THE PUT PRINCIPLE

A UNIFIED THEORY OF CAPITAL MARKET PRICING.

ABSTRACT

This paper develops a unified theory for the pricing of capital market assets that is consistent across all classes of assets: equities, bonds, currencies and derivatives.

A forward-looking approach to the determination of the Risk Premium is first postulated and an appropriate simple mathematical model is developed. This provides insight into both the mathematical form of the Risk Premium and the importance of investor risk behaviour as an element in its formulation.

The techniques of stochastic calculus are then utilised to derive an option pricing model which explicitly allows for the inclusion of the Risk Premium in the form derived.

Initial empirical testing with US equity-index option data indicates that the model is extremely robust. Of significant interest is that, as a result of solving to find the appropriate Risk Premium, the "volatility smile" is virtually eliminated.

Although only equities and derivatives are included in this paper, the theory has also been extended to the development of bond and currency valuation models. The bond model provides a means to estimate the Term Premium inherent in the yield structure, thus enabling the true underlying term structure of interest rates to be extracted from market data. The Currency model also offers valuable insight into the dynamics of exchange rate movements.

Also of key importance is that the Put Principle applies to individual assets and groups of assets. It therefore provides a direct challenge to the concepts underlying much of modern portfolio theory.

KEYWORDS

Risk Premium; Option Pricing; Safety First; Volatility Smile; Downside Risk; Market Price of Risk.

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1. INTRODUCTION

1.1 Background

1.1.1 The purpose of this paper is to provide an alternative model for the pricing of investment assets. Conceived initially as an alternative approach to estimating equity Risk premia, it has been developed into a much broader investment theory which covers equities, options, currencies and bonds. Only the first two of these are covered in this paper, with a subsequent paper dealing with currency and bond assets.

1.1.2 Traditional approaches to the estimation of Risk Premia have been based primarily on *ex post* analyses of return series in order to identify the relative excess returns earned by the different asset classes or sub-classes. This approach, whilst clearly providing insight into the relative magnitude of the returns, does however suffer from a number of inherent drawbacks: -

> Firstly, it is dependent on the actual time period over which the sampling is conducted which can thus introduce a start-point or end-point bias.
>
> Secondly, and of greater importance, it assumes that historic return differences provide a true measure of the underlying market risk premia.

1.1.3 Whilst recently-published research work has sought to overcome some of these problems, it has still only proved possible to provide longer-run estimates of the likely ranges of risk premia

1.1.4 In this paper, an alternative approach is used to estimate the forward-looking risk premia which are applied by the market in the formulation of asset prices. The derivation is based on the assumption that investors require additional expected returns to compensate them for taking on additional levels of perceived risk, and that this requirement forms an intrinsic part of the market pricing mechanism.

2. THEORETICAL DEVELOPMENT

2.1 Real Downside Risk and "Safety First"

2.1.1 The theoretical development in this paper is based on the usual simplifying assumptions regarding the existence of frictionless markets; the absence of transaction costs and tax considerations; and the efficient pricing of information.

2.1.2 An additional key assumption is that investors are primarily concerned with the risk of failing to earn a required minimum level of return on their investments. In this paper, we assume that this hurdle rate is equal to the Risk-free rate of return which could be earned on riskless assets by the investors. We define this risk as Real Downside Risk and the concern with Real Downside Risk is assumed to have a significant role in the pricing of assets purchased by the investors.

2.1.3 The concept that investors may be more concerned about Downside Risk was first propounded by A D Roy who used the term "Safety First" to describe the attitude of such investors.

2.1.4 In his paper "Safety First and the Holding of Assets" he argued that, given

i) An expected return on assets of $\boldsymbol{m}$,
ii) A constant standard deviation of returns of $\boldsymbol{\sigma}$, and
iii) A minimum required return of $\boldsymbol{d}$;

then the "Safety First" requirement is equivalent to minimising the probability that the expected return from the assets $\boldsymbol{m}$ is less than the required return $\boldsymbol{d}$.

2.1.5 He then demonstrated that this requirement, expressed as $\mathbf{Prob}(\boldsymbol{d} \leq \boldsymbol{m})$, is equivalent to resolving the expression

$$\mathbf{Max}\frac{(\boldsymbol{m}-\boldsymbol{d})}{\boldsymbol{\sigma}} \tag{2.1}$$

2.1.6 As a means of achieving this, we can rank different portfolios in order of attractiveness by setting

$$\frac{(\boldsymbol{m}-\boldsymbol{d})}{\boldsymbol{\sigma}} = \boldsymbol{K}\ , \tag{2.2}$$

where $\boldsymbol{K}$ is a constant for each level of attractiveness.

2.1.7 Re-arranging Equation **(2.2)**, we have

$$\boldsymbol{m} = \boldsymbol{d} + \boldsymbol{K}.\boldsymbol{\sigma} \tag{2.3}$$

the equation of a straight line whose slope is dependent upon the value of $\boldsymbol{K}$.

2.1.8 In particular if we assume, as outlined above, that the minimum return requirement is the Risk-free rate $\boldsymbol{r}$, then we have

$$\boldsymbol{m} = \boldsymbol{r} + \boldsymbol{K}.\boldsymbol{\sigma} \tag{2.4}$$

2.1.9 Thus, the Safety First investor, with such a minimum return objective, would require his preferred portfolios to provide a return which is equal to the Risk-free rate plus an additional amount which is a function of the riskiness of the portfolio.

2.1.10 Whilst the above analysis provides an interesting insight into the likely behaviour of "Safety First" investors, it does not actually provide us with a clear enough definition of the likely range of values for $\boldsymbol{K}$.

2.1.11 We can, however, obtain a clearer insight into this aspect by adopting a different approach to the problem.

2.2 An Alternative Approach

2.2.1 We first assume that "Safety First" investors are dominant in the market-place, and hence are effectively the "price-setters" rather than "price-takers" in the market.

2.2.2 We also assume, as outlined above, that they are primarily concerned with minimising their exposure to Real Downside Risk.

2.2.3 Since these investors are assumed to be price-setters in the market, we need to be able to construct a methodology for pricing assets which is based around their risk requirements.

2.2.4 As we know that these investors are more concerned with the risk of loss, we can see that one approach would be to start by calculating the cost of purchasing Real Downside Risk protection. This will clearly be related to the riskiness of the assets, and to the time horizon(s) of the investor(s).

2.2.5 This cost, or premium, will be equivalent to the cost of purchasing a Put option with an expiry date set at the investor's time horizon, and the strike price equal to the forward price of the asset at that time. (i.e. the current spot price accumulated at the Risk-free rate of interest).

2.2.6 Whilst the purchasing of such downside protection ensures that the Risk-free rate of interest is earned on the initial assets, the investor also will require to recover the cost of purchasing the Put options from the expected return from those assets.

2.2.7 We now make a further temporary stringent assumption (but one that can readily be relaxed), that the investors are unwilling to make any allowance for the existence of Upside Risk. (i.e. the potential for the volatility of the asset to render additional positive returns).

2.2.8 In such circumstances, we can see that the required return from the assets will be the amount necessary to return the Risk-free rate on the initial assets and to cover the cost of the Downside Risk insurance premium. This is because the investors have no downside or upside risk.

2.2.9 Expressed mathematically, this would require that the following equation be satisfied: -

$$\mathbf{E}(\boldsymbol{A_t}) - \boldsymbol{P_t} = \boldsymbol{R_t} \qquad \textbf{(2.5)}$$

Where:

Expected return over time **t** from Asset A_t $= \mathbf{E}(\boldsymbol{A_t})$

Premium rate for downside risk insurance $= \boldsymbol{P_t}$

Risk-free Rate for period t $= \boldsymbol{R_t}$

2.2.10 Rearranging equation **(2.5)**, we have

$$\mathbf{E}(\boldsymbol{A_t}) = \boldsymbol{R_t} + \boldsymbol{P_t} \qquad \textbf{(2.6)}$$

2.2.11 Thus the expected return is equal to the sum of the Risk-free Rate, plus the cost of insuring the Real Downside Risk.

2.2.12 However, as noted above, this equation was based on the assumption that the investors made no allowance for the potential for Upside Risk, so we need to weaken this assumption to provide a closer reflection of reality.

2.2.13 We can note that, in the same manner that we can estimate the premium for Real Downside Risk using the price of a Put option on the forward price of the asset, so we can also estimate a value to be placed on the Real Upside Risk (being the potential for an excess return above the Risk-free rate) as the price of a Call option on the forward.

2.2.14 If we therefore relax our stringent assumption to some degree and now assume that the investors will be willing to allow some offset for the Real Upside Risk, we can rewrite Equation **(2.5)** as

$$\mathbf{E}(\boldsymbol{A_t}) - \boldsymbol{P_t} + \boldsymbol{k.C_t} = \boldsymbol{R_t} \qquad \mathbf{(2.7)}$$

where $\boldsymbol{C_t}$ is the cost of the Call option and $\mathbf{1} \geq \boldsymbol{k} \geq \mathbf{0}$ is a scaling constant.

2.2.15 Since these theoretical options are only effectively exercisable at expiry, they are equivalent to European style options and, consequently.The Put-Call parity relationship holds, and the values of the Put and Call options on the forward price are identical. We can therefore rewrite equation **(2.7)** as

$$\mathbf{E}(\boldsymbol{A_t}) = \boldsymbol{R_t} + (\mathbf{1} - \boldsymbol{k})\boldsymbol{P_t} \qquad \mathbf{(2.8)}$$

or in more simplified form, replacing $(\mathbf{1} - \boldsymbol{k})$ with $\mathbf{1} \geq \boldsymbol{K} \geq \mathbf{0}$

$$\mathbf{E}(\boldsymbol{A_t}) = \boldsymbol{R_t} + \boldsymbol{K.P_t} \qquad \mathbf{(2.9)}$$

2.2.16 We can also note here that, for a broader sphere of investors, whose risk profile might be somewhat different from that of the Safety First investor, it would seem quite rational to assume that they too would have an excess return requirement which could be represented in the form $(-\boldsymbol{j.P_t} + \boldsymbol{k.C_t})$, where $\mathbf{1} \geq \boldsymbol{j} \geq \mathbf{0}$ is a second scaling constant.

2.2.17 As can readily be seen, such a profile is also encapsulated in Equation **(2.9)**, and the bounds of $\boldsymbol{K}$ remain the same. This is because the upper bound of $\boldsymbol{K} = \mathbf{1}$ represents the extreme case where investors make no allowance for Real Upside Risk, whilst the lower bound of $\boldsymbol{K} = \mathbf{0}$ represents the Risk Neutral case, where investors regard the upside and downside risks as equivalent and therefore have a return expectation equal to the Risk-free rate.

2.3 Defining the Nature of the Risk Premium

2.3.1 We have thus developed a possible valuation methodology for pricing risky assets, which suggests that the expected return from those assets should be a sum of the Risk-free rate plus a constant multiplied by the value of a Put option.

2.3.2 Of importance here is the fact that the Put option is specifically defined as

i) A European-style option, exercisable only at expiry.

ii) An option on the forward price of the asset

iii) Having the price for the option payable at the expiry date, since it must be paid for from the return earned on the asset.

2.3.3 It is interesting at this stage to look closely at the characteristics of such an option.

2.3.4 In Appendix 1, the standard Black-Scholes Call option pricing formula is developed, using an expectations approach within a stochastic calculus framework. This is a fairly standard approach but the derivation is included both for the sake of completeness and because the same technique is employed later in this paper to develop an alternative option pricing formula.

2.3.5 We adapt the standard Black-Scholes equation for the pricing of options on stock prices into the form required for pricing options on forwards. Thus the basic formula

$$C = S.N(d_1) - e^{-rt}.X.N(d_2) \qquad \textbf{(2.10)}$$

Where $d_1 = \dfrac{\ln(S/X) + (r + ½\sigma^2)t}{\sigma\sqrt{t}}$ **(2.11)**

And $d_2 = \dfrac{\ln(S/X) + (r - ½\sigma^2)t}{\sigma\sqrt{t}}$ **(2.12)**

becomes

$$C\,.e^{rt} = e^{rt}.S.N(d_1) - F.N(d_2) \qquad \textbf{(2.13)}$$

where F is the forward price; S is the spot price; and $F = S.e^{rt}$ by definition.

(The formula for the Call option has been used here, but as explained above, this is identical to the value of the Put option when the options are calculated on the forward price).

2.3.6 We can then write

$$d_1 = \frac{\ln(S/F) + (r + ½\sigma^2)t}{\sigma\sqrt{t}}$$

$$= ½\sigma\sqrt{t}$$

As $\ln\left(S/F\right) = -rt$

And, similarly $d_2 = -½\sigma\sqrt{t}$

Giving, $P\,.e^{rt} = e^{rt}.S.[N(½\sigma\sqrt{t}) - N(-½\sigma\sqrt{t})]$

So $P\,.e^{rt} = F.[2N(½\sigma\sqrt{t}) - 1]$

$$\approx F.\frac{\sigma\sqrt{t}}{\sqrt{2\pi}} \qquad \textbf{(2.14)}$$

Or, $$\frac{P}{S}.e^{rt} \approx e^{rt}.\frac{\sigma\sqrt{t}}{\sqrt{2\pi}} \qquad \textbf{(2.15)}$$

(See Brenner & Subrahmanam, FAJ 1988, V44 "A simple formula to compute the implied standard deviation")

2.3.7 Hence, if we define the Risk Premium over a time horizon T as $P_T^* = K.P_T.e^{rT}$

Then:

$$P_T^* \approx K.e^{rT}.\frac{\sigma\sqrt{T}}{\sqrt{2\pi}} \qquad \textbf{(2.16)}$$

and equation **(2.9)** can be written as

$$\mathrm{E}(A_t) = R_t + K.e^{rT}.\frac{\sigma\sqrt{T}}{\sqrt{2\pi}}_t \qquad \textbf{(2.17)}$$

2.3.8 As we can see, in its approximated form, the equation for the expected return on the assets is virtually identical to that proposed by Roy. There are, however, two significant differences.

Firstly, we have now defined this formula as an intrinsic element in the pricing of securities, as opposed to it being a tool for selecting assets which meet the criteria of the Safety First investor.

Secondly, we have established clear boundaries for the constant, which approximate to $0.4 \geq K \geq 0$.

2.3.9 Additionally, we can note the following result which arises when we use this approximation. If we define the expected return of the asset as the drift rate μ, then we can write

$$\mu t \approx rt + K.e^{rt}.\frac{\sigma\sqrt{t}}{\sqrt{2\pi}}$$

So that

$$K.e^{rt}.\frac{1}{\sqrt{2\pi}} \approx \frac{\mu t - rt}{\sigma\sqrt{t}} \qquad \textbf{(2.18)}$$

2.3.10 And we can see that the right hand side is identical to the standard definition of λ_t, the Market Price of Risk for the time period under consideration, thus giving us:-

$$\lambda_t \approx K.e^{rt}.\frac{1}{\sqrt{2\pi}} \qquad \textbf{(2.19)}$$

So that $$P_t^* \approx \lambda_t.\sigma\sqrt{t} \qquad \textbf{(2.20)}$$

And $$\mathbf{E}(A_t) = \boldsymbol{R_t} + \lambda_t.\sigma\sqrt{\boldsymbol{t}} \qquad \textbf{(2.21)}$$

And more generally we can write

$$\mathbf{E}(A_t) = \boldsymbol{R_t} + \lambda_t. \qquad \textbf{(2.22)}$$

2.3.11 Thus the expected return on the asset, priced into the market, is equal to the Risk-free rate, plus the market price of risk multiplied by the volatility.

2.3.12 This development of an alternative form for the Risk Premium, one that is dependent upon $\sqrt{\boldsymbol{t}}$, rather than $\boldsymbol{t}$, creates a very different perspective on the nature of market structures and the manner in which asset prices are formed.

2.3.13 If it is correct, it will enable investors to understand the actual pricing mechanism of the markets with a much greater degree of clarity.

2.3.14 It is intuitively an attractive concept, since it provides a more rational basis for the determination of asset prices. For example, if the Risk Premium discounted into asset prices for a horizon of a single year is of the order of 4%, then we can see that the required Risk Premium for 25 years would be 20%, $\textbf{i.e.}\left(\mathbf{4\%.\sqrt{25}}\right)$. This would appear to be much more rational as, on a prospective basis, it equates to a required excess return of less than 1% per annum.

2.3.15 It can also be noted that this approach creates a more even balance between the risk - return profiles of equities and bonds, (since the bond markets also embody a Risk Premium, commonly referred to as the Term Premium), thus removing the distortions that arise from assuming fixed annual levels for the equity Risk Premium.

2.3.16 However, in order to be able to utilise this theory, we need to be able to analyse existing market prices and structures in order to understand what levels of underlying volatility and market price of risk have been assumed in the formulation of prices.

2.3.17 A means of undertaking such analyses and of demonstrating the validity of the Put Principle is undertaken in the next section of this paper.

3. An Alternative Option Pricing Formula

3.1 Defining the Approach

3.1.1 The most obvious source of information on the issues of underlying volatilities and the market price of risk would appear to be the traded options market. There are, however, some significant problems to be overcome.

3.1.2 As is well-known, analysis of the price structures in the options markets reveals that the underlying volatility assumptions are not constant, and vary according to strike price and expiry date. This, therefore, does not appear to provide a very stable basis upon which to build estimates of risk premia and hence of market prices.

3.13 The contention of this paper, however, is that there exists a consistent theoretical valuation basis which spans all asset classes.

3.1.4 Thus we postulate that, if an equity risk premium exists within the pricing structure of equities, then this must also be reflected in the pricing of the related derivatives.

3.1.5 This is, of course, at odds with the standard option pricing approach, which assumes a risk-neutral framework. It is this risk-neutral assumption, it is contended that actually leads to the apparent existence of the volatility "smile".

3.1.6 As discussed in Appendix 1, Black & Scholes found that, using the more standard form for the Risk Premium, this term could be eliminated from the final form of the partial differential equation which they required to solve. They were thus able to infer that this was not a required element, thus leading to the well-established "risk-neutral" solution.

3.1.7 However, we have determined that we wish to incorporate the Risk Premium in the form of a function of $\sqrt{t}$ rather than t. This means that there is an additional term, other than the stochastic term, which is a function of this variable and this causes some additional complications in using the partial differential equation approach

3.1.8 A solution can be determined, however, applying stochastic calculus and using an expectations-based approach. This applies the same technique as is demonstrated in Appendix 1 in the development of the standard Black-Scholes equation.

3.1.9 Subsequent to the development of this formula, it is also demonstrated that the revised solution is also consistent with the original Black-Scholes solution to the partial differential equation.

3.2 The Revised Pricing Formula

3.2.1 If, as outlined in Appendix 1, we consider a $\mathbb{P}$ – Brownian motion W_t as being representative of the stochastic nature of asset returns. The change in price of an asset S_t, allowing for the existence of the risk premium P_t^* in the form now defined, with a linear drift rate of r, the risk-free rate of interest, can be represented by the equation:

$$dS_t = S_t.(r.dt + dP_t^*) + S_t.\sigma.dW_t \qquad \textbf{(3.1)}$$

where dP_t^* is the first differential of P_t^*.

then $$d(\log S_t) = r.dt + dP_t^* + \sigma.dW_t \qquad \textbf{(3.2)}$$

So $$\log S_t = r.t + P_t^* + \sigma.W_t$$

And $$S_t = \exp(r.t + P_t^* + \sigma.W_t) \qquad \textbf{(3.3)}$$

3.2.2 We now calculate the price of a European call option on S_t, with a strike price X **,** exercisable at time T with an unknown claim value at that time of Y **.**

3.2.3 As described above, we require to transform this $\mathbb{P}-$Brownian motion W_t with drift $(r.t + K.P_t)$, into an alternative $\mathbb{Q}-$Brownian motion $\hat{W}_t$ which is a pure martingale, and demonstrate that we can construct a self-financing replicating portfolio.

3.2.4 For the construction of the portfolio, we define the value of the riskless bond B_t such that:-

$$B_t = e^{r.t}$$

3.2.5 In order to establish an appropriate martingale measure under which we can calculate the expected value of the claim E_t, we next eliminate the drift term in the asset price equation. In this case, we discount both the asset price and the claim by the bond rate, to eliminate the growth due to the risk-free rate, and also by the risk premium rate to eliminate this additional source of drift. Hence we define the discounted asset function Z_t such that:-

$$Z_t = B_t^{-1}.K_t^{-1}.S_t \qquad \textbf{(3.4)}$$

Where

$$K_t = e^{P_t^*}$$

And the corresponding discounted claim as

$$E_T = B_T^{-1}.K_T^{-1}.Y \qquad \textbf{(3.5)}$$

3.2.6 As before, we now consider the function

$$L_t = \log(Z_t) \qquad \textbf{(3.6)}$$
$$= \sigma.W_t$$

So

$$dL_t = \sigma.dW_t \qquad \textbf{(3.7)}$$

And applying Ito's Lemma, we derive the stochastic differential equation for $\mathbf{Z_t}$ as

$$dZ_t = Z_t.(\sigma.dW_t + \tfrac{1}{2}\sigma^2.dt) \qquad \textbf{(3.8)}$$

3.2.7 Applying the Cameron-Martin-Girsanov theorem, we can transform the $\mathbb{P}-$Brownian motion W_t into $\mathbb{Q}-$Brownian motion $\hat{W}_t$ by introducing a drift of $\frac{1}{2}\sigma$ into the original Brownian motion so that

$$dZ_t = Z_t.\sigma.d\hat{W}_t \qquad \textbf{(3.9)}$$

And hence Z_t under $\mathbb{Q}$ is driftless and is a martingale.

3.2.8 It is also a necessary condition that the pricing is conditional only upon the history of the asset up to the present time.
3.2.9 We therefore define a filtration $\boldsymbol{F_t}$ representing the history of the asset up until time $\boldsymbol{t}$.
3.2.10 We define the conditional expectation process $\boldsymbol{E_t}$ under measure $\mathbb{Q}$ and subject to the filtration $\boldsymbol{F_t}$ such that

$$\boldsymbol{E_t = \mathbb{E}_{\mathbb{Q}}[B_T^{-1}.K_T^{-1}.Y \mid F_t]} \tag{3.10}$$

Then $\boldsymbol{E_t}$ is also a $\mathbb{Q}-$martingale.

3.2.11 Since $\boldsymbol{Z_t}$ is a $\mathbb{Q}-$martingale process with volatility greater than zero, it follows from the Martingale representation theorem (see Appendix 2) that there exists an $\boldsymbol{F_t}$-previsible process, $\boldsymbol{\varphi_t}$ such that:

$$\boldsymbol{dE_t = \varphi_t.dZ_t} \tag{3.11}$$

3.2.12 We can now seek to create a self financing replicating portfolio to ensure that an arbitrage price exists at all times.
We define a portfolio $\boldsymbol{(\varphi_t, \psi_t)}$ which consists of

$\boldsymbol{\varphi_t}$ Units of the security $\boldsymbol{S_t}$ at time $\boldsymbol{t}$, and
$\boldsymbol{\psi_t}$ Units of the bond $\boldsymbol{B_t}$

Where $\boldsymbol{\psi_t}$ is defined by the equation:-

$$\boldsymbol{K_t^{-1}.\psi_t = E_t - \varphi_t.Z_t}$$

And $\boldsymbol{\varphi_t}$, $\boldsymbol{\psi_t}$ and $\boldsymbol{K_t}$ are each previsible (i.e., can be determined at the start of each period) and are constant for each period $\boldsymbol{dt}$.

3.2.13 Then the value $\boldsymbol{V_t}$ of the portfolio $\boldsymbol{(\varphi_t, \psi_t)}$ is given by

$$\begin{aligned}\boldsymbol{V_t} &= \boldsymbol{\varphi_t.S_t + \psi_t.B_t}\\ &= \boldsymbol{\varphi_t.S_t + K_t.(E_t - \varphi_t.Z_t).B_t}\\ &= \boldsymbol{K_t.B_t.E_t + \varphi_t.S_t - K_t.B_t.\varphi_t.Z_t}\end{aligned}$$

Or

$$\boldsymbol{V_t = K_t.B_t.E_t} \tag{3.12}$$

And

$$\boldsymbol{dV_t = K_t(B_t.dE_t + E_t.dB_t)}$$

Because $\boldsymbol{K_t}$ is a constant for the tick-time $\boldsymbol{t}$ to $\boldsymbol{t+dt}$.
3.2.14 Substituting, we obtain,

$$\boldsymbol{dV_t = K_t(B_t.\varphi_t.dZ_t + (K_t^{-1}.\psi_t + \varphi_t.Z_t)dB_t)}$$

$$= K_t.\varphi_t.B_t.dZ_t + K_t.\varphi_t.Z_t.dB_t + \psi_t.dB_t$$
$$= \varphi_t(K_t.B_t.dZ_t + K_t.Z_t.dB_t) + \psi_t.dB_t$$
$$= \varphi_t.d(K_t.B_t.Z_t) + \psi_t.dB_t$$
$$= \varphi_t.dS_t + \psi_t.dB_t \qquad \textbf{(3.13)}$$

3.2.15 Thus the change in value of the portfolio is due only to changes in the value of the assets. In addition, we note that at time T

$$V_T = K_T.B_T.E_T$$
$$= Y$$

3.2.16 Hence we have a replicating self-financing portfolio which ensures that there is an arbitrage price at all times.

3.2.17 We are therefore able to calculate the price of the option using an expectations approach but under the measure $\mathbb{Q}$.

The price of the option is given by V_0 , the value of the portfolio at time $\mathbf{0}$.

3.2.18 As we have seen above this is given by:-

$$K_0.B_0.E_0 = K_0.B_0.\mathbb{E}_{\mathbb{Q}}[B_T^{-1}.K_T^{-1}.Y \mid F_0]$$
$$= B_T^{-1}.K_T^{-1}.\mathbb{E}_{\mathbb{Q}}[Y \mid F_0]$$

Since $\mathbf{K_0}$ and $\mathbf{B_0}$ are both defined to be equal to unity at time $\mathbf{0}$, we therefore have

$$V_0 = \exp(-rT - P_T^*).\mathbb{E}_{\mathbb{Q}}[Y \mid F_0] \qquad \textbf{(3.14)}$$

3.2.19 For a European-style call option with strike price of X , the value of the claim at expiry is only dependent upon the stock price at that time. The value of the claim will therefore be $(S_t - X)$ if this is positive, and zero otherwise. i.e. $\max(S_t - X, 0)$

3.2.20 Hence we merely need to know the marginal distribution of the stock price under measure $\mathbb{Q}$ to be able to determine the expectation value of the claim.

3.2.21 If we rewrite the process equation for S_t in terms of the $\mathbb{Q}-$Brownian motion $\hat{W}_t$, recalling that we require to eliminate the drift term of $\frac{1}{2}\sigma^2.dt$

Equation **(3.2)** therefore becomes

$$d(\log S_t) = dP_t^* + \sigma.d\hat{W}_t + r.dt - \tfrac{1}{2}\sigma^2.dt$$

(3.15)

3.2.22 If we denote the stock price at time zero as S , then we have

$$\log S_t = \log S + P_t^* + \sigma.\hat{W}_t + (r - \tfrac{1}{2}\sigma^2).t$$

So $$S_t = S.\exp(P_t^* + \sigma.\hat{W}_t + (r - \tfrac{1}{2}\sigma^2).t$$

Or

$$S_t = [S.\exp(P_t^*)].[\exp(\sigma.\hat{W}_t + (r - \tfrac{1}{2}\sigma^2).t)] \qquad \textbf{(3.16)}$$

3.2.23 For ease of manipulation, we can alternatively write:-

$$S_t = [S.\exp(P_t^* + rt)].[\exp(\sigma.\hat{W}_t - \tfrac{1}{2}\sigma^2.t)] \qquad \textbf{(3.17)}$$

3.2.24 And, for any random variable Y which is a normal $N(\mu,\sigma^2)$

$$\mathbb{E}_{\mathbb{Q}}[\exp(\theta Y)] = \exp(\theta\mu + \tfrac{1}{2}\theta^2\sigma^2)$$

For all real θ.

3.2.25 If, as in Appendix 1, we therefore consider the variable

$$y = (\sigma.\hat{W}_t - \tfrac{1}{2}\sigma^2.t)$$

Then, $$\mathbb{E}_{\mathbb{Q}}[\exp(Y)] = \exp(-\tfrac{1}{2}\sigma^2 t + \sigma.\hat{W}_t)$$

$$= \exp(-½\sigma^2 t + ½\sigma^4 t)$$

3.2.26 Since this is the moment-generating function of a Normal $N(-½\sigma^2 T, \sigma^2 T)$, we can see that the marginal distribution of S_T under the measure $\mathbb{Q}$ is given by $S.\exp(P_T^* + rT)$ multiplied by the exponential of a Normal with mean $(-½\sigma^2 T)$ and variance $\sigma^2 T$.

3.2.27 If we rewrite Equation **(3.17)** in the form

$$S_t = [S.\exp(P_t^* + rt)].[\exp(y)]$$

So $$y = \ln S_t - \ln S - (P_t^* + rt)$$

Thus we can write the value V_0 as :-

$$V_0 = \frac{e^{-\mu t}}{\sqrt{2\pi\sigma^2 t}}\left[\int_a^\infty (S.e^{\mu t}.e^y - X).e^{-\frac{(y+½\sigma^2 t)^2}{2\sigma^2 t}}.dy\right] \qquad \textbf{(3.18)}$$

Where $a = \ln X - \ln S - (P_t^* + rT)$

3.2.28 From which, by mathematical manipulation (see Appendix 1), we can derive the following equation for the value of the option

$$V_0 = S.N(d_1) - e^{-\mu t}.X.N(d_2) \qquad \textbf{(3.19)}$$

Where $$d_1 = \frac{\ln(S/X) + (\mu + ½\sigma^2)t}{\sigma\sqrt{t}}$$

(3.20)

$$d_2 = \frac{\ln(S/X) + (\mu - ½\sigma^2)t}{\sigma\sqrt{t}} \qquad \textbf{(3.21)}$$

And $$\mu T = P_t^* + rt$$

3.3 Formula Consistency

3.3.1 Before proceeding to the actual testing of the revised pricing formula, it is clearly important that we review the consistency of the Put-principle Option Pricing formula ('POP') with reference to the theoretical development work pioneered by Black-Scholes.

3.3.2 We note, as an initial point that when $\mathbf{K}' = \mathbf{0}$, the POP equation simplifies to the standard Black Scholes valuation model, indicating that the Black-Scholes model possibly represents a special case (the Risk-Neutral solution) of a more general formula derived above.

3.3.3 However, we can also review the POP equation to test its consistency with the partial differential equation approach that lay behind the Black-Scholes development of their pricing formula.

3.3.4 We can rearrange Equation **(3.19)** into the form

$$V^* = S^*.N(d_1) - e^{-rt}.X.N(d_2) \qquad \textbf{(3.22)}$$

Where

$$V^* = V.\exp(P_t^*)$$
$$S^* = S.\exp(P_t^*)$$

And

$$d_1 = \frac{\ln(S^*/X) + (r + ½\sigma^2)t}{\sigma\sqrt{t}}$$
$$d_2 = \frac{\ln(S^*/X) + (r - ½\sigma^2)t}{\sigma\sqrt{t}}$$

3.3.5 We can see clearly that Equation **(3.22)** is merely the Black-Scholes formula applied to the asset S^* rather than the asset S.

3.3.6 We also note that, at any time that we know the price of asset S, we also know the price of asset S^*, since $\exp(P_t^*)$ is merely a function of the volatility; the time to expiry; and the market price of risk. All of these are known (or assumed to be constants in the case of volatility and the market price of risk).

3.3.7 We can therefore deduce that Equation **(3.22)** is consistent with the Black-Scholes partial differential equation

3.3.8 It therefore follows, that the POP equation **(3.19)** is also consistent and that, since we can construct a hedge portfolio from V^* and S^*, then we can also construct a hedge portfolio for V and S.

3.3.9 We can demonstrate that this is the case, by first considering the hedge portfolio Π^*, comprising a unit of the option V^* and an amount $-\Delta$ of the underlying asset S^*, so that

$$\Pi^* = V^* - \Delta.S^* \qquad \textbf{(3.23)}$$

The jump in value in one time-tick is then given by

$$d\Pi^* = dV^* - \Delta.dS^* \qquad \textbf{(3.24)}$$

3.3.10 If we define a new portfolio Π, such that $\Pi^* = \Pi\,\mathbf{exp}(\boldsymbol{P_t^*})$, then we can write Equation **(3.23)** as

$$\Pi\boldsymbol{.}\mathbf{exp}(\boldsymbol{P_t^*}) = \boldsymbol{V}\boldsymbol{.}\mathbf{exp}(\boldsymbol{P_t^*}) - \Delta\boldsymbol{.S}\boldsymbol{.}\mathbf{exp}(\boldsymbol{P_t^*}) \qquad \textbf{(3.25)}$$

Giving

$$\Pi = \boldsymbol{V} - \Delta\boldsymbol{.S} \qquad \textbf{(3.26)}$$

Since we can divide both sides by $\mathbf{exp}(\boldsymbol{P_t^*})$

3.3.11 We can also rewrite Equation **(3.24)** as

$$\boldsymbol{d}(\Pi\boldsymbol{.}\mathbf{exp}(\boldsymbol{P_t^*})) = \boldsymbol{d}(\boldsymbol{V}\boldsymbol{.}\mathbf{exp}(\boldsymbol{P_t^*})) - \Delta\boldsymbol{.d}(\boldsymbol{S}\boldsymbol{.}\mathbf{exp}(\boldsymbol{P_t^*})) \qquad \textbf{(3.27)}$$

But since $\mathbf{exp}(\boldsymbol{P_t^*})$ is unaffected by the stochastic differentiation process, since this uses values at the start of each time period, and $\mathbf{exp}(\boldsymbol{P_t^*})$ is a previsible constant in this respect, we can again eliminate this term from the equation, giving

$$\boldsymbol{d}\Pi = \boldsymbol{dV} - \Delta\boldsymbol{.dS} \qquad \textbf{(3.28)}$$

3.3.12 Thus, as stated above, we can see that, since we can construct a hedged portfolio for $\boldsymbol{V^*}$ and $\boldsymbol{S^*}$ then we can also construct a hedge portfolio for $\boldsymbol{V}$ and $\boldsymbol{S}$.

3.3.13 We therefore have an alternative option pricing formula incorporating a market-based Risk Premium, that we can use to conduct detailed analyses of actual market prices and can hence test the validity of the POP formula.

4. TESTING THE OPTION PRICING FORMULA

4.1 Implied Volatilities

4.1.1 The options market is clearly a logical source of data on forecast volatilities although, historically, the levels of implied volatilities estimated using the Black-Scholes-Merton formulae have been quite diverse in terms of levels and have varied significantly across differing expiry dates and across different strike prices for the same option expiry date.

4.1.2 These implied volatilities appear as a broadly consistent curve for each expiry date, known as the 'volatility smile' or 'smirk'.

4.1.3 From both a practical and theoretical basis, it is difficult to rationalize the existence of the volatility smile, since it seems extremely unlikely that the market could operate efficiently with such a diverse range of estimated volatilities.

4.1.4 Hence it seems more likely that the 'smile' is, at least in major part, a result of the risk-neutral assumptions embedded in the standard option pricing models, rather than a fundamental part of the market valuation process.

4.1.5 If this is the case, then we will expect to observe a more stable pattern of implied volatilities when we fit live option data to the POP model.

4.2 Methodology

4.2.1 The newly derived POP model was tested with samples of actual market data with the following objectives:

1. To assess the validity of the formula in providing a better fit to market-determined option prices
2. To determine whether option prices could be modelled using a single estimate of volatility across all strike prices for a given option expiry date
3. To obtain market-driven estimates of investor risk-aversion parameters for different expiry terms and at different times in the market cycle.

4.2.2 Option pricing data for the S&P 500 was utilized for the empirical testing as the breadth and liquidity of the US markets provides a more consistent source of data on a regular basis and this is therefore the data utilized in this paper.

4.2.3 Even within the relatively liquid US market, however, it was found to be important to ensure that the testing was conducted on strikes and expiries which had actual trades occurring on the dates of the testing.

4.2.4 Accordingly, where possible, the POP model was fitted to data series for which there was associated data for actual traded volumes for that day. For certain of the price series where such data was not available, the POP model was fitted to a set of call options which extended from the 'at-the-money' strike together with 5 or six adjacent 'out of the money' strikes.

4.2.5 The data series that have been tested cover the time period from 11th January 2000 to 30th May 2003, although the majority of the data tested falls within the period 10th January 2002 to 30th May 2003.

4.2.6 The data series used are the end of day settlement prices provided by the Chicago Board Options Exchange (CBOE) for the S&P500 contract

4.2.7 Bid and Ask prices for each strike price for which data was available were averaged to provide a mid-price for the contracts.

4.2.8 A number of alternative methodologies are available for estimating the time to expiry for each contract. The methodology adopted in this paper is to compute ‘actual workdays’ as a fraction of a 260 day year, as this appeared to provide a slighter better fit to market prices.

4.2.9 For each contract series, the Put-Call parity relationship was tested to ensure that the data series appeared consistent as to the allocation of the relevant Put and Call option prices relative the market Strike prices.

4.2.10 As dividends accrue to the index, but not to the option-holder, the Put-Call parity relationship was also used to estimate the present value of dividends discounted by the market. This value was checked against the anticipated dividend flow rate for the period to ensure consistency.

4.2.11 Given that Put-Call parity was generally found to hold in the US market, it was only necessary to validate the Call pricing equation, using the variables established above.

4.2.12 For each expiry date on the selected trading days, the POP model was fitted to the actual market settlement prices, using the Solver routine in Excel. The fitting process was a simple minimisation of least squares based on the differences between the actual option pricing curve and the POP model solution to provide the best fit.

4.2.13 The data points used for this purpose were selected as described above.

4.2.14 In general, it was found that the bulk of the trading volume was carried in ‘at-the-money’ or ‘out-of-the-money’ strikes within a relatively narrow range.

4.2.15 In addition, as a further check on the ‘quality’ of the fit between the curves, the POP model was fitted at each strike price, keeping the derived value of the Market Price of Risk constant, in order to generate a set of ‘implied volatilities’ for comparison with those generated from the Black-Scholes-Merton model.

4.3 Empirical Results

4.3.1 For each expiry date it was found that it was possible to obtain a good fit between the actual option market price curve and the POP model, using a single estimate of volatility and a single estimate of the Market Price of Risk (‘MPR’) parameter.

4.3.2 Of equal significance was the fact that the volatility estimates were markedly similar across a range of expiry dates for each of the trading dates tested.

4.3.3 The comparison tests of implied volatilities also appeared to demonstrate a high level of consistency across the major part of the option pricing curve, although it was noted that the implied volatilities for options which were deep ‘in-the-money’ were significantly lower than those in which active trading was taking place.

4.3.4 The implication of this latter finding, on the assumption that the POP model is valid, is that deep ‘in-the-money’ options are being mis-priced by the standard curve-fitting models currently being utilised.

4.3.5 It was also observed that the MPR factor tended to move by relatively small amounts over time, with the more significant changes occurring in the level of underlying volatility being assumed by the market.

4.3.6 There was clearly a marked increase in both the level of volatility and the level of Market Price of Risk, during periods of stock market weakness. These factors, of course combine to form the Risk Premium. This also therefore, as anticipated, was observed to rise during periods of market decline.

4.4 Test Results

4.4.1 Detailed data and graphical representations of samples of the results are contained within Appendix 3 to this paper and a summary of the results is shown below.

4.4.2 Sample data for single expiry months for each of the following trading dates are provided.

11th January 2000
17th January 2001
17thJanuary 2003
10th March 2003
27th May 2003

4.4.3 The tables below show the summary data for all of the tested expiry dates for each of the above trading dates.

S&P 500		Close	1438	11-Jan-2000
Expiry Date	16-Mar-2000	15-Jun-2000	14-Sep-2000	14-Dec-2000
Days to expiry	47	112	177	242
Volatility	15.19%	15.38%	15.08%	16.66%
Risk Premium	4.12%	4.31%	4.68%	3.96%
Risk Aversion	0.68	0.70	0.78	0.60

S&P 500			Close	1326.7	17-Jan-2001
Expiry Date	17-Feb-2001	17-Mar-2001	16-Jun-2001	22-Sep-2001	22-Dec-2001
Days to expiry	22	42	107	177	242
Volatility	19.00%	18.10%	16.73%	17.08%	17.18%
Risk Premium	3.02%	4.98%	4.14%	3.94%	4.17%
Risk Aversion	0.40	0.69	0.62	0.58	0.61

S&P 500		Close	900.8	17-Jan-2003
Expiry Date	21-Mar-2003	20-Jun-2003	19-Sep-2003	19-Dec-2003
Days to Expiry	45	110	175	240
Volatility	20.53%	19.16%	18.85%	19.26%
Risk Premium	3.61%	5.65%	6.13%	6.13%
Risk aversion	0.44	0.74	0.81	0.80

S&P 500		Close	807.48	10-Mar-2003
Expiry Date	21-Mar-2003	20-Jun-2003	19-Sep-2003	19-Dec-2003
Days to expiry	9	74	139	204
Volatility	27.86%	20.64%	19.48%	19.08%
Risk Premium	2.47%	6.20%	5.75%	5.65%
Risk Aversion	0.22	0.75	0.74	0.74

S&P 500			Close	951.48	27-May-2003
Expiry Date	20-Jun-2003	18-Jul-2003	19-Sep-2003	19-Dec-2003	19-Mar-2004
Days to expiry	18	38	83	148	213
Volatility	15.94%	15.47%	15.21%	14.27%	14.51%
Risk Premium	1.39%	2.31%	2.92%	3.90%	3.95%
Risk Aversion	0.22	0.37	0.48	0.68	0.68

5. CONCLUSIONS

5.1 Summary

5.1.1 The primary purpose underlying this research was to improve upon the techniques available for understanding market valuations of assets.

5.1.2 This purpose would appear to have been achieved, as we have developed a means of extracting additional information from the options market on the underlying Risk Premium that is being discounted, together with its component parts of volatility and Market price of Risk.

5.1.3 That, as a result of this research, we have also developed the POP valuation model for pricing options is clearly an added benefit.

5.1.4 The fact that the model effectively removes the 'volatility smile' would also appear to support its use as a more natural tool for the purpose of pricing both traded and 'over-the-counter' options.

5.1.5 The definition of the underlying Risk Premium incorporated into current market prices is clearly an important one, since it allows us to break away from estimations based solely on historic data.

5.1.6 The fact that the Risk Premium is defined in terms of a theoretical option, rather than a simple additional annual excess return is also a major step in the process of understanding the dynamics of the markets.

5.2 Other Implications

5.2.1 The empirical results from testing the POP model pricing formula in the US options market appear to provide strong support for the use of this model as a standard option pricing tool.

5.2.2 The model has also been tested on limited data for the UK, German and Japanese markets. These tests have generated similar results to those observed in the US market in terms of the improved fit compared to the Black-Scholes-Merton model.

5.2.3 The demonstration of the effectiveness of the POP model in pricing options, and the associated appreciation that there is a single underlying estimate of volatility used by the market, does provide us with the necessary information to move forward in attempting to build more effective, forward-looking models of the equity market.

5.2.4 It should also, however, be noted that the theory developed in this paper is applicable to individual assets and to groups of assets.

5.2.5 This does provide a significant challenge to large areas of existing investment theory, since these have been based on different assumptions

5.2.6 This issue and the application of the theory to the currency and bond markets, and to the development of equity pricing models, will be addressed in a companion paper to be published later in the year.

THE PUT PRINCIPLE - APPENDIX I

THE BLACK-SCHOLES OPTION PRICING EQUATION

A Stochastic Calculus Approach to the Black Scholes Equation

In this section, we develop the standard risk-neutral option pricing model using a stochastic calculus approach. This is included for the sake of completeness and also because we utilise the same methodology when we incorporate the Risk Premium into the process. It is also recognised that this is an unfamiliar area for many people.

The classical theory, as developed in the Black-Scholes models, is developed from a hedged portfolio approach from which a partial differential equation is developed. Interestingly, this equation contains no explicit terms reflecting effect of the risk premium, even though a risk premium, or additional drift term, was assumed at the outset.

As a result, Messrs Black and Scholes were able to solve the partial differential equation by effectively assuming a risk-neutral environment. i.e., that the risk premium element did not influence the actual pricing of options.

A key element of the Black-Scholes approach lay in establishing that the price of a derivative was determined by its arbitrage price. This price is equal to the value of a notional portfolio consisting of appropriate proportions of the underlying asset and a riskless bond. The portfolio is constructed so that its value replicates the pay-off characteristics of the derivative at each time point. Additionally the necessary rebalancing of the assets requires no additional cash inflows or outflows to be made. Such a portfolio is described as a self-financing replicating portfolio**.**

It has also been shown that, where the price equation of an asset can be represented by a pure martingale, (i.e. a driftless martingale), then the arbitrage price and the expectation value will be equal. (See Baxter & Rennie. "Financial Calculus")

This approach potentially provides a tractable means of deriving the required derivative equation, subject to our being able to meet the required stringent tests.

We first consider a $\mathbb{P}-$Brownian motion $\boldsymbol{W_t}$ as being representative of the stochastic nature of asset returns and assume that a Risk-Neutral environment pertains. Then the change in price of an asset $\boldsymbol{S_t}$, can be represented by the equation

$$\boldsymbol{dS_t = S_t.r.dt + S_t.\sigma.dW_t} \qquad \textbf{(6.1)}$$

Or

$$\boldsymbol{d(\log S_t) = r.dt + \sigma.dW_t} \qquad \textbf{(6.2)}$$

Where
$\boldsymbol{r}$ = drift rate = Risk-free rate
$\boldsymbol{t}$ = Time to expiry
$\boldsymbol{S_t}$ = Asset Price at time $\boldsymbol{t}$
$\boldsymbol{\sigma}$ = Standard measure of risk of asset
$\mathbb{P}$ = Measure for the Brownian motion

Integrating, we have $\boldsymbol{\log S_t = r.t + \sigma.W_t}$
Assuming $\boldsymbol{S_0 = 1}$

And hence $$S_t = \exp(r.t + \sigma.W_t) \qquad \textbf{(6.3)}$$

We now wish to calculate the price of a European call option on S_t, with a strike price X , exercisable at time T with an unknown claim value at that time of Y .

As described above, if we are to be able to use an expectations approach to compute the present value of the option, we need to be able to transform this $\mathbb{P}-$Brownian motion W_t with drift $r.t$ into an alternative $\mathbb{Q}-$Brownian motion $\hat{W}_t$ which is a pure martingale.

If we can then construct a self-financing replicating portfolio with value V_t equal to the expected present value of the claim E_t, within that $\mathbb{Q}-$Brownian motion framework, then we can calculate the price of the options using the expected value of the claim.

For the construction of the portfolio, we also need to define the value of the riskless bond B_t such that:-

$$B_t = e^{r.t}$$

In order to establish an appropriate martingale measure under which we can calculate the expected value of the claim E_t, we need to be able to eliminate the drift term in the asset price equation.

An obvious way to achieve this is to discount both the asset price and the claim by the bond rate, to eliminate the growth due to the risk-free drift rate.

Hence we define the discounted asset function Z_t such that:-

$$Z_t = B_t^{-1}.S_t \qquad \textbf{(6.4)}$$

And the corresponding discounted claim as

$$E_T = B_T^{-1}.Y \qquad \textbf{(6.5)}$$

If we now consider the function

$$L_t = \log(Z_t) \qquad \textbf{(6.6)}$$

$$= \sigma.W_t$$

So

$$dL_t = \sigma.dW_t \qquad \textbf{(6.7)}$$

$$dL_t^2 = \sigma^2.dt \qquad \textbf{(6.8)}$$

Applying Ito's Lemma to $\boldsymbol{Z_t = \exp(L_t)}$, we can derive the stochastic differential equation for $\mathbf{Z_t}$ as

$$\boldsymbol{dZ_t = Z_t.(\sigma.dW_t + \tfrac{1}{2}\sigma^2.dt)} \qquad \textbf{(6.9)}$$

Applying the Cameron-Martin-Girsanov theorem for changes of measure, we can transform the $\mathbb{P}-$Brownian motion $\boldsymbol{W_t}$ into $\mathbb{Q}-$Brownian motion $\boldsymbol{\hat{W}_t}$ by introducing a drift of $\boldsymbol{\frac{1}{2}\sigma}$ into the original Brownian motion so that

$$\boldsymbol{dZ_t = Z_t.\sigma.d\hat{W}_t} \qquad \textbf{(6.10)}$$

And hence $\boldsymbol{Z_t}$ under $\mathbb{Q}$ is driftless and is a martingale.

It is also a necessary condition that the pricing is conditional only upon the history of the asset up to the present time. We therefore define a filtration $\boldsymbol{F_t}$ representing the history of the asset up until time $\boldsymbol{t}$.

We can then define the conditional expectation process $\boldsymbol{E_t}$ under measure $\mathbb{Q}$ and subject to the filtration $\boldsymbol{F_t}$ such that

$$\boldsymbol{E_t = \mathbb{E}_{\mathbb{Q}}[B_T^{-1}.Y \mid F_t]} \qquad \textbf{(6.11)}$$

Then $\boldsymbol{E_t}$ is also a $\mathbb{Q}-$martingale.

Since $\boldsymbol{Z_t}$ is a $\mathbb{Q}-$martingale process with volatility greater than zero, it follows from the Martingale representation theorem (see Appendix) that there exists an $\mathbf{F_t}$-pre-visible process, $\boldsymbol{\varphi_t}$ such that:

$$\boldsymbol{dE_t = \varphi_t.dZ_t} \qquad \textbf{(6.12)}$$

We can now seek to create a self financing replicating portfolio to ensure that an arbitrage price exists at all times.

If we define a portfolio $\boldsymbol{(\varphi_t, \psi_t)}$ which consists of

$\boldsymbol{\varphi_t}$ Units of the security $\boldsymbol{S_t}$ at time $\boldsymbol{t}$, and

$\boldsymbol{\psi_t}$ Units of the bond $\boldsymbol{B_t}$

Where $\boldsymbol{\psi_t}$ is defined by the equation:-

$$\boldsymbol{\psi_t = E_t - \varphi_t.Z_t}$$

And $\boldsymbol{\varphi_t}, \boldsymbol{\psi_t}$ are each previsible (i.e., can be determined at the start of each period) and are constant for each period $\boldsymbol{dt}$.

Then the value $\boldsymbol{V_t}$ of the portfolio $\boldsymbol{(\varphi_t, \psi_t)}$ is given by

$$\begin{aligned} V_t &= \varphi_t.S_t + \psi_t.B_t \\ &= \varphi_t.S_t + (E_t - \varphi_t.Z_t).B_t \\ &= B_t.E_t + \varphi_t.S_t - B_t.\varphi_t.Z_t \end{aligned}$$

Or

$$V_t = B_t.E_t \tag{6.13}$$

And

$$dV_t = B_t.dE_t + E_t.dB_t$$

Substituting, we obtain,

$$\begin{aligned} dV_t &= (B_t.\varphi_t.dZ_t + (\psi_t + \varphi_t.Z_t)dB_t) \\ &= \varphi_t.B_t.dZ_t + \varphi_t.Z_t.dB_t + \psi_t.dB_t \\ &= \varphi_t(B_t.dZ_t + Z_t.dB_t) + \psi_t.dB_t \\ &= \varphi_t.d(B_t.Z_t) + \psi_t.dB_t \\ &= \varphi_t.dS_t + \psi_t.dB_t \end{aligned} \tag{6.14}$$

Thus the change in value of the portfolio is due only to changes in the value of the assets.
In addition, we note that at time T

$$\begin{aligned} V_T &= B_T.E_T \\ &= Y \end{aligned}$$

Hence we have a replicating self-financing portfolio which ensures that there is an arbitrage price at all times.
We are therefore able to calculate the price of the option using an expectations approach but under the measure $\mathbb{Q}$.

The price of the option is given by V_0 , the value of the portfolio at time 0.
As we have seen above this is given by:-

$$\begin{aligned} B_0.E_0 &= B_0.\mathbb{E}_{\mathbb{Q}}[B_T^{-1}.Y \mid F_0] \\ &= B_T^{-1}.\mathbb{E}_{\mathbb{Q}}[Y \mid F_0] \end{aligned}$$

Since $\mathbf{B_0}$ is defined to be equal to unity at time 0, we therefore have

$$V_0 = \exp(-rT).\mathbb{E}_{\mathbb{Q}}[Y \mid F_0] \tag{6.15}$$

Given that we are seeking to price a European-style call option with strike price of X, we can see that the value of the claim at expiry is only dependent upon the stock price at that time. The value of the claim will therefore be $(S_t - X)$ if this is positive, and zero otherwise. i.e. $\max(S_t - X, 0)$

Hence we merely need to know the marginal distribution of the stock price under measure $\mathbb{Q}$ to be able to determine the expectation value of the claim.

If we rewrite the process equation for S_t in terms of the $\mathbb{Q}-$Brownian motion $\hat{W}_t$, recalling that we require to eliminate the drift term of $\frac{1}{2}\sigma^2.dt$
Equation **(6.2)** therefore becomes

$$d(\log S_t) = \sigma.d\hat{W}_t + r.dt - \tfrac{1}{2}\sigma^2.dt \qquad \textbf{(6.16)}$$

If we denote the stock price at time zero as S, then we have

$$\log S_t = \log S + \sigma.\hat{W}_t + (r - \tfrac{1}{2}\sigma^2).t$$

So $\qquad S_t = S.\exp(\sigma.\hat{W}_t + (r - \tfrac{1}{2}\sigma^2).t$

Or $\qquad S_t = S.[\exp(\sigma.\hat{W}_t + (r - \tfrac{1}{2}\sigma^2).t)] \qquad$ **(6.17)**

And for ease of manipulation, we can alternatively write:-

$$S_t = [S.\exp(rt)].[\exp(\sigma.\hat{W}_t - \tfrac{1}{2}\sigma^2.t)] \qquad \textbf{(6.18)}$$

If we now consider the moment-generating functions of normal variables, we know that, for any random variable Y which is a normal, $N(\mu,\sigma^2)$,

$$\mathbb{E}_{\mathbb{Q}}[\exp(\theta Y)] = \exp(\theta\mu + ½\theta^2\sigma^2) \qquad \text{For all real } \theta.$$

If we thus consider the variable

$$y = (\sigma.\hat{W}_t - ½\sigma^2.t)$$

We see that

$$\mathbb{E}_{\mathbb{Q}}[\exp(y)] = \mathbb{E}_{\mathbb{Q}}[\exp(-½\sigma^2 t + \sigma.\hat{W}_t)]$$

Or $\qquad \mathbb{E}_{\mathbb{Q}}[\exp(y)] = \exp(-½\sigma^2 t).\mathbb{E}_{\mathbb{Q}}[\exp(\sigma.\hat{W}_t)]$

$$= \exp(-½\sigma^2 t).\exp(\sigma.0 + ½\sigma^2.\sigma^2 t)]$$

$$= \exp(-½\sigma^2 t + ½\sigma^4 t) \qquad \textbf{(6.19)}$$

Since $\hat{W}_T$ Is a normal $N(0,\sigma^2 T)$ with respect to $\mathbb{Q}$.

Equation **(6.19)** is the moment-generating function of a Normal $N(-½\sigma^2 T, \sigma^2 T)$, so the marginal distribution of S_T under the measure $\mathbb{Q}$ is given by $S.\exp(rT)$ multiplied by the exponential of a Normal with mean **(-½σ²T)** and variance **σ²T.**

If we rewrite Equation **(6.18)** in the form

$$S_t = [S.\exp(rt)].[\exp(y)]$$

Then

$$y = \ln S_t - \ln S - rt \tag{6.20}$$

And we can write the value V_0 as :-

$$V_0 = \frac{e^{-rt}}{\sqrt{2\pi\sigma^2 t}}\left[\int_a^{\infty}(S.e^{rt}.e^y - X).e^{-\frac{(y+½\sigma^2 t)^2}{2\sigma^2 t}}.dy\right] \tag{6.21}$$

Where $a = \ln X - \ln S - rT$

Then $V_0 = \frac{1}{\sqrt{2\pi\sigma^2 t}}\left[\int_a^{\infty}(S.e^y - X.e^{-rt}).e^{-\frac{(y+½\sigma^2 t)^2}{2\sigma^2 t}.dy}\right]$

$$= \frac{1}{\sqrt{2\pi\sigma^2 t}}\left[S.\int_a^{\infty}e^y.e^{-\frac{(y+½\sigma^2 t)^2}{2\sigma^2 t}}.dy - X.e^{-rt}\int_a^{\infty}e^{-\frac{(y+½\sigma^2 t)^2}{2\sigma^2 t}}.dy\right]$$

But $y - \frac{(y+½\sigma^2 t)^2}{2\sigma^2 t} = \frac{2\sigma^2 t.y - (y^2 + \sigma^2 t.y + ¼\sigma^4 t^2)}{2\sigma^2 t}$

$$= -\frac{(y-½\sigma^2 t)^2}{2\sigma^2 t}$$

Hence the value V_0 of the call option can be written as:

$$V_0 = S.N(d_1) - e^{-rt}.X.N(d_2) \tag{6.22}$$

Where $d_1 = \frac{\ln(S/X) + (r+½\sigma^2)t}{\sigma\sqrt{t}}$ **(6.23)**

And $d_2 = \frac{\ln(S/X) + (r-½\sigma^2)t}{\sigma\sqrt{t}}$ **(6.24)**

Which is the standard Black-Scholes call option pricing formula.

We can similarly derive the Black-Scholes-Merton model, which allows for the payment of dividends.

As these do not accrue to the benefit of the derivative owner, but only to the holder of the underlying security, we can see that, if we assume a continuous dividend stream of qt, then the claim value will be given by

$$\max(e^{-qt}.S_t - X, 0)$$

As the strike price X will not be affected by the removal of the stream of dividend payments, whereas the return on the security up to the time of expiry will be impacted.

We also require to adjust the drift rate to allow for the payment of dividends, and this therefore becomes $(rt - qt)$.

Substituting these into Equation **(6.22)**, we derive the B-S-M equation

$$V_0 = e^{-qt} S.N(d_1) - e^{-rt}.X.N(d_2) \qquad \textbf{(6.25)}$$

Where

$$d_1 = \frac{\ln(S/X) + (r - q + ½\sigma^2)t}{\sigma\sqrt{t}} \qquad \textbf{(6.26)}$$

And

$$d_2 = \frac{\ln(S/X) + (r - ½\sigma^2)t}{\sigma\sqrt{t}} \qquad \textbf{(6.27)}$$

THE PUT PRINCIPLE - APPENDIX 2

CAMERON-MARTIN GIRSANOV THEOREM

MARTINGALE REPRESENTATION THEOREM

Cameron-Martin-Girsanov Theorem

The Cameron-Martin-Girsanov theorem provides a means with which to interpret a change from a measure $\mathbb{P}$ to an equivalent measure $\mathbb{Q}$ as having the effect of changing the drift of the underlying Brownian motion.

Equivalent measures are defined as being measures (probability-sets of all possible outcomes) under which the set of events having zero probabilities are identical

The theorem is stated as follows:

Let W_t be a $\mathbb{P}-$Brownian motion and let ν_t be a previsible process under the filtration F_t, so that the value of ν_t at any time t is dependent only upon the history of the process up until time $t-1$. We also define the process ν_t to be bounded such that the expectation $\mathbb{E}_{\mathbb{P}}$ under measure $\mathbb{P}$ satisfies the inequality:

$$\mathbb{E}_{\mathbb{P}} \exp\left(\frac{1}{2}\int_0^T \nu_t^2 .dt\right) < \infty$$

Then we can define a measure $\mathbb{Q}$ equivalent to $\mathbb{P}$, defined by the relationship

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = \exp\left(-\int_0^T \gamma_t dW_t - \frac{1}{2}\int_0^T \nu_t^2 dt\right)$$

Under which there exists a Brownian motion $\hat{W}_t$, such that

$\hat{W}_t = W_t + \int_0^t \nu_x dx$ is a $\mathbb{Q}$-Brownian motion.

Thus $\hat{W}_t$ has a drift of $-\nu_t$ relative to W_t.

Martingale Representation Theorem

If X_t is a martingale with volatility σ_t, under some measure $\mathbb{P}$, then any other $\mathbb{P}$-martingale Y_t can be represented in terms of X_t by means of an F_t-previsible process ν_t, defined such that:

$$\int_0^T \nu_t^2 \sigma_t^2 .dt < \infty$$

Subject only to the requirement that $\sigma_t > 0$ at all times.

Then Y_t can be represented by the equation

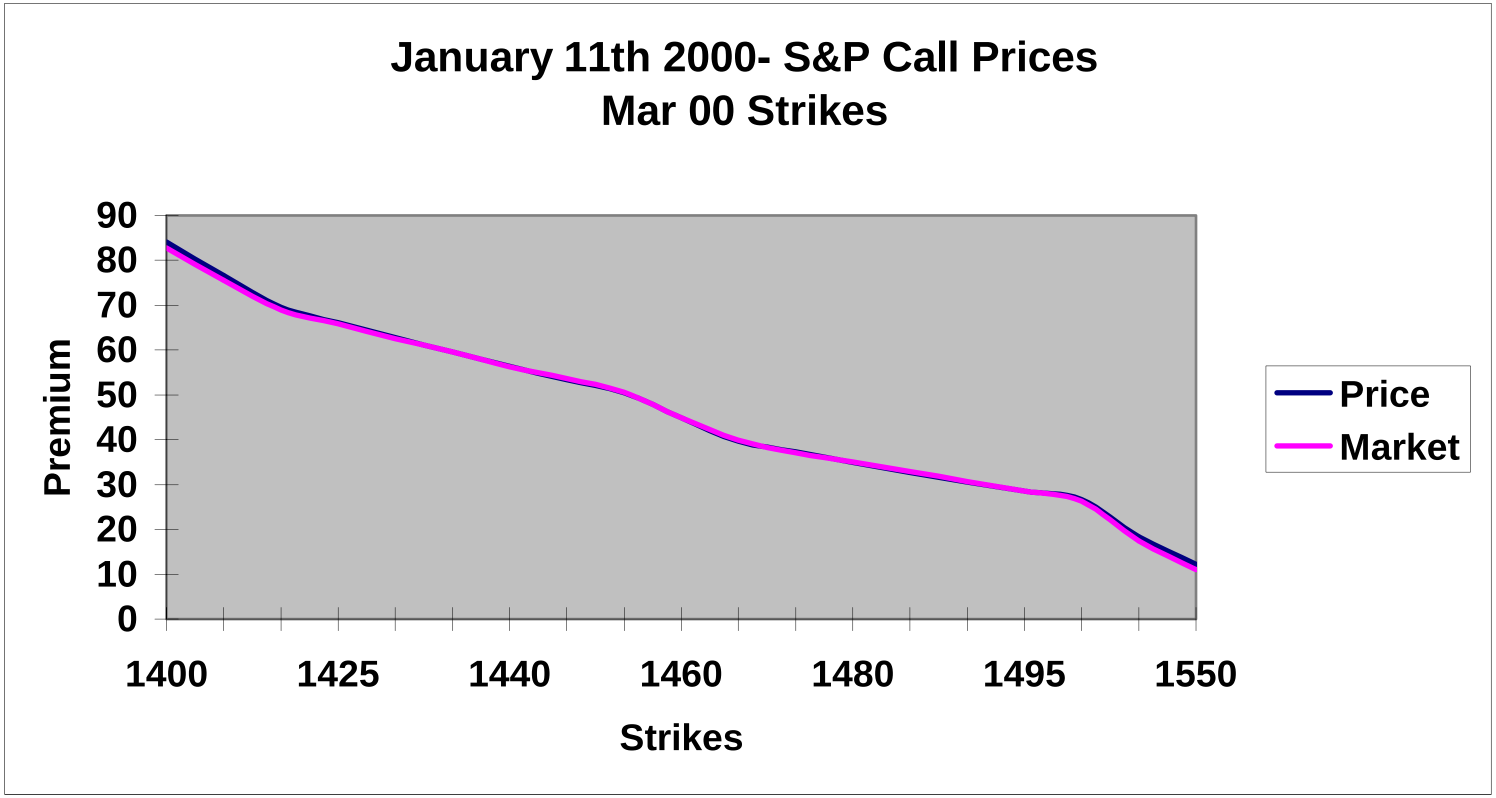
$$Y_t = Y_0 + \int_0^t \nu_s dX_s$$

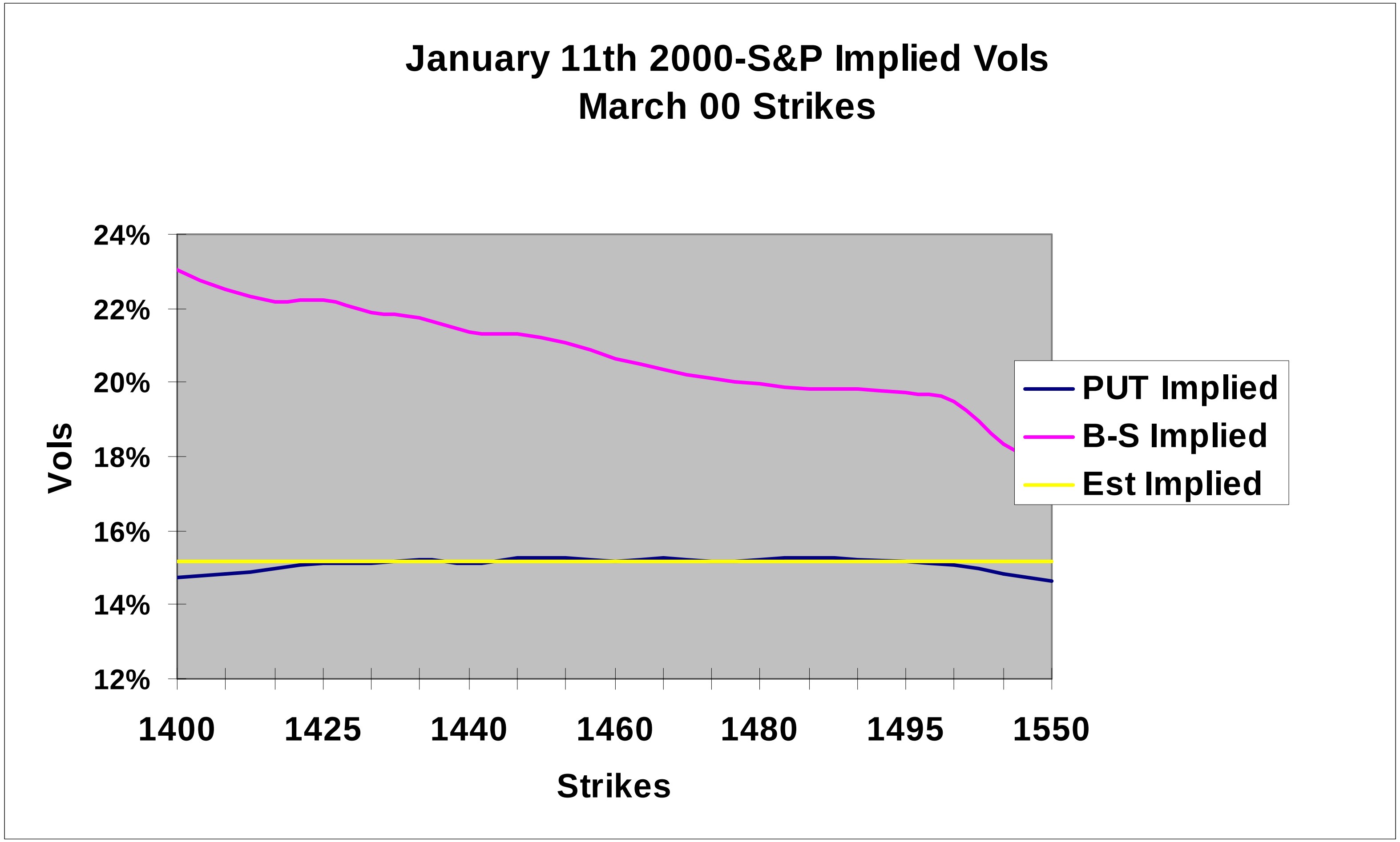
THE PUT PRINCIPLE - APPENDIX 3

CONTENTS

S&P 500	**1438.16**				
DATE	11-Jan-2000		σ	15.19%	19.31%	**4.12%**
Depo Rate	5.88%	5.71%	**K'**	0.679462		
Div Yld	1.25%	1.24%				

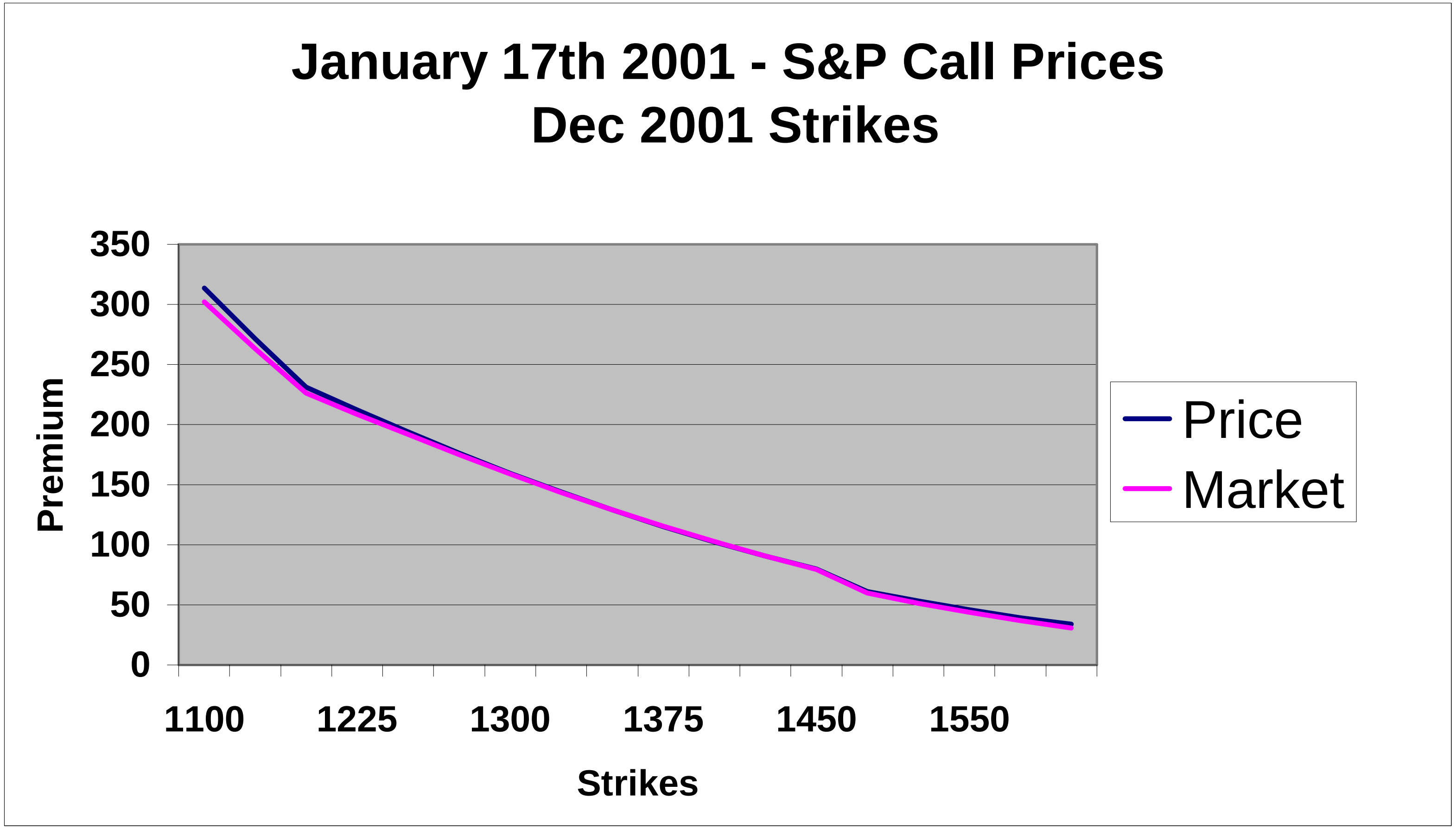
Strike	Expiry	Put/Call	Market	Days	ln(S/K)	d1	d2	new Price	Market	%Diff	Abs diff	PUT Implied	B-S Implied
1400	16-Mar-2000	CALL	82.75	47	0.02689228	0.846889	0.782305665	84.110	82.75	1.64%	1.360	14.73%	23.06%
1410	16-Mar-2000	CALL	75.5	47	0.01977481	0.736682	0.672099262	76.659	75.5	1.53%	1.159	14.81%	22.52%
1420	16-Mar-2000	CALL	68.875	47	0.01270765	0.627255	0.562671709	69.546	68.875	0.97%	0.671	14.98%	22.16%
1425	16-Mar-2000	CALL	65.875	47	0.00919271	0.57283	0.508246576	66.124	65.875	0.38%	0.249	15.11%	22.20%
1430	16-Mar-2000	CALL	62.5	47	0.00569007	0.518595	0.454012075	62.795	62.5	0.47%	0.295	15.10%	21.87%
1435	16-Mar-2000	CALL	59.625	47	0.00219967	0.46455	0.399966874	59.560	59.625	-0.11%	-0.065	15.21%	21.74%
1440	16-Mar-2000	CALL	56.25	47	-0.0012786	0.410693	0.346109657	56.422	56.25	0.31%	0.172	15.14%	21.38%
1445	16-Mar-2000	CALL	53.625	47	-0.0047448	0.357022	0.292439121	53.382	53.625	-0.45%	-0.243	15.26%	21.31%
1450	16-Mar-2000	CALL	50.625	47	-0.008199	0.303537	0.238953975	50.442	50.625	-0.36%	-0.183	15.25%	21.07%
1460	16-Mar-2000	CALL	44.875	47	-0.0150719	0.197118	0.132534764	44.864	44.875	-0.02%	-0.011	15.19%	20.62%
1470	16-Mar-2000	CALL	39.875	47	-0.0218979	0.091425	0.026841969	39.693	39.875	-0.46%	-0.182	15.25%	20.37%
1475	16-Mar-2000	CALL	37.125	47	-0.0252935	0.038848	-0.025735108	37.261	37.125	0.37%	0.136	15.15%	20.09%
1480	16-Mar-2000	CALL	35	47	-0.0286776	-0.01355	-0.078134259	34.931	35	-0.20%	-0.069	15.21%	19.95%
1485	16-Mar-2000	CALL	32.875	47	-0.0320503	-0.06577	-0.130356684	32.702	32.875	-0.53%	-0.173	15.24%	19.84%
1490	16-Mar-2000	CALL	30.625	47	-0.0354116	-0.11782	-0.182403571	30.573	30.625	-0.17%	-0.052	15.21%	19.84%
1495	16-Mar-2000	CALL	28.5	47	-0.0387617	-0.16969	-0.234276096	28.543	28.5	0.15%	0.043	15.18%	19.75%
1500	16-Mar-2000	CALL	26.25	47	-0.0421006	-0.22139	-0.285975424	26.611	26.25	1.37%	0.361	15.07%	19.50%
1525	16-Mar-2000	CALL	17.375	47	-0.0586299	-0.47733	-0.541914049	18.351	17.375	5.62%	0.976	14.82%	18.31%
1550	16-Mar-2000	CALL	11	47	-0.0748904	-0.72911	-0.793690886	12.211	11	11.01%	1.211	14.64%	17.64%

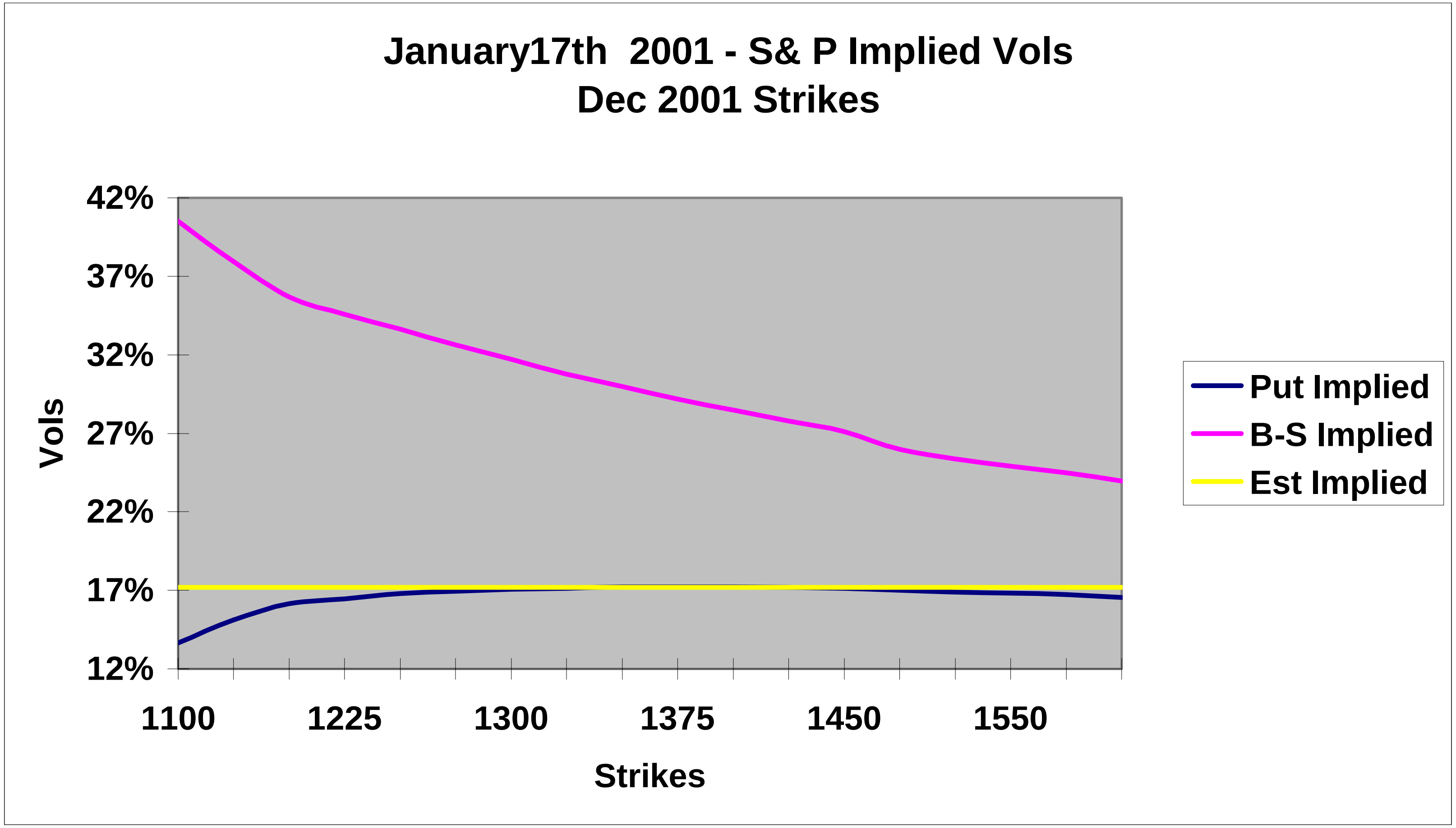

January 11th 2000- S&P Call Prices
Mar 00 Strikes
Premium
90
80
70
60
50
40
30
20
10
0
1400
1425
1440
1460
1480
1495
1550
Strikes
Price
Market


January 11th 2000-S&P Implied Vols
March 00 Strikes
24%
22%
20%
18%
16%
14%
12%
Vols
1400
1425
1440
1460
1480
1495
1550
Strikes
PUT Implied
B-S Implied
Est Implied

S&P 500	**1326.7**					
DATE	17-Jan-2001		σ	17.18%	21.36%	**4.17%**
Depo Rate	5.71%	5.55%	**K'**	0.608926		
Div Yld	1.24%	1.23%				

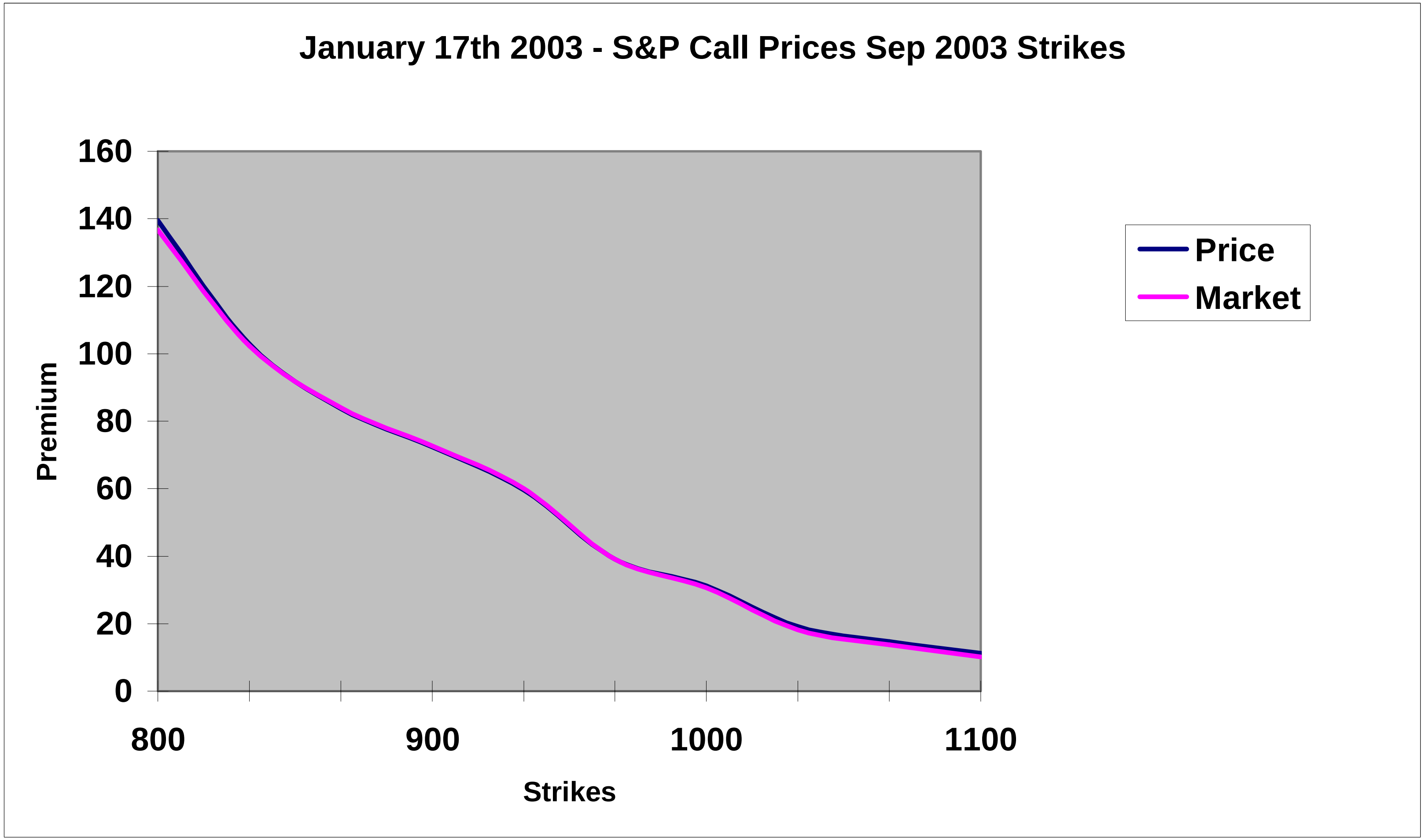
Strike	**Expiry**	**Put/Call**	**Market**	**Days**	**ln(S/K)**	**d1**	**d2**	**new Price**	**Market**	**%Diff**	**Abs diff**	**Put Implied**	**B-S Implied**	**Est Implied**
1100	22-Dec-2001	CALL	302.25	242	0.228975	1.959464	1.79367791	313.524	302.25	3.73%	11.274	13.64%	40.51%	17.18%
1150	22-Dec-2001	CALL	263.25	242	0.184523	1.691338	1.52555138	271.362	263.25	3.08%	8.112	15.11%	37.95%	17.18%
1200	22-Dec-2001	CALL	226.375	242	0.141963	1.434625	1.26883801	231.248	226.375	2.15%	4.873	16.14%	35.69%	17.18%
1225	22-Dec-2001	CALL	208.5	242	0.121344	1.310252	1.14446549	212.155	208.5	1.75%	3.655	16.46%	34.59%	17.18%
1250	22-Dec-2001	CALL	191.625	242	0.101141	1.188392	1.02260571	193.806	191.625	1.14%	2.181	16.78%	33.63%	17.18%
1275	22-Dec-2001	CALL	174.875	242	0.081339	1.068946	0.90315916	176.269	174.875	0.80%	1.394	16.94%	32.62%	17.18%
1300	22-Dec-2001	CALL	159	242	0.061921	0.951819	0.78603211	159.604	159	0.38%	0.604	17.08%	31.71%	17.18%
1325	22-Dec-2001	CALL	143.5	242	0.042872	0.836923	0.67113619	143.861	143.5	0.25%	0.361	17.13%	30.77%	17.18%
1350	22-Dec-2001	CALL	129.25	242	0.02418	0.724174	0.55838797	129.078	129.25	-0.13%	-0.172	17.21%	29.99%	17.18%
1375	22-Dec-2001	CALL	115.5	242	0.005831	0.613495	0.44770866	115.282	115.5	-0.19%	-0.218	17.22%	29.19%	17.18%
1400	22-Dec-2001	CALL	102.75	242	-0.01219	0.50481	0.33902367	102.484	102.75	-0.26%	-0.266	17.22%	28.47%	17.18%
1425	22-Dec-2001	CALL	90.75	242	-0.02989	0.398049	0.23226241	90.686	90.75	-0.07%	-0.064	17.19%	27.78%	17.18%
1450	22-Dec-2001	CALL	79.5	242	-0.04728	0.293144	0.12735796	79.876	79.5	0.47%	0.376	17.13%	27.11%	17.18%
1500	22-Dec-2001	CALL	60	242	-0.08118	0.088655	-0.0771312	61.120	60	1.87%	1.120	17.01%	25.96%	17.18%
1525	22-Dec-2001	CALL	51.25	242	-0.09771	-0.01105	-0.1768336	53.101	51.25	3.61%	1.851	16.88%	25.38%	17.18%
1550	22-Dec-2001	CALL	43.75	242	-0.11397	-0.10913	-0.2749147	45.927	43.75	4.98%	2.177	16.81%	24.92%	17.18%
1575	22-Dec-2001	CALL	37	242	-0.12997	-0.20564	-0.3714264	39.546	37	6.88%	2.546	16.73%	24.48%	17.18%
1600	22-Dec-2001	CALL	30.625	242	-0.14572	-0.30063	-0.4664182	33.903	30.625	10.70%	3.278	16.56%	23.97%	17.18%

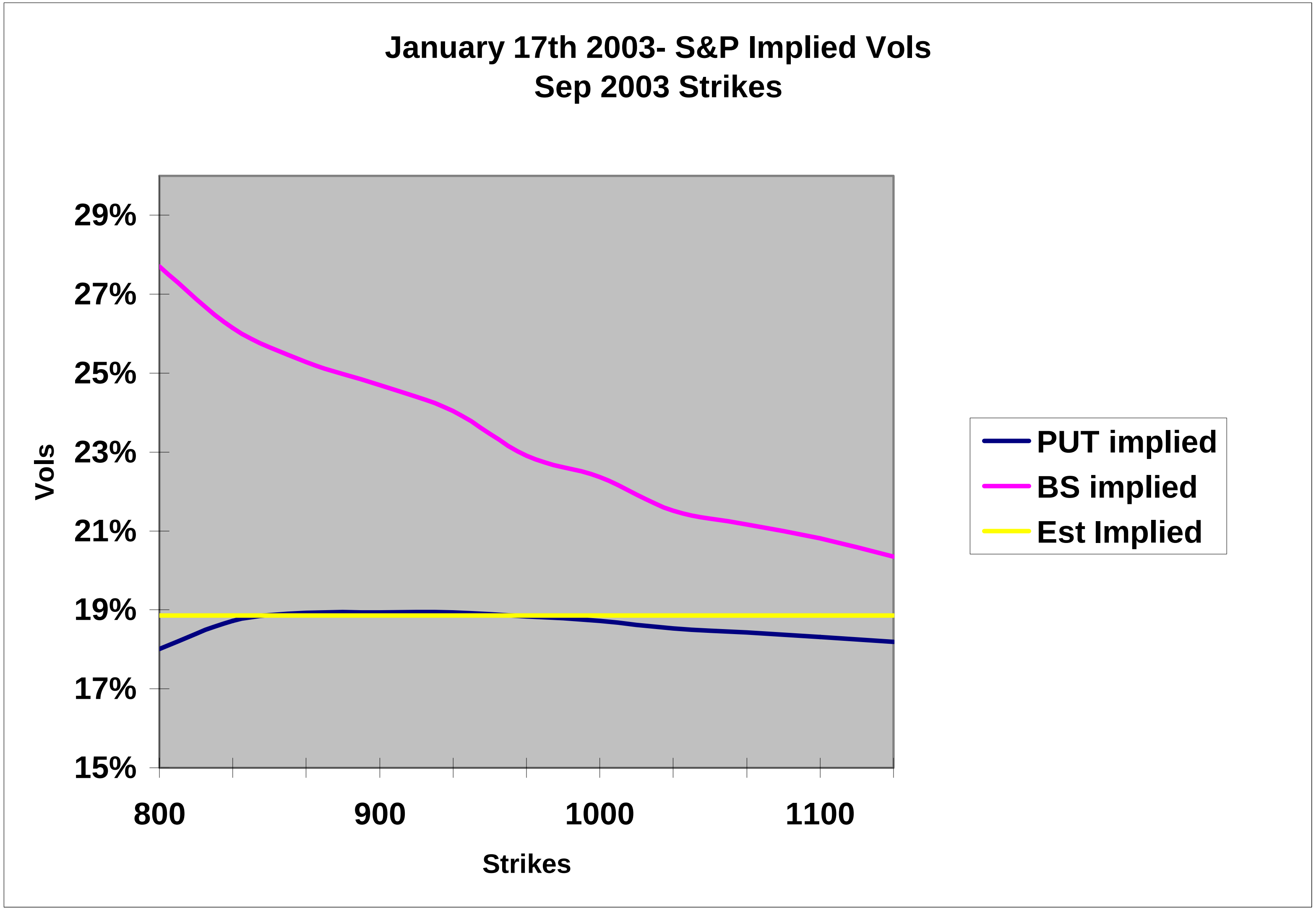

January 17th 2001 - S&P Call Prices
Dec 2001 Strikes
350
300
250
200
150
100
50
0
Premium
1100
1225
1300
1375
1450
1550
Strikes
Price
Market


January17th 2001 - S& P Implied Vols
Dec 2001 Strikes
42%
37%
32%
27%
22%
17%
12%
Vols
1100
1225
1300
1375
1450
1550
Strikes
Put Implied
B-S Implied
Est Implied

S&P 500	**901.5**						
DATE	17-Jan-2003			**σ**	18.85%	24.98%	**6.13%**
Depo Rate	1.45%	1.44%	0.990357	**K'**	0.814		
Div Yld	1.95%	1.93%	0.987113				

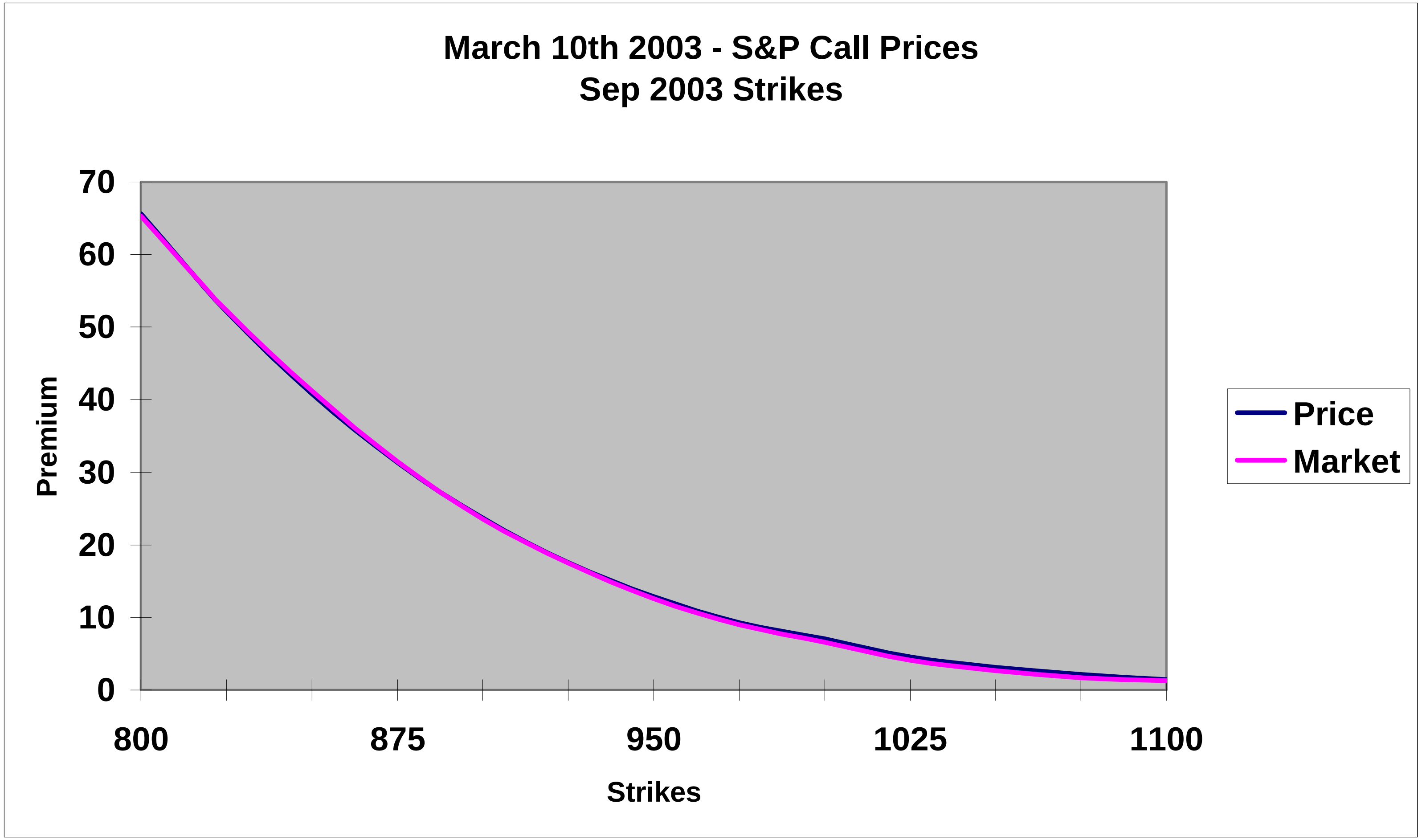
Strike	Expiry	Put/Call	Settle	Days	ln(S/K)	d1	d2	new Price	Market	Diff	PUT implied	BS implied
800	19-Sep-2003	CALL	136.7	175	0.11944831	1.149703183	0.995026	139.464	136.7	2.764	18.01%	27.70%
850	19-Sep-2003	CALL	102.3	175	0.05882369	0.757759776	0.603083	102.818	102.3	0.518	18.72%	26.14%
880	19-Sep-2003	CALL	84	175	0.02413814	0.533514653	0.378838	83.754	84	-0.246	18.92%	25.29%
900	19-Sep-2003	CALL	72.7	175	0.00166528	0.388225702	0.233549	72.368	72.7	-0.332	18.94%	24.69%
925	19-Sep-2003	CALL	60	175	-0.02573369	0.211088971	0.056412	59.651	60	-0.349	18.94%	24.04%
975	19-Sep-2003	CALL	39.1	175	-0.07837743	-0.1292573	-0.28393	39.138	39.1	0.038	18.84%	22.91%
1000	19-Sep-2003	CALL	30.7	175	-0.10369524	-0.29293912	-0.44762	31.166	30.7	0.466	18.72%	22.36%
1050	19-Sep-2003	CALL	18.2	175	-0.1524854	-0.60837173	-0.76305	19.126	18.2	0.926	18.53%	21.52%
1075	19-Sep-2003	CALL	13.7	175	-0.1760159	-0.76049843	-0.91518	14.749	13.7	1.049	18.43%	21.17%
1100	19-Sep-2003	CALL	10.1	175	-0.19900542	-0.90912765	-1.0638	11.262	10.1	1.162	18.31%	20.81%
1150	19-Sep-2003	CALL	5.4	175	-0.24345718	-1.19651212	-1.35119	6.383	5.4	0.983	18.19%	20.35%

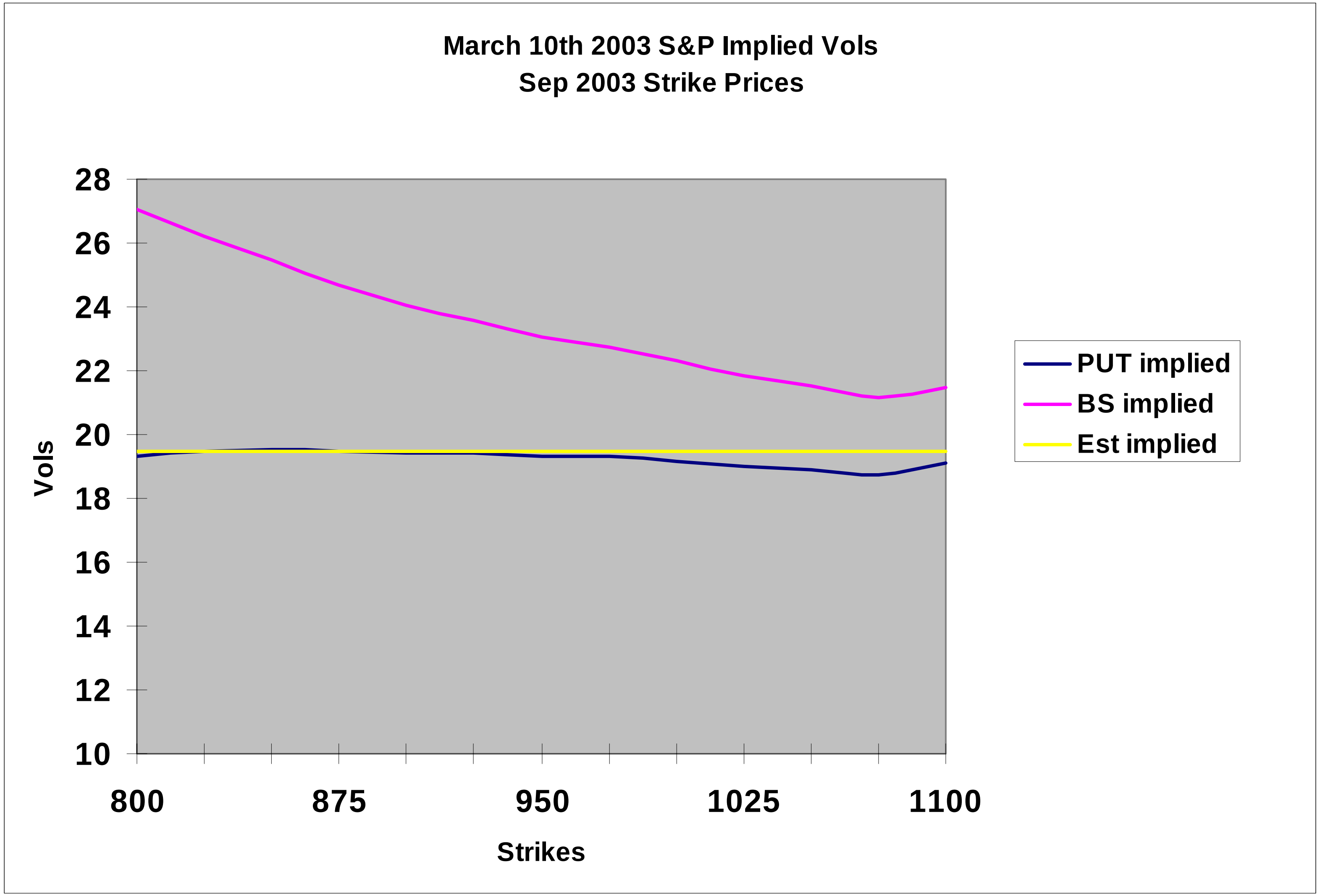

January 17th 2003 - S&P Call Prices Sep 2003 Strikes
160
140
120
100
80
60
40
20
0
Premium
800
900
1000
1100
Strikes
Price
Market


January 17th 2003- S&P Implied Vols
Sep 2003 Strikes
29%
27%
25%
23%
21%
19%
17%
15%
Vols
800
900
1000
1100
Strikes
PUT implied
BS implied
Est Implied

S&P 500	**807.48**					
DATE	10-Mar-2003		σ	19.48%	25.23%	**5.75%**
Depo. Rate	1.21%	1.21%	**K'**	0.739775		
Div Yld	1.81%	1.79%				

Strike	Expiry	Put/Call	Market	Days	ln(S/K)	d1	d2	Price	Market	%Diff	Abs diff	PUT implied	BS implied
650	19-Sep-2003	CALL	171.3	139	0.216946	1.866181	1.72373	181.889	171.30	6.18%	10.589	14.17	32.69
700	19-Sep-2003	CALL	131.6	139	0.142838	1.345931	1.20348	137.790	131.60	4.70%	6.190	16.85	30.81
725	19-Sep-2003	CALL	113.3	139	0.107747	1.099584	0.95714	117.322	113.30	3.55%	4.022	18.05	29.92
750	19-Sep-2003	CALL	96.2	139	0.073845	0.86159	0.71914	98.307	96.20	2.19%	2.107	18.63	29.05
775	19-Sep-2003	CALL	80	139	0.041055	0.6314	0.48895	81.002	80.00	1.25%	1.002	19.06	28.02
800	19-Sep-2003	CALL	65.3	139	0.009307	0.408518	0.26607	65.600	65.30	0.46%	0.300	19.31	27.05
825	19-Sep-2003	CALL	52.3	139	-0.02147	0.192496	0.05005	52.202	52.30	-0.19%	-0.098	19.45	26.19
850	19-Sep-2003	CALL	41.2	139	-0.05132	-0.01708	-0.15952	40.815	41.20	-0.93%	-0.385	19.55	25.48
875	19-Sep-2003	CALL	31.5	139	-0.08031	-0.22057	-0.36302	31.360	31.50	-0.44%	-0.140	19.48	24.69
900	19-Sep-2003	CALL	23.6	139	-0.10848	-0.41834	-0.56078	23.686	23.60	0.36%	0.086	19.4	24.03
925	19-Sep-2003	CALL	17.5	139	-0.13588	-0.61068	-0.75313	17.593	17.50	0.53%	0.093	19.4	23.56
950	19-Sep-2003	CALL	12.6	139	-0.16254	-0.7979	-0.94035	12.857	12.60	2.04%	0.257	19.32	23.07
975	19-Sep-2003	CALL	9	139	-0.18852	-0.98025	-1.1227	9.251	9.00	2.79%	0.251	19.3	22.73
995	19-Sep-2003	CALL	6.6	139	-0.20882	-1.1228	-1.26524	7.033	6.60	6.55%	0.433	19.15	22.32
1025	19-Sep-2003	CALL	4.1	139	-0.23853	-1.33133	-1.47378	4.582	4.10	11.76%	0.482	19	21.86
1050	19-Sep-2003	CALL	2.7	139	-0.26263	-1.5005	-1.64295	3.159	2.70	16.99%	0.459	18.9	21.53
1075	19-Sep-2003	CALL	1.7	139	-0.28616	-1.66569	-1.80814	2.150	1.70	26.46%	0.450	18.73	21.15
1100	19-Sep-2003	CALL	1.275	139	-0.30915	-1.82708	-1.96953	1.445	1.28	13.35%	0.170	19.1	21.46

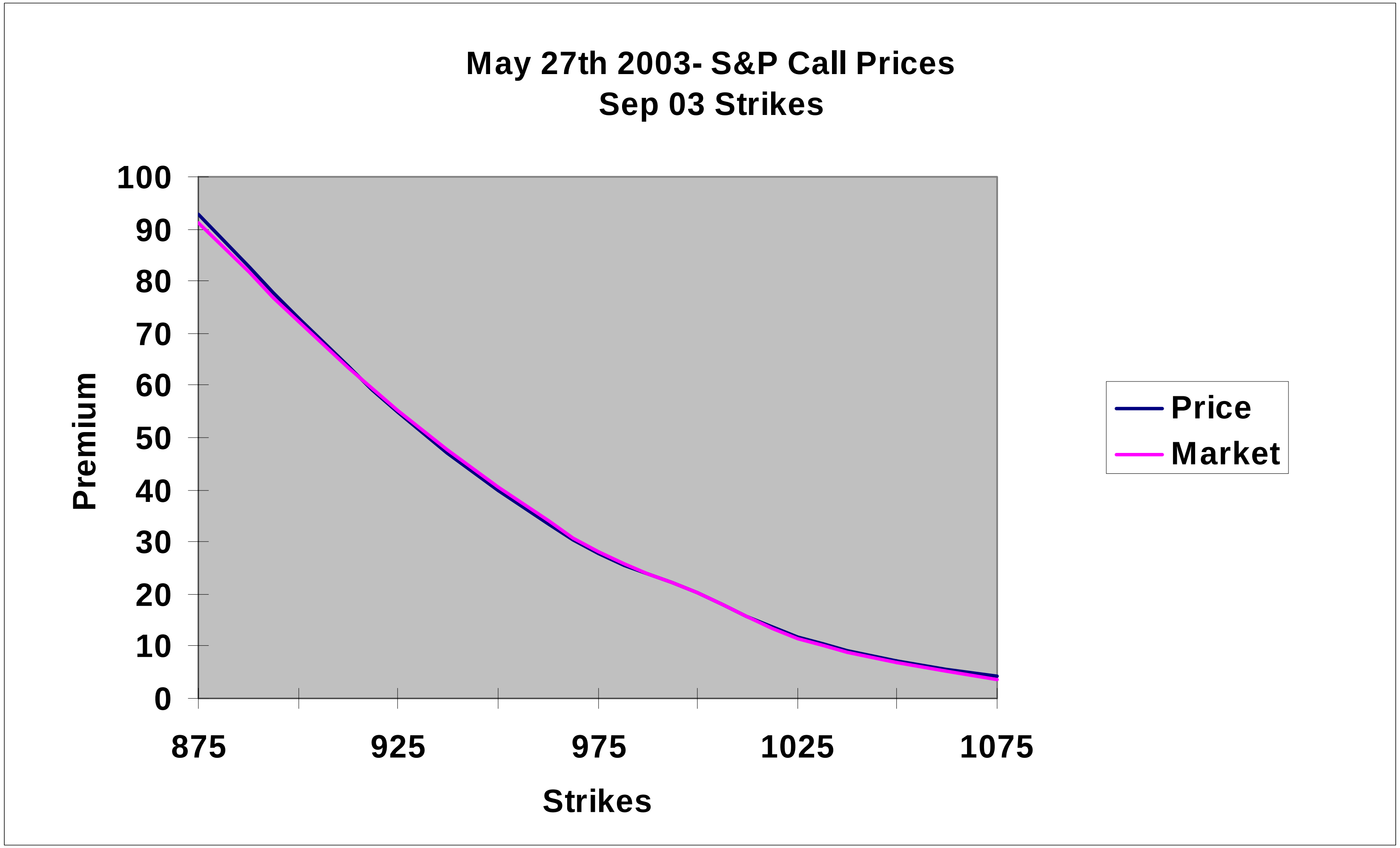

March 10th 2003 - S&P Call Prices
Sep 2003 Strikes
70
60
50
40
30
20
10
0
Premium
800
875
950
1025
1100
Strikes
Price
Market

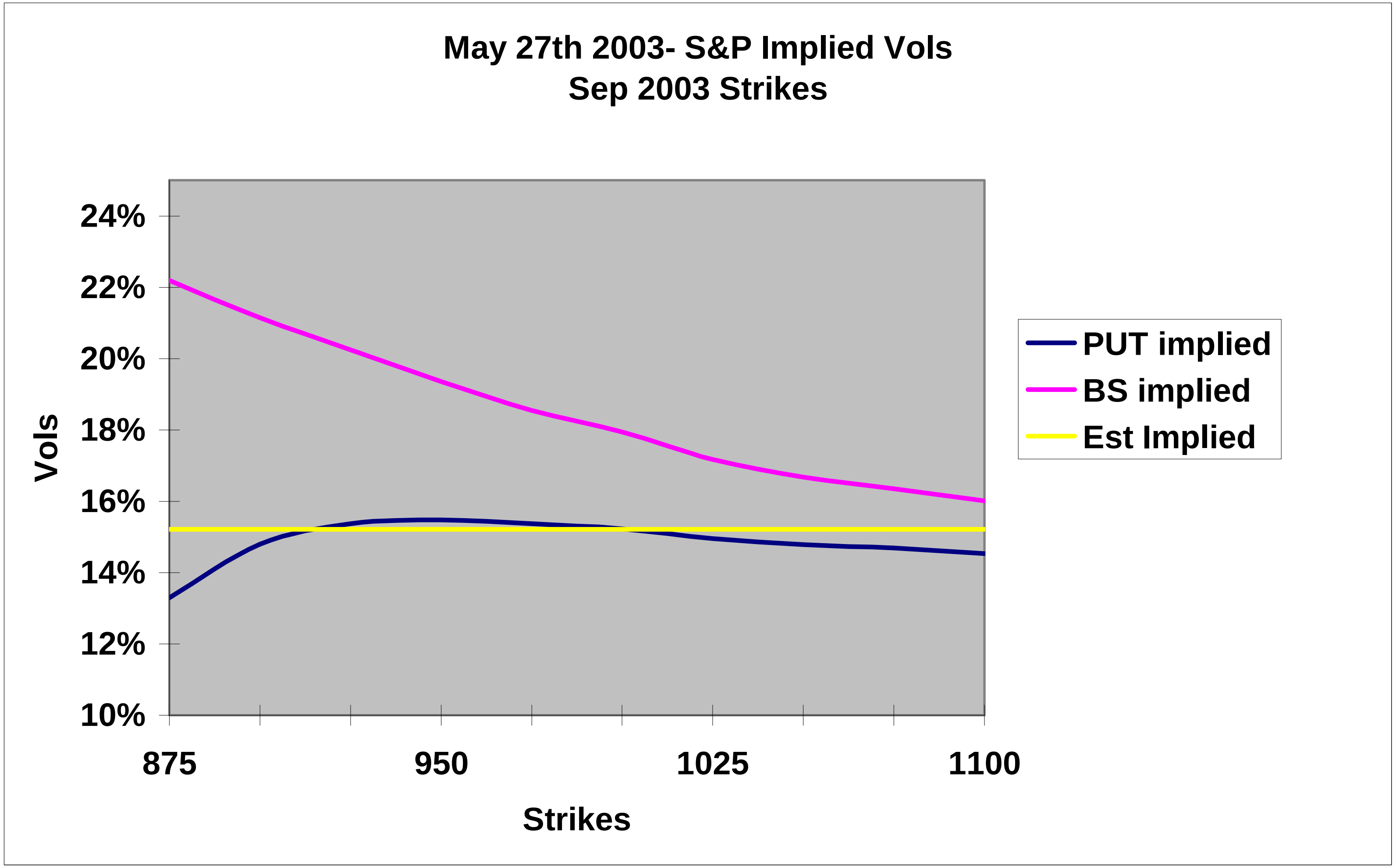


March 10th 2003 S&P Implied Vols
Sep 2003 Strike Prices
28
26
24
22
20
18
16
14
12
10
Vols
800
875
950
1025
1100
Strikes
PUT implied
BS implied
Est implied

S&P 500	**951.48**						
DATE	27-May-2003			σ	15.21%	18.13%	**2.92%**
Depo	1.22%	1.21%	0.996136	**K'**	0.480568		
Div Yld	2.20%	2.18%	0.993077				1.405

Strike	Expiry	Put/Call	Market	Days	ln(S/K)	d1	d2	Price	Market	%Diff	Abs diff	PUT implied	BS implied
875	19-Sep-2003	CALL	91.1	83	0.100882	1.181091	1.09515	92.956	91.10	2.04%	1.856	13.29%	22.19%
900	19-Sep-2003	CALL	72.2	83	0.072712	0.853282	0.76734	72.811	72.20	0.85%	0.611	14.79%	21.15%
925	19-Sep-2003	CALL	55.3	83	0.045313	0.534454	0.44852	55.006	55.30	-0.53%	-0.294	15.37%	20.24%
950	19-Sep-2003	CALL	40.5	83	0.018644	0.22413	0.13819	39.958	40.50	-1.34%	-0.542	15.47%	19.35%
975	19-Sep-2003	CALL	28.2	83	-0.00733	-0.07813	-0.16407	27.850	28.20	-1.24%	-0.350	15.37%	18.54%
995	19-Sep-2003	CALL	20.25	83	-0.02764	-0.31441	-0.40035	20.231	20.25	-0.09%	-0.019	15.22%	17.94%
1025	19-Sep-2003	CALL	11.45	83	-0.05734	-0.66008	-0.74601	11.890	11.45	3.84%	0.440	14.95%	17.17%
1050	19-Sep-2003	CALL	6.7	83	-0.08144	-0.94049	-1.02643	7.278	6.70	8.63%	0.578	14.78%	16.68%
1075	19-Sep-2003	CALL	3.75	83	-0.10497	-1.2143	-1.30024	4.268	3.75	13.80%	0.518	14.69%	16.35%
1100	19-Sep-2003	CALL	1.95	83	-0.12796	-1.48182	-1.56775	2.399	1.95	23.03%	0.449	14.54%	16.01%


May 27th 2003- S&P Call Prices
Sep 03 Strikes
100
90
80
70
60
50
40
30
20
10
0
Premium
875
925
975
1025
1075
Strikes
Price
Market


May 27th 2003- S&P Implied Vols
Sep 2003 Strikes
Vols
24%
22%
20%
18%
16%
14%
12%
10%
875
950
1025
1100
Strikes
PUT implied
BS implied
Est Implied

