start with something more straightforward like non profit business, we require an accurate gross premium calculation, allowing for lapses, mortality improvement and interest rate structures

Closed form solutions – non profit

- Our first problem, we would like to use commutation functions to perform these calculation, but they don't handle future lapses, complex interest rate structures and mortality improvements

- Hence we are looking for something that is a function of age, term, the yield curve at the projected valuation date, policy characteristics, mortality and lapse rates, x, n, r(n), χ, q, and s, respectively.

GPV(t, χ)	= F1(x, n, r(n), χ, q, s)	Annuity
	+ F2(x, n, r(n), χ, q, s)	Maturity benefit
	+ F3(x, n, r(n), χ, q, s)	Surrender benefit
	+ F4(x, n, r(n), χ, q, s)	Death benefit
	+ F5(x, n, r(n), χ, q, s)	Future Expenses
	- F6(x, n, r(n), χ, q, s)	Future Premiums
	+ F7(x, n, r(n), χ, q, s)	Future charges

$$\text{Reserve} = \sum_{k=1}^{7} F(k) = \sum_{k=1}^{7} \Big(\sum_{j=1}^{n} probability * cashflow * ZCB(j) \Big)$$

Closed form solutions – with profit

- Turning to with profits business – things get a bit trickier
- The equation that summaries the realistic liability of with profits business is :

  Realistic Liabilities = $\sum$ AS + $\sum$ Put(AS, G(b), r, σ)

where

AS = Asset Share

Put = the Black-Scholes price of a put option with parameters

AS as the share price

G{b} = guarantee at event (including a future bonus assumptions {b})

r = the risk free rate for that duration derived from ZCB's

σ is the volatility assumption

Closed form solutions – with profit

- Another way of considering this is to re-write the equation in terms of the guarantee with a call option :

Realistic Liabilities = ∑ GPV(G(b), b{n}, r) + ∑ C(AS, G(b), r, σ)

where

GPV is a gross premium valuation at the risk free rate, liabilities are the guaranteed element only including future reversionary bonus and exclude terminal bonus.

C is a call option with strike G, it pays out when the asset share at maturity exceeds the guaranteed element.

$$\text{Realistic Liabilities} = \sum_{k=1}^{7} F(k) = \sum_{k=1}^{7} \Big(\sum_{j=1}^{n} probability * cashflow * ZCB(j) \Big)$$

$$+ \sum_{k=1}^{7} \Big(\sum_{j=1}^{n} probability * call(j, as, G\{b\}) \Big)$$

Closed form solutions – GAO's

- GAO reserving can be thought of as having two components
  - the intrinsic value of the option
  - the time value of the option.

- The intrinsic plus time value is an interest rate derivative and is usually priced using a formula similar to that for a swaption.
- It should be noted also that both the time value and intrinsic value of the option are influenced by expected mortality rates.
- If mortality varies stochastically in the model then the values will change.

Closed form solutions – GAO's intrinsic value

- If we take a single premium endowment or UWP contract as an example the projected maturity benefit is:
- K, the guaranteed benefit at vesting = *SA * (1+b)^n*
- Guaranteed conversion rate = a^g
- Value of an annuity at vesting = $a^i = 1/(ZCB(n) * survival(n)) \sum_{j=n}^{110} probability * ZCB(j+1)$
- The intrinsic value = $K * ZCB(n) * probability(n) * Max[1/a^g - 1/a^i, 0] * a^i$

Summed to the point at which the survival probability is zero

Closed form solutions – GAO's time value

- To obtain the time value we need to value the total value using a derivative formula
- Such formula are usually based on the premise that the ratio a^i / a^g as a process can be modelled.

- Black-Scholes does not allow for the term structure of interest rates. The information about the term structure is available in a^i / a^g as is information about expected mortality which is another stochastic variable.

- Such an option would return the value of the GAO on the guarantee on vesting.

Closed form solutions – GAO's time value

Value of GAO (time & instrinsic) =

K * ZCB(n) * probability(n) *

{N(($\ln(a^g / a^i) / \sigma_m \sqrt{n} + \sigma_m \sqrt{n} / 2$) + a^i / a^g *

N($\ln(a^i / a^g) / \sigma_m \sqrt{n} + \sigma_m \sqrt{n} / 2$) -1}

- Where σ_m = annualised volatility of market annuity price.
- This volatility term also includes volatility due to mortality improvement / deterioration as well as changes to the interest rate structure

Closed form solutions – GAO's further sophistication

- The previous valuation of the GAO only considered existing guaranteed benefits
- When you consider the potential for future addition to the existing guarantees then a more sophisticated calculation is required
- The formula is set out in Sheldon & Smith (2004)

GAO value = probability(n)*{ ZCB(n) *K* $N_2[a_1, b_1,\rho]$

$+ as''$ * $N_2[a_2, b_2 , -\rho]$

+ ZCB(n) *K * (a^i / a^g)* $N_2[a_3, b_3 , -\rho]$

$+ as''$ * (a^i / a^g)*$\exp(\rho\sigma_s\sigma_m)$* $N_2[a_4, b_4 , -\rho]$}

Where:

$a_1 = \ln(K / as'') / \sigma_s \sqrt{n} + \sigma_s \sqrt{n} / 2$

$a_2 = \ln(as'' / K) / \sigma_s \sqrt{n} + \sigma_s \sqrt{n} / 2$

$a_3 = \ln(K / as'') / \sigma_s \sqrt{n} + (\sigma_s - \rho\sigma_m)\sqrt{n} / 2$

$a_4 = \ln(as'' / K) / \sigma_s \sqrt{n} + (\sigma_s + \rho\sigma_m)\sqrt{n} / 2$

$b_1 = \ln(a^g / a^i) / \sigma_m \sqrt{n} + \sigma_m \sqrt{n} / 2$

$b_2 = \ln(a^g / a^i) / \sigma_m \sqrt{n} + (\sigma_m - \rho\sigma_s)\sqrt{n} / 2$

$b_1 = \ln(a^i / a^g) / \sigma_m \sqrt{n} + \sigma_m \sqrt{n} / 2$

$b_2 = \ln(a^i / a^g) / \sigma_m \sqrt{n} + (\sigma_m + \rho\sigma_s)\sqrt{n} / 2$

$N_2(a, b, \rho)$ denotes the standard cumulative bi-variate normal distribution with correlation ρ.

Where σ_s = annualised volatility for the asset share / unit fund.

And ρ= correlation of unit fund and market annuity.

Closed form solutions - summary

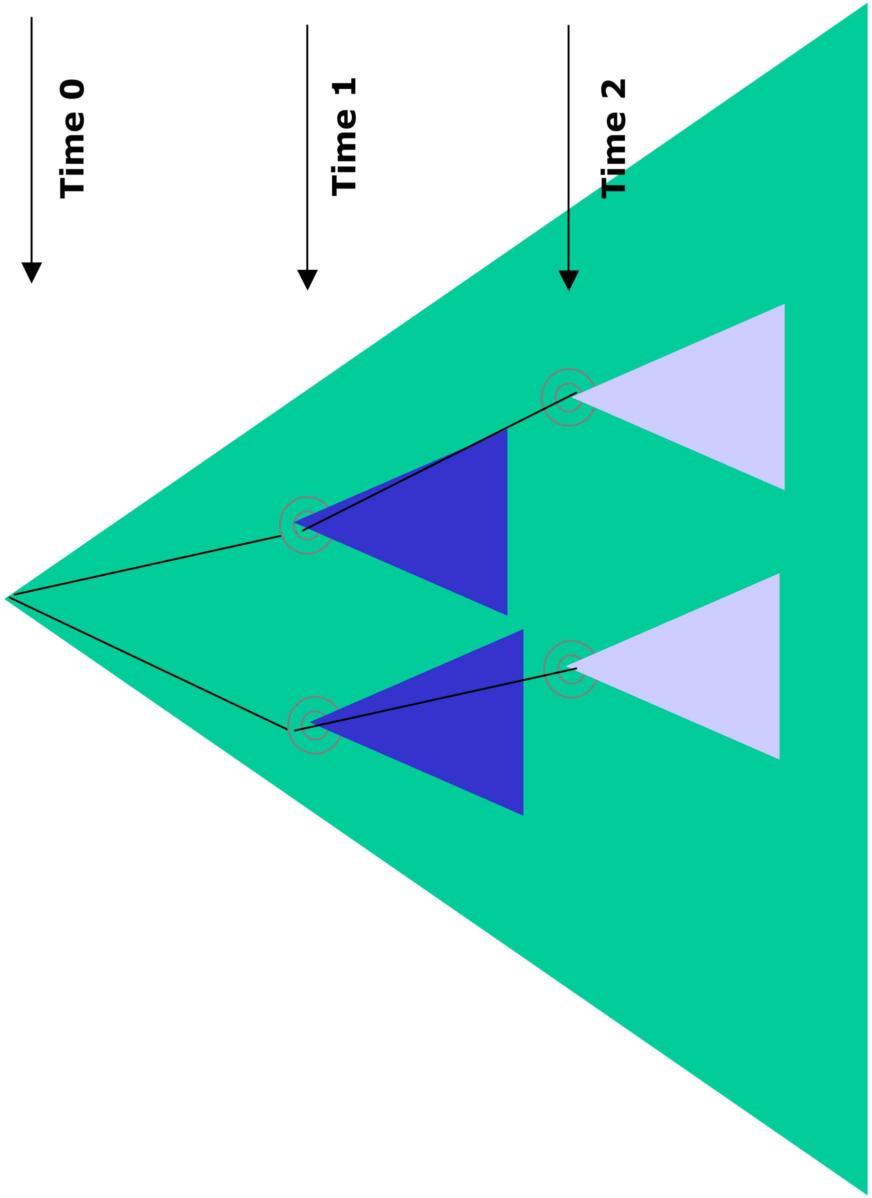
- A close consideration of the appropriate valuation formula required to reflect the different benefit structures written by companies is needed
- A number of these formulae are in the public domain
- When things get complex it does require some grey matter to ensure that the valuation is okay
- Some simplifying assumptions are used just to keep the algebra manageable
- Not everyone can immediately confirm / validate the approach taken as being reasonable

Simulation within simulation – the fan method

- We use a Monte Carlo technique to draw up the market to market balance sheet at time zero. This can reflect the complexities required like management actions, dynamic bonus or lapse decisions.

- The fan method allows you to use a Monte Carlo technique to draw up the balance sheet at future intervals

The fan method



- Run 1000 simulations to get a time zero marked to market balance sheet
- For every simulation in a pre-defined time horizon we perform a further simulation to get the marked to market balance sheet at that time
- This balance sheet can be calculated using 500 simulations
- The simulation sets used for valuation reflect the changes to the yield curve over the time horizon
- Runtime would increase significantly

The fan method – runtime performance

- 1,000 simulations
- Fan of 500
- 1 period projection
- Effectively 500,000 simulations

- 1 PC = 500 hours
- 10 PCs = 50 hours = 1 weekend
- 40 PCs = 12.5 Hours = overnight

- This is the type of PC power being used by companies currently to get the time zero balance sheet

The fan method – runtime performance

Solvency wars

Calculating the new "realistic" solvency required of life assurers by the Financial Services Authority is getting serious. Standard Life has linked 45 personal computers to do the necessary "stochastic" modelling – checking what would happen to its assets and liabilities in a wide range of circumstances, including terrifying scenarios involving equity market crashes and interest rate gyrations.

Prudential this week told analysts it was using 75 PCs to crunch its numbers. Aviva then escalated this Weapons of Mass Computation contest by revealing it had no fewer than 200 PCs linked up for the same task. That raises the question: just how many PCs will the FSA watchdogs need at Canary Wharf to check all these computations?

Source : Financial Times 27 February 2004

- Weapons of mass computation
- Standard Life : 45
- Prudential : 75
- Aviva : 200

The fan method - summary

- Very heavy reliance on PC hardware
- Is the most transparent, despite having many thousands of simulations
- Can be used to demonstrate the strength / weakness of a closed form approach

Projecting the Realistic Balance Sheet - Summary

- A necessary evil
- A range of techniques are available to do so
- The most transparent technique, simulation within simulation is now a practical possibility
  - even of getting the results before we all retire

QUESTIONS ?