The Actuarial Profession
making financial sense of the future

Reserving Risk and the Impact on the Mean
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Reserving Risk and the Impact on the Mean

Contents

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Introduction

- Often actuaries are required to estimate the “mean of possible outcomes”
- This presentation discusses the impact of both process and parameter uncertainty upon the mean estimate **not** just the spread of potential outcomes
- The presentation considers this impact in a “Bayesian world”
- The implications of both GRIT and the ICA work => more consideration of process and parameter uncertainty within actuarial estimates
- We have put forward a “devil’s advocate” case that the impact of considering uncertainties surrounding actuarial modeling should result in actuaries increasing their mean selections
- We have shown both some practical examples and theory supporting these examples

Parameter Uncertainty - Introduction

- Most of the distributions we deal with are right skewed.
- These distributions do not necessarily behave intuitively.
- The relationship between the parameters and the mean of these distributions is highly nonlinear.
- Small changes in these parameters can result in large movements of the resultant mean.
- By their very nature there is more room for upwards (adverse) movement than downwards movement.

Parameter Uncertainty – Example 1

Example 1 – Gamma Distribution

- If we assume $X \sim Gamma(\alpha, \beta)$ with both fitted mean and standard deviation of 1,000.
- $E(X) = \dfrac{\alpha}{\beta}$
- This has the parameters $\alpha = 0.001, \beta = 0.000001$.
- Let us assume that the parameters are equally likely to be 5% higher, 5% lower than the fitted parameters.

Parameter Uncertainty - Example 1

Example 1 cont. – Gamma Distribution

- Gamma Distribution Means with different parameters.

		Alpha		
		-5%	0%	+5%
	-5%	1,000	1,053	1,105
	0%	950	1,000	1,050
Beta	+5%	905	952	1,000

- This has an overall mean of 1,002 > 1,000.

Parameter Uncertainty – Example 2

Example 2 – Lognormal Distribution

- If we assume $X \sim LogNormal(\mu, \sigma)$ with both fitted mean and standard deviation of 1,000.
- $E(X) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$
- This has the parameters $\mu = 6.561, \sigma = 0.833$.
- Let us assume that the parameters are equally likely to be 5% higher, 5% lower and the fitted parameters.

Parameter Uncertainty – Example 2

Example 2 cont. – Lognormal Distribution

- LogNormal Distribution Means with different parameters.

		Mu		
		-5%	0%	+5%
	-5%	696	967	1,342
	0%	720	1,000	1,388
Sigma	+5%	746	1,036	1,438

- This has an overall mean of 1,037 > 1,000.

Parameter Uncertainty Theory

- Let us now look at this on a more technically robust basis.
- Taking example 2 from above (LogNormal) with a fitted mean and standard deviation of 1,000 we shall now examine the effect on the mean of a specific fit.
- Firstly we simulate 18 independent values from the LogNormal with the fitted parameters previously given.
- The mean of our sample happens to be 996.

Parameter Uncertainty Theory

- Bayes Theorem states:
  $f(\mu,\sigma|x_i) = L(x_i|\mu,\sigma)\,g(\mu,\sigma)$
- Where f is the posterior distribution, L is the likelihood and g is the prior distribution.
- The log of our observations is Normally distributed, so assuming the Jeffreys prior we get:
  $$f(\mu,\sigma|y_i=\ln(x_i)) \propto \frac{1}{(\sqrt{2\pi})^n\sigma^n}\exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i-\mu)^2\right)\times\frac{1}{\sigma^2}$$

Parameter Uncertainty Theory

- Using the following hierarchy it is easy to use Monte Carlo simulation to look at the effect of this parameter uncertainty on the mean:

$$\sigma|x_i \sim \frac{1}{\sqrt{Gamma\left(\frac{n-1}{2},\frac{2}{\sum_{i=1}^{n}\ln(x_i)^2-\frac{1}{n}\left(\sum_{i=1}^{n}\ln(x_i)\right)^2}\right)}}$$

$$\mu|\sigma,x_i \sim Normal\left(\frac{1}{n}\sum_{i=1}^{n}\ln(x_i),\frac{\sigma^2}{n}\right)$$

Parameter Uncertainty Theory

- The effect on the mean in this case is to give us a mean of 1,066.
- In this example if we fit the parameters by method of moments, (ie unbiased sd) we obtain a mean of 1,016.
- If we fit the parameters by ML, (ie biased sd for Nomal) we obtain a mean of 980.
- By modelling parameter uncertainty we obtain a mean greater than that obtained through merely fitting the parameters.

Impact on Reserve Risk

- For simplicity let us consider a single development period of a simple triangle:

	Dev 1	Dev 2	Development Factors
Year 1	10,000	20,000	2.00
Year 2	10,000	30,000	3.00
Year 3	10,000	40,000	4.00
Year 4	10,000	50,000	5.00
Year 5	10,000	25,000	2.50
Year 6	10,000	30,000	3.00
Year 7	10,000	35,000	3.50
Year 8	10,000	40,000	4.00
Year 9	10,000	45,000	4.50
Year 10	10,000	50,000	5.00
Year 11	10,000		
Volume Weighted			3.65

Impact on Reserve Mean

- If we start by making assumptions akin to that for the ODP model, namely:

$$E(C_{i2}) = (\lambda - 1)D_{i1} \text{ and}$$
$$\mathrm{Var}(C_{i2}) = \phi(\lambda - 1)D_{i1}.$$

- We will now assume that the model is LogNormal with these as the means and variances.
- The log of the incremental claims over the cumulative claims in the previous development period are thus distributed as a normal distribution with parameters:

$$\mu_i = \ln(\lambda - 1) - \frac{1}{2}\ln\left(1 + \frac{\phi}{(\lambda - 1)D_{i1}}\right) \text{ and}$$
$$\sigma^2 = \ln\left(1 + \frac{\phi}{(\lambda - 1)D_{i1}}\right)$$

Impact on Reserve Mean

- Due to our careful choice of cumulative claims in the first development period, these are i.i.d.
- Our claims distribution assuming a Jeffreys prior as before is:

$$\sigma | x_i \sim \left(Gamma\left(\frac{n-1}{2}, \frac{2}{\sum_{i=1}^{n} \ln\left(\frac{C_{i2}}{D_{i1}}\right)^2 - \frac{1}{n}\left(\sum_{i=1}^{n} \ln\left(\frac{C_{i2}}{D_{i1}}\right)\right)^2} \right) \right)^{-1}$$

$$\mu | \sigma, x_i \sim Normal\left(\frac{1}{n}\sum_{i=1}^{n} \ln\left(\frac{C_{i2}}{D_{i1}}\right), \frac{\sigma^2}{n} \right)$$

Impact on Reserve Mean

- The table below compares the ODP Bootstrap approach for this set of data with the method described:

	Volume Weighted CL	ODP Bootstrap	LogNormal
Mean	36,500	36,307	38,008
SD		1,009	18,323
5%		34,637	20,845
10%		35,040	23,104
20%		35,460	26,495
40%		36,055	31,825
50%		36,309	34,444
60%		36,578	37,609
80%		37,176	46,452
90%		37,573	54,305
95%		37,907	62,377
97.5%		38,298	75,146
99%		38,578	103,817
99.5%		38,783	127,367

Impact on Reserve Mean

- The bootstrap approach assumes that the selected mean is the mean of the distribution post parameter uncertainty.
- In our example, the lack of volatility from the ODP Bootstrap is derived from the fact that the expected incremental claims for the historic are backward looking. They are calculated by differencing the cumulative claims in period 2 and the cumulative claims in period 2 divided by the volume weighted development factor. This is clearly a limitation of the ODP bootstrap approach.
- Of more interest to us is the difference in the means of both approaches.
- Strictly the mean should increase when we factor in parameter uncertainty.

Process Uncertainty

- To illustrate the point with regards to process uncertainty I shall slightly modify the example data, so as to increase both the parameter and process uncertainties.
- This table highlights the differences

	Dev 1	Dev 2	Develpoment Factors
Year 1	10,000	**15,000**	**1.50**
Year 2	10,000	30,000	3.00
Year 3	10,000	40,000	4.00
Year 4	10,000	50,000	5.00
Year 5	10,000	25,000	2.50
Year 6	10,000	30,000	3.00
Year 7	10,000	35,000	3.50
Year 8	10,000	40,000	4.00
Year 9	10,000	45,000	4.50
Year 10	10,000	**55,000**	**5.50**
Year 11	10,000		
Volume Weighted			3.65

Process Uncertainty

- The table below compares the results for the example with greater uncertainty with that for less certainty.

	More Uncertainty			Less Uncertainty	
	Volume Weighted CL	ODP Bootstrap	LogNormal	ODP Bootstrap	LogNormal
Mean	36,500	36,240	42,038	36,307	38,008
SD		1,390	39,733	1,009	18,323
5%		33,924	17,036	34,637	20,845
10%		34,464	19,946	35,040	23,104
20%		35,068	23,274	35,460	26,495
40%		35,878	29,658	36,055	31,825
50%		36,281	33,194	36,309	34,444
60%		36,652	37,381	36,578	37,609
80%		37,373	50,922	37,176	46,452
90%		38,082	67,947	37,573	54,305
95%		38,509	86,602	37,907	62,377
97.5%		38,839	118,544	38,298	75,146
99%		39,217	194,323	38,578	103,817
99.5%		39,439	236,170	38,783	127,367

Other Issues

- The BF method does not hold up to scrutiny in the context of mean claims reserving. Clearly the expected value of the inverse of the factor to ultimate is not the same as the inverse of the expected value of the factor to ultimate.
- If the consideration of risk really should affect the mean then diversification credit will mitigate some of this.
- Model Uncertainty.
- In our examples we have assumed that the loss distributions are infinite in range, which is not true. Allowing for the curtailing of these distributions would offset some of the downside risk and reduce the increase in the mean.

Summary

- From the above Parameter and Process uncertainty lead to a potential increase in the mean estimate.
- Some of these results are often already allowed for :
  - Actuaries have traditionally loaded reserves where either the parameters are more uncertain or the class is known to be more unstable than the data suggests.
  - It would appear that this practice is justified on a statistical level.

Conclusion and Comment

- In order to illustrate the points we have made in this presentation we have taken liberties with both the data we have used and the techniques.
- Most actuaries, including us, are using some form of, mean invariant, bootstrapping to quantify the reserve uncertainty. However, although this is a generally accepted actuarial technique there are limitations with the approach.

Conclusion and Comment

- There don't appear to be many approaches which allow for effects as we have described.
- There appears to be some way to go before we have settled these issues.
- This presentation implies that the most important factor in terms of claims reserving and understanding the volatilities is the historic triangles. This is rarely the case.
- However, we hope that this presentation has been food for thought.

Questions and Discussion