REINSURANCE AND RETENTIONS

A London Market Actuaries' Group Paper

VOLUME I

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Section 1

INTRODUCTION AND CONCLUSIONS

Introduction

The working party adopted the following terms of reference:

1. To provide a review of some current practices in the field of reinsurance retentions.

2. To investigate and discuss those aspects of general insurance operations which we believe should influence the reinsurance decision process.

3. To present a synopsis of practical methods that may be used in order to translate the identified objectives of reinsurance into an explicit programme and retention policy.

We have defined the retention of a general insurance operation as all business which is not ceded including coinsured layers of excess of loss reinsurance, and any unplaced parts of the operation's reinsurance programme. We stress that we have used the word retention in its literal sense, namely, an amount retained. We consider that a company which has, for example, reinsured itself £90 million excess of £10 million has decided to retain claims excess of £100 million.

The remainder of the paper is divided into three sections. Section 2 covers some aspects of the current reinsurance market, Section 3 a discussion of the factors that influence the reinsurance programme and retention philosophy, and Section 4 summarises the practical methods for estimating aggregate claim distributions and retentions that we have reviewed. Detailed documentation of the application of these methods is contained in the appendices.

We have attempted to address the problems of retentions separately for all types of general insurer, including proprietary and mutual companies and Lloyd's syndicates.

We have assumed throughout that companies reserve accurately for claims since reserving problems and their effects on reinsurance strategy are, properly, the subject of a separate paper. We have not addressed the question of reinsurance security. In practice there is likely to be a trade-off between the cost and the quality of any reinsurance that is to be purchased.

Conclusions

During the last decade computer technology has leapt forward, but, reinsurance practices do not appear to have kept pace. This revolution enables insurance companies to store previously unimagined amounts of data. It also allows the technicians within those companies to experiment with much more ambitious risk management procedures. Therefore, it is likely that many opportunities exist for organisations who exploit the new technology to gain competitive advantage. This is because, historically, reinsurance practice must have applied unnecessary caution in the face of inadequate data and methodology.

A point of particular importance is that a seller of reinsurance will require a return on capital. The purchaser of the reinsurance must be aware of this fact. This is discussed further in Section 3. We have avoided use of the term "probability of ruin" because of the unhelpful connotations of the word ruin. We think that words such as "the probability of a £10 million reduction in earnings" are of more use and importance.

We can try to summarise this paper in one paragraph. First, we believe a retention should be defined as all business that is written but not ceded. Second, an insurer should review its objectives, and from this base develop a retention strategy. The insurer should view reinsurance as a benefit which will incur a cost. The aim must, therefore, be to use reinsurance as efficiently as possible. The retention strategy should be considered from the top down. The requirements of the entire operation must be determined and from this the implications for internal operating units should follow. Third, the estimation of the aggregate retained claims distribution is essential input into the retention process. This is an area where the actuary in particular can add considerable value. In the paper we present a number of methods which can be helpful in calculating these aggregate claim distributions and determining retentions.

Section 2

SOME ASPECTS OF THE CURRENT REINSURANCE MARKET

2.1 Introduction

This section reviews current practices and some of the rules of thumb for determining retentions.

There are a wide variety of reinsurance products. These range from a straightforward Quota Share treaty for a small proprietary insurance company to a financial reinsurance arrangement for a Lloyd's syndicate. We have not attempted here to cover the market practice across the whole field, but rather have concentrated on those aspects which we believe are important to the market as a whole.

Many insurance companies consider their retentions at three levels, "individual account" level, "company" level and "group" level. The overall retention that results is often built from the bottom up.

2.2 Retentions in Practice

It is worth pointing out that despite the increasing array of mathematical techniques available, decisions regarding retention levels are still based on rules of thumb, and a desire to conform to market norms. This is due, in part, to the impractical data requirements of some theoretical methods, and their often unrealistic assumptions (for example, independence of risks).

Companies may, for commercial reasons, also purchase more (or less) reinsurance than they need, or that various theories might imply. The practical importance of these commercial factors needs to be borne in mind when considering the validity of any methods, or theories, for setting retention levels.

In many instances, the choice of retention level is made by the underwriter of the account under consideration. He will use his skill and judgement, based on his knowledge of the account, to decide the best retention level. The aim, in deciding on this level is more likely to be to balance the relationship between profits and stability, rather than to reduce the risk that capital is exhausted. The probability of ruin is not a concept which underwriters are likely to consider.

A survey of U.S. insurance companies conducted by the Munich Re in 1976, showed that the main factors which were then considered when setting retention levels were, (in order of priority) level of capital, cost of reinsurance and smoothing of earnings fluctuations.

We are not aware of any more up to date surveys, but some previous studies (References 6 and 10) had highlighted the commonly held belief that retention levels should be positively correlated with the size of the company (as measured by premium income or capital/reserves). It is however thought that some composite insurers hold much lower retention levels than their size would indicate, perhaps due to the relatively low cost of reinsurance during a soft market, the risk aversion of the company, or other commercial reasons such as reciprocity. Also, a company which operates a profit-centre approach for each of its categories of business, without any central rationalisation, will probably have lower retentions than one which looks at its retentions on a more global basis.

Retentions considerations should focus on the amount of cover purchased as well as the size of the deductible. This is particularly true for event covers such as catastrophe excess of loss. Several insurance and reinsurance companies have developed their own loss accumulation systems which help them to decide how much catastrophe reinsurance to purchase. These systems can also prove useful in deciding the level of the catastrophe deductibles.

In practice, deciding on the deductible is only part of the process. The structure of the reinsurance programme will affect how much protection is provided. Factors such as the number of reinstatements purchased, inclusion of any drop-down facilities in the contract, vertical versus horizontal cover, and the availability of back-up covers will need to be considered. Underwriters look for continuity of cover: changes are gradual rather than sudden and will generally be in one direction (that is, upwards). There is often reluctance to increase the retention voluntarily.

Other important factors include the risk willingness of the company's management and the capacity (and, therefore, price) of the reinsurance market. Regardless of what retention may be theoretically correct, the market conditions may be such that cover is simply not available. An example of this was the upheaval of the retrocessional market which occurred following the windstorms in Europe in early 1990.

The extent to which companies/syndicates use brokers for advice about retention levels is unclear, but their use to provide alternative quotations for different reinsurance programmes is one way in which a company can obtain help to decide on the best retention. It should be noted, however, that a broker has traditionally earned a living from the placing of reinsurance rather than advising clients to retain risk.

2.3 Rules of Thumb for Setting Retention Levels.

2.3.1 Risk Theory Approach

This approach, which is based on a Normal approximation, assumes that the optimum retention is defined in terms of a per risk excess. Reduction of the probability of ruin to a certain minimum is the target. The theory is developed in Reference 1 and leads to formulae relating the retention, premium loading and free reserves.

These formulae, in turn, lead to a rule of thumb described below, where the maximum retention should not exceed a certain percentage of the free reserves.

Other risk theory approaches involve modelling the aggregate claims distribution. The effect of different forms of reinsurance and different retentions is assessed by analysing the changes in the net retained aggregate claims distribution. The aggregate claims distribution can be modelled by combining the claims severity and claims frequency distributions using a range of possible techniques.

2.3.2 Rules Based on Maximum Percentages

Perhaps the most commonly quoted rules of thumb are those which link the retention level, again a per risk excess, with items such as free reserves and premium income:-

TABLE 1 - RETENTION RULES OF THUMB

As a percentage of:-	Retention
Capital and free reserves	1 - 5%
Retained premium income (by class)	1 - 10%
Liquid assets	400 - 600%

These rules assume that the aim of the reinsurance programme is to smooth out fluctuations in the net retained account. This is achieved by setting the retention so that a single large claim cannot impact the company by more than, say 5% of its free capital or 10% of premium. By measuring the retention against its liquid assets a company can try to ensure that it has enough cash available to meet a single claim.

Claim in this context means either a single large claim affecting a single risk or an accumulation of relatively small claims arising out of a single event.

These rules of thumb can be expressed differently. The company can determine what percentage of the profits of a class of business they are prepared to lose. This amount combined with estimates of the maximum operating ratio and written premium of the Quota Share treaty will imply a retention.

For excess of loss reinsurance, the retention can be based at the level at which claims become very infrequent or alternatively the level at which the average claim up to that point starts to show significant variation year on year. The basis for this method is that if a claim of a certain severity occurs frequently then claims of that severity are not giving rise to significant variation in results.

For property portfolios, the common practice when designing a Surplus treaty is to compile a table of limits which shows the company's retention for different risk categories.

This could be constructed by firstly deciding on a minimum retention. The retentions for each risk category are then calculated by scaling this minimum in relation to the relative premium loadings for each risk category (Reference 4). In practice, of course, the individual underwriter's experience and judgement will play a major part in determining the retention levels in the table of limits.

Companies do, in practice, vary their retention levels both by risk category within a class, and between classes of business. It is common practice for underwriters to fix their Surplus retention levels so that they are, broadly, inversely proportional to the original premium rates which they charge (In other words, they keep more of the less hazardous risks). It is preferable that retention levels should be based on some assessment of the quality of the risk (for example, as measured by the construction type for Fire insurance) rather than in direct proportion to the actual premium rates.

Section 3

RETURN TO FIRST PRINCIPLES

3.1 Introduction

This section sets out the general considerations relevant to determining reinsurance retentions. Our intention is to return to first principles and consider why companies require reinsurance. We believe that it is from this point that a reinsurance strategy should be built.

The first key point is that the aims of the general insurer in its entirity must be the starting point for a retention policy. As we have seen, in many instances individual units within a general insurer develop their own retention strategy. The retention of the total is the sum of the pieces and may, or may not, be appropriate. In other words retention strategy develop from the bottom up; it should be designed from the top down.

We now consider the major influences in determining the retention of general insurers at the top level. Many of the ideas presented are equally relevant when determining retention strategy for individual business units based upon a global strategy.

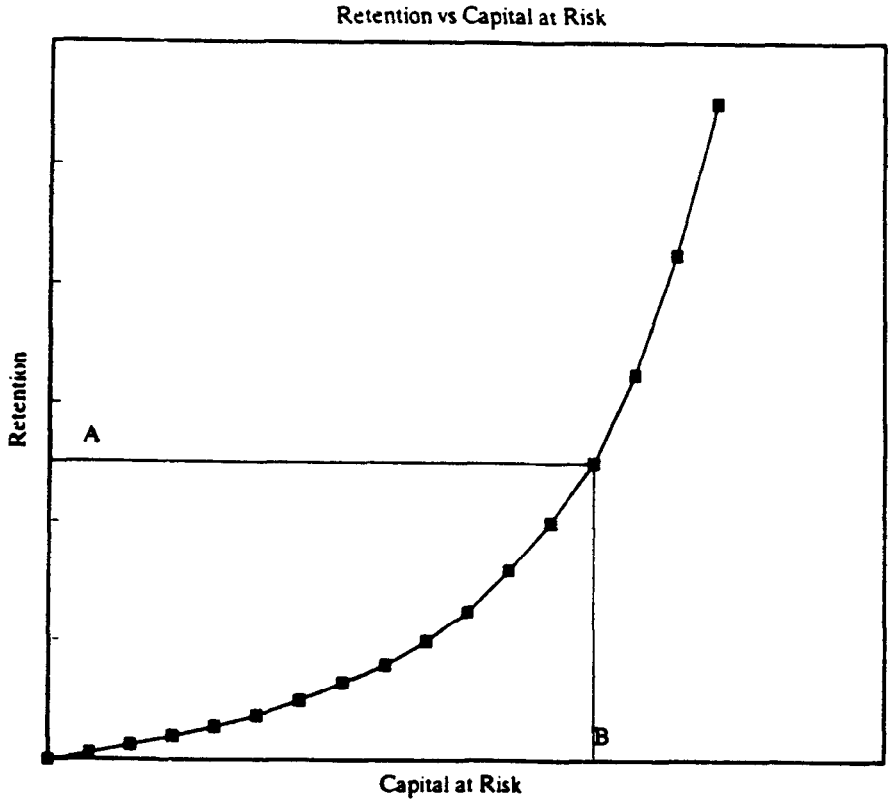
The process of setting a retention level is related to the control of exposure. The control of exposure is the last part of a three stage process.

1. Identify Exposure
2. Quantify Exposure
3. Control Exposure

For an employers' liability insurer, the process of exposure identification should focus both on large claims, and aggregation arising either from industrial disease, or an explosion. A property insurer may have exposure to aggregation from one natural catastrophe in addition to aggregation from adjacent sites and exposure to total loss on one risk. These identified exposures represent potential claims for which insurance may be required.

The second step in the process is the quantification of the severity of potential loss from the identified exposures together with their associated probabilities. Some techniques for achieving this are described in Section 4.

We have adopted a standard presentation of the results of these techniques, which is to show the effect on free reserves of having different retentions. An example of these graphs is shown below:



The graph is to be read as follows: if the retention is set at Level A there is a likelihood of 5% of losing an amount equal to B of free reserves. The actuary can use these graphs to help management quantify a subjective assessment of risk.

The objectives of a company play an important part in determining its retention. Some of these are discussed below for each type of insurer. We then review two general considerations which should effect retentions, namely, the underwriting cycle and the cost of reinsurance.

3.2 Exposure Control

We feel it is important to stress that an insurer's retention should be as much a reflection of its perceived risk aversion as of the underlying distribution of its claims or of conditions in the reinsurance market. Risk aversion depends on the financial condition of the company, and its corporate culture, and is reflected in the reinsurance protection it purchases.

In determining retentions, we need to consider measures by which to quantify unacceptable claim deviation. Possible measures, at a "group level", are the effect on earnings, the effect on shareholders funds, on share price or on Names. We have only presented results in terms of the effect on shareholders' funds.

The insurer must consider its objectives. These objectives may be different for the following three groups:

1. Proprietary insurance companies
2. Mutual insurance companies
3. Lloyd's syndicates.

Proprietary Insurance Companies

For a proprietary company the objective must broadly be to produce a long-term return on capital employed commensurate with the risks involved, and, in the short term, to distribute part of this return as a smoothly increasing dividend.

For a publicly quoted proprietary company, there is also a need to maintain the market share price. This price, to a great extent, is influenced by the return on capital and dividends. Other influences include analysts' comments and market perception of the company.

Some companies form part of conglomerates which have higher quality earnings streams from other activities which may allow the general insurance operation greater variability in results without jeopardising the overall corporate objectives.

Some proprietary companies are set up as captives to write the insurance risks of a larger parent company. In such a case, setting profit objectives is purely an internal or tax accounting process. The objectives of the captive will be aimed at controlling the variability of the results, thus protecting solvency, and developing the captive.

Companies can attempt to control the emergence of profit in the following ways:-

1. Via alterations in reserve surplus.
2. By realising investment gains.
3. Using reinsurance.

At the start of any trading period, the status of the company's reserve surplus and unrealised investment gains must be taken into account. The first two methods of smoothing are cost effective for the company, however, it is only the third that has an elastic supply. The company may determine its retention by examining:-

1. The expected profit in the ensuing period.
2. The variability associated with that expected profit.
3. The desired variability in profit in the ensuing period.
4. The availability of reserve surplus and unrealised investment gains to smooth the difference between the actual and desired variability.

Mutual Insurance Companies

It is likely that the main objective of a mutual is to build up the solvency of the company in order to enable it to write more risk. The control of variability will be pitched at a level that protects solvency rather than annual earnings.

As a result, the mutual is more likely to focus on the maximum amount it wishes to lose in one year. For a large well established mutual the Estimated Maximum Loss from one event may be very small in comparison to the financial resources. In such a case reinsurance is probably not required.

For a small mutual, such as the one of the professional indemnity vehicles that have become commonplace during the last ten years, incurring gross claims in excess of called capital may be a very real possibility. To reinsure very heavily defeats the object of the mutual. The managers might focus on the maximum capital the members wish to have at risk in any year (which may well be much greater than the called capital) at given levels of probability.

The mutual may determine its retention by considering:-

1. The variability associated with the claims costs.
2. The desired capital at risk during the ensuing period.

The retention should be fixed to ensure that items 1. and 2. are consistent. The reserve surplus and unrealised capital gains do not feature directly because revenue account profit is not of overwhelming importance. However, in determining the desired capital at risk, the members will consider the capital already available in the mutual which should include the above items. A small mutual provides an example of where a desired retention profile might be achieved by alteration of the gross portfolio rather than by via reinsurance.

Lloyd's Syndicates

Lloyd's syndicates are different from insurance companies in two ways. First, the shareholders on each underwriting year are separately identified. Second, investment income is only earned on insurance funds which are invested in similar assets for every syndicate until the underwriting year is closed. The investments are generally risk free in nature. Thus, the underwriting result becomes the major source of variation in results between different syndicates and different years of account on the same syndicate. This differs from proprietary companies in two respects, first investment income is of secondary importance and second separate cohorts are considered rather than the change in the overall financial state of the company during the period.

The retention philosophy must focus on controlling the variability of the underwriting result for the individual underwriting year during the three year period prior to closure. It is fair to assume that all underwriters work on the basis that they will close the year in the normal fashion after thirty-six months and set their retention accordingly.

If we suppose that all names require the same variability then a further complication arises from Names participating in varying numbers of "independent" syndicates. Even if all syndicates have identically distributed underwriting results, different Names would experience different variability due to different participations.

Reserve surplus and unrealised capital gains should not have a role in the control of variability at Lloyd's. The syndicate may determine its retention by examining:-

1. The expected result of the underwriting year.
2. The variability associated with that expected result.
3. The desired variability in the underwriting result.

Since Names are generally risk averse, we believe that the retention is primarily aimed at obtaining the desired level of variability. The Lloyd's syndicate can be faced with a unique problem since attaining the desired level of variability could imply purchasing so much reinsurance that the expected profits will be unacceptably low. The underwriter is faced with a dilemma, either reduce the profit or increase the variability.

Variability in Claims Costs

Variability in claims costs are dependent on the amount and nature of the business written. For a major composite insurance group the gross book of business may very nearly conform to that which is desired. For a small company writing LMX business, the gross distribution is likely to be extremely unsuitable and require considerable alteration.

Variability can be reduced by reciprocal reinsurance with another insurer. We define a reciprocal reinsurance as one where the quantum of risk ceded and accepted are equal. The point of this contract is to reduce the variability in the book of business via diversification. Many large insurance operations will already have optimised their diversification via world wide operations and will not add value via reciprocity.

After effecting the reciprocal reinsurance the insurer is left with a redefined book of business. If the characteristics of this business are still incompatible with the objectives then reinsurance can be utilised.

If reinsured and reinsurer both accept that "reinsurance costs money", then long term good relationships with reinsurers can be very valuable. Once this relationship exists and the purpose of reinsurance is established, there should be no barriers to the type of reinsurance cover available provided both parties are satisfied. This, in turn, might allow a simplification of current reinsurance programmes and thus savings on the administration side.

3.3 The Underwriting Cycle

We have not yet discussed the affects of the insurance cycle. An analogy can be drawn between the general insurer and a geared investment trust. Premiums represent borrowed funds. In this analogy a softening market leads to an increase in the cost of borrowing. Usually, there will be no correlated or predictable change in the investment return, and hence, the unit profitability is squeezed. In this situation most types of general insurer will become more variation averse. The expected profit is low, and hence, the acceptable downside is reduced. A priori, the insurer will wish to change the retention to reduce variability.

Under these circumstances the company may cede business at unprofitable rates (for the reinsurer) and in this way improve the short term profitability without loss of business. The cedent should acknowledge that a pay back to the reinsurer will be required in the future. However, this will occur at times of greater unit profitability and so the objective will have been achieved. This is the second way in which the insurance cycle may affect the retention.

This concept is particularly true of the London Market where the rates at the bottom of the cycle can be extremely soft, but each player in the market is supported by equally cheap reinsurance. However, historically there have been reinsurers of London Market companies who have been "fair game" and not received a pay back. The London Market operation of the Insurance Corporation of Ireland is one such company.

The London Market may be considered from a different perspective - as one insurance entity, with each company or syndicate a "department", often the last retrocessionaire for much of the world's market. The reinsurance rates that individual "departments" charge each other are unimportant to the entity as a whole since these merely constitute internal accounting. If we view the market from this perspective, the entity suffers from the cycle when the rates it receives for business ceded into the market are too low. It overcomes the cycle by reducing the profit of each department and by "cannibalising" one or two departments. In other words, the market cedes much of it's loss to these "departments" who never recover. The LMX spiral partly arises out of each "department's" desire not to be one of the "cannibalised".

3.4 The Cost of Reinsurance

Any purchaser of reinsurance needs to bear in mind that the reinsurer is a commercial enterprise and requires a return on capital. The cedent should expect reinsurance premiums to exceed recoveries in the long term and, as such, this represents a cost. The purchase of reinsurance, therefore, reduces profits in the long term. In return the reinsurance provides some stability of claims costs to the cedent.

A principle that we consider should underlie any discussion of an appropriate retention for a company is that the company should avoid purchasing any unnecessary reinsurance.

Section 4

MATHEMATICAL MODELS

4.1 Introduction

At whatever level within a general insurer while investigating retention philosophy, understanding the variability associated with the relevant aggregate claim distribution is essential. In this section we demonstrate some methods that can be used when estimating aggregate claim distributions and investigating retentions. Where possible, we have demonstrated the use of these methods on three case studies. The details of the calculations are given in Appendices 1 to 4.

The three case studies consist of aviation, liability and property risks. Exhibit 1 contains the underlying severity distributions used to derive the aggregate claims distributions on which our analysis is based.

We express the effect of different retention levels as reductions in free reserves together with associated probabilities. Equally, results could be expressed in terms of premium income, earnings or other measures. An increase in retention should not necessarily be seen as increasing the probability that a company will face ruination. It can more usefully be seen as increasing the probability of a specified reduction in free assets or earnings. This increased variability is compensated for by an increase in the expected profitability.

We have used four methods to quantify these effects. The methods used are not intended to be exhaustive, nor, to be necessarily the best methods available. They are methods which have either been used by the members of the working party or which are believed to be commonly used.

We would like to stress that the results of these methods are only as good as the assumptions underlying them which may, in some instances, be very limited. In particular, the assumptions concerning the tail of the probability distribution can be critical when examining retentions.

4.2 Straub's Method of Calculating Retention Levels

This method is based on the theory developed in Erwin Straub's book "Non-life Insurance Mathematics" (Reference 18). Straub develops a mathematical representation of the following intuitively reasonable relationship:-

$$\text{RETENTION} = \frac{\text{CAPITAL} \times \text{RISK WILLINGNESS} \times \text{PROFIT MARGIN}}{\text{UNBALANCEDNESS}}$$

If four of the elements of the equation are known then the fifth is implied. The formula can be used to investigate the relations between capital and retention. A different formula is developed for each of the common types of reinsurance. The method takes the classical risk theory approach and considers an infinite future time period. This is different from the approaches presented in the next three sections which consider a finite future period.

The capital item refers to the free reserves backing the class of business under consideration. Risk willingness is expressed as a function of the tolerated ruin probability (or probability in the examples of Appendix 1). The smaller the tolerated ruin probability, the lower the risk willingness of the company. Unbalancedness is dependent on the type of business written and is determined essentially by the distribution of total aggregate claims.

The relationship follows certain intuitive rules. For example, if we increase the unbalancedness of the portfolio, then ceteris paribus, we would expect the retention to decrease. Alternatively, as the risk willingness of the insurer increases then so should the retention.

In its most general form, Straub's formula relies on very few assumptions about the risk process which is being considered. However, for the purposes of the examples used to demonstrate the method in this paper, we have assumed that:-

1. There are equal loadings used by the insurer and reinsurer. (This makes the mathematics easier!).
2. The claim amount distributions can be approximated by discrete distributions.
3. The claim count distribution is Poisson.
4. Either Quota Share or Risk Excess reinsurance is used.

After fixing the various components of the formula, the method calculates either the Quota Share or the Risk Excess retention. By varying key components such as risk willingness and capital, graphs may be drawn to summarise their inter-relationship.

This method has the advantage that it allows explicitly for all of the important items when setting retentions. The items are linked together in a neat formula.

In addition to calculating a retention level, it is also possible to use the method to calculate a measure of the need for reinsurance. This is clearly an important consideration before deciding what retention to hold. However, given that a particular company needs reinsurance, the method provides little help in deciding what form of reinsurance is the most efficient.

4.3 Heckman and Meyers' Method for the Calculation of Aggregate Loss Distributions (Appendix 2)

The basis of this method is published in a paper entitled "The Calculation of Aggregate Loss Distributions From Claim Severity and Claim Count Distributions" published in 1983 (Reference 11). The method works by convoluting the severity distributions of individual claims. This is achieved by the use of characteristic functions and then inverting the resulting integral by means of numerical integration techniques as described in the paper.

This gives a powerful and practical tool for calculating probability points on the aggregate claim distribution together with excess pure premiums (that is, stop loss risk premiums). Furthermore, the method allows aggregate distributions to be calculated for the combination of a number of lines of business.

Once the method has been set up on a computer, it is quick to use. For example, it is easy to amend the severity distribution to allow for changes in retentions and then recalculate the aggregate claim distributions. By reading off the sizes of aggregate claims at various retentions and probability levels, the effect of various retention strategies can be assessed.

The assumptions underlying the method are:-

Claim Count Distribution

The method can be constructed on a Poisson, Binomial or Negative Binomial claim count distribution. The distribution is, thus, described by two parameters, namely, the expected number of claims and the contagion or contamination parameter. If this second parameter is zero then the Poisson distribution is assumed. If it is positive then we have the Negative Binomial or Polya distribution and if it is negative, then we have the Binomial distribution.

Use of positive contagion is helpful in practice as it makes some allowance for non independence of claims, that is, a higher than expected number of claims in one period can increase the expected number of claims in a future period.

Claim Severity Distribution

The method requires a cumulative probability distribution that is piecewise linear. This results in a great deal of flexibility because any distribution can be represented to any desired degree of accuracy by increasing the number of points in the approximation.

In contrast to the recursive method (Section 4.5), this approach does not require equally spaced intervals. The approach facilitates the use of empirical distributions as exhibited by the underlying data without the need to fit a standard distribution.

The analysis of claim severity is relatively straightforward. In practice, though, it is often helpful to pay special attention to the upper tail of the distribution. In most cases, use of a distribution fitted only to the largest claims can be of value, particularly when coupled with an examination of the underlying claims process and exposures.

Parameter Uncertainty

In practical situations, parameter uncertainty can far outweigh the variation that can occur from randomness within known frequency and severity distributions. The Heckman and Meyers' approach can reflect both sources of variability by introducing a mixing parameter which has an Inverse Gamma distribution and is applied to rescale the claim severity distributions, increasing the level of variability. The effect of this parameter may be removed from the method by setting it to zero.

4.4 A Simulation Method for Retention Determination (Appendix 3)

The essence of the method is to simulate both gross and net aggregate claims distributions in order to assess the effectiveness of different reinsurance programmes. Here a retention is defined as in Section 3 to be everything that is not ceded.

Simulation is very flexible and facilitates the examination of the distribution of claim costs on a per claim, per event or per year basis. Even if the probability distribution of the severity of an individual catastrophe claim is a standard one that can be treated analytically, the distribution of the aggregate annual catastrophe costs to an insurer can be very complex.

Some of the alternative methods used for calculating aggregate claims distributions rely on assumptions such as the independence of individual claims. There are many instances in general insurance where such an assumption is invalid. A strength of the simulation approach is that it does not require this assumption. All this work is based around the use of simple spreadsheet models on a personal computer.

Any random variable with a known density function can be simulated provided that random samples from the uniform distribution over the unit interval (0,1) are available. (U(0,1) random variables) The practitioner can therefore define any empirical distribution for gross claims. Similarly, the effects of most reinsurance programs on the gross claims can be defined parametrically.

The example given in Appendix 3 considers all aspects of a model for UK property catastrophes. The limitations of the analysis are as important as the results themselves. In particular, the use of the standard deviation as a variability measure needs investigation.

The simulation in Appendix 3 depends on claim distribution assumptions. Claims are, of course, the result of random events such as hurricanes. Models can be built for catastrophes where the underlying natural phenomena themselves are simulated, and a separate stage is required to calculate the impact of the event on the insurer. This allows the modeller to use larger and more credible data, such as meteorological records, and thus improve the reliability of the simulations.

A particularly fine example of this, in our opinion, is a methodology for estimating US windstorms claims described in "A Formal Approach to Catastrophe Risk Assessment and Management" by Karen M. Clark (Reference 7) contained in the 1986 Casualty Actuarial Society discussion paper programme.

In this model, windpaths are represented by frequency and severity probability distributions which vary by location. The derivation of these distributions depends on an understanding of the dynamics of hurricanes and the use of historical meteorological data.

Insured properties are classified by location, age and structure. The connection between the windstorm and insured risks made by applying damage and vulnerability factors to the insured values. These factors are based on engineering studies.

Monte Carlo simulation is then used to produce two thousand years of experience. Each simulation results in a hurricane severity at each location (which is zero if the hurricane does not reach the location). The combination of simulated severities and insured values produces simulated claims at each location. Aggregated claims for each simulation gives a distribution of catastrophe claims.

The methodology has certain attractive features. It combines a practical understanding of meteorology, of engineering and of the distribution of insured risks and it has particular value where historical claim experience is limited or where external factors (for example, climatic changes) are considered important. The method does, however, require the insurer to maintain an extensive and detailed exposure database.

4.5 The Recursive Method for the Calculation of Aggregate Claim Distributions (Appendix 4)

The objective of the method is to estimate the aggregate claims generated by an insurance portfolio. The approach is to assume the aggregate claims can be represented as the sum of a number of individual claims where the number of claims is, itself, a random variable. The aggregate claim distribution can be calculated directly from a straightforward recursive formula.

To make the model more tractable, two assumptions are made:-

1. The individual claim severities are identically distributed random variables.
2. The number of claims and the individual severities are independent random variables.

If the mass function assumed for the claim frequency is of the type where successive values are related by a recursive relationship (Reference 1 eq^n 2.9.13) then the formula is easily manipulated. The model is referred to as the Collective Risk Model in risk theory. In the special case where number of claims has a Poisson distribution, claims are said to have a Compound Poisson distribution.

The mass function of the aggregate claims can be found by direct numerical calculation if the severity distribution of individual claims is a discrete equi-distant distribution according to which only the values

$$Z_i = iZ_1 \qquad i = 1, 2, 3 \ldots$$

can occur. In the simplest case, this reduces to a subset of the natural numbers.

The required aggregate claims mass function can then be calculated using the recursive formula (Reference 1). The effects of different per risk retentions are reflected in the distribution selected for the individual claim severities. Repetition of the calculations with different retentions facilities a comparison of the effects of these retentions on the aggregate claims distribution.

Section 5

GLOSSARY OF TERMS

Aggregate claim distribution - The distribution function of total claims during the specified period for example, a year.

Annual aggregate stop loss - A reinsurance cover capping the aggregate claims incurred in a period.

Coefficient of variation - The ratio of the standard deviation of a random variable to its mean.

Convolution - The combination of the density functions of two or more random variables to yield the density function of the combined variable.

Deductible - The amount of risk retained below the attachment point of a reinsurance cover.

Density function - The function representing the probability mass of a continuous random variable.

Distribution function - The function representing the cumulative probability mass of a random variable.

Drop-down cover/Top and drop - Excess of loss reinsurance cover with flexible attachment points and limits.

Financial reinsurance - Reinsurance where the quantum of recovery is known and only the timing of payment is uncertain.

LMX - London Market Excess, that is, reinsurance of a London Market reinsurer.

Mass function - The function representing the probability mass of a discrete random variable.

Per risk excess - Excess of loss reinsurance for individual insured risks.

Polya - An alternative name for the Negative Binomial distribution.

Probability of ruin - The probability that the free reserves of an insurer are exhausted.

Profit centre - An individual unit within an organisation with separate financial objectives.

Reinstatement - The process of replacing an excess of loss reinsurance once a claim has been made.

Unbalancedness - The degree of fluctuation inherent in the profitability of a portfolio of business.

Section 6

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