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# RESERVES IN LLOYD'S AND THE LONDON MARKET

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## PART 1

### 1. *Introduction*

1.1 Lloyd's and the rest of the London market underwrite a significant part of the world's insurance and are a dominating influence on insurance world-wide. This paper looks at the data which is collected primarily for reserving purposes inside the market—or rather, the paper looks at some new and unusual but very simple ways of looking at that data. Experience is showing that these simple methods are proving to be more powerful aids to underwriters and auditors than was envisaged when they were first invented. They are graphical and hence easy to 'see'.

1.2 We shall start by describing some new work done by one author[1] (S. Benjamin) about four years ago to design a new method for Lloyd's to set its 'minimum reserving percentages' at the end of each year which syndicates must observe. The Audit Committee set up a small working party to investigate the application of the method more widely and asked certain underwriters and auditors to cooperate. The working party reported[2] that the method was worth further experimentation and it is now on a full experimental basis in the Lloyd's market.

1.3 There are two ways of looking at the method. The first way is to look at it entirely within the context of minimum reserving inside Lloyd's. The method may then be thought of, and used, as one which is both more easily justified and more flexible than the present method, requires only a minimal and trivial amount of extra calculation, and uses only the data which is already collected. The new method merely gives two figures to use where the present method gives one—an apparently trivial difference.

1.4 The second and alternative way of looking at the method is to recognize that it is of general application to almost any insurance data and that it introduces quite new powerful ideas of practical value. The spin-off from the introduction of the idea may well be of greater value than its original purpose. In order to encourage this wider use, the method is presented by way of a set of graphs prepared centrally each year which allows each syndicate to plot its own data on the same graphs for direct comparison.

1.5 A further report has been prepared[3], but not yet presented, which looks to the method in order to develop an interpretation of the insurance results as a 'return on capital'. We shall explore this later in the paper.

1.6 In fact there were two methods invented. The second, which is also very simple and visual, was not explored in detail but appears to have considerable potential and we discuss it later.

1.7 Following the description of the work for Lloyd's we shall show how we widened its application yet further, taking advantage of its potential to analyse any data which is in the form of a run-off triangle.

1.8 We shall describe how this complemented work we had been doing for a number of years on reserving other London Market accounts. Furthermore we have been able to use and refine ideas put forward by Mr David Craighead in his papers[4][5] to the Institute of Actuaries, resulting in a complete computerized system which we use to evaluate statistically London Market claim reserves and to monitor their development over successive periods. This system is fully operational on our main frame computer; it has been used very many times; it is stable—using the computer jargon—and there is a User's Manual.

2. *The proposed method*

2.1 As stated above, the new proposed method uses two figures instead of one. An actual example will make this clear.

2.2 In one instance when the present method stated that the minimum reserve should be 78% of premiums the new method gave the two figures 3·4 and 33%. The calculations are illustrated and can be compared as follows. Suppose that the paid loss ratio to date is, say, 10%.

*Present method*

| | | |
|---|---|---|
| Paid Loss Ratio | = 10% | |
| Reserve | = 78% | |
| (Implied) Ultimate Loss Ratio | = 88% | |

*New method*

| | | |
|---|---|---|
| Ultimate Loss Ratio | = 3·4 × Paid Loss Ratio + 33% | |
| | = 3·4 × 10% + 33% | = 67% |
| Paid Loss Ratio | = | = 10% |
| (Implied) Reserve | = | = 57% |

In this particular case the reserve under the present method was 78%. Under the new method it would have been 57%. If the paid loss ratio to date had been 20% the new method would have imposed a higher minimum reserve.

3. *Visual patterns*

3.1 We look at data in order to see patterns in it. We want to turn the data into information. The data relate to the past and we hope the discovery of patterns will help to assess the likely future. It is useful to see what the continuation of past patterns would imply; it provides a benchmark for considering the future. Persons experienced in the field can then apply judgement as to whether the future is likely to diverge from the benchmark, or even whether the benchmark should be thrown away as useless or misleading.

3.2 In the insurance world there is a lot of data, and there is a lot of people looking at that data; that part of the insurance world known as the London Market is no exception. This paper is about ways of seeing patterns in data which are typical of the London Market, although, of course, the same methods can be applied to data from any other insurance market.

3.3 The human eye is a very powerful instrument for absorbing information. That is why visual and graphical displays are so useful. The methods we show here are graphical and the ideas are simple but we know they are unusual and a little strange at first to practitioners in the London Market.

4. *The data*

4.1 Risks are underwritten in the London Market. The cover given is usually for one year. The premiums are received over a period of typically three years. The incidents which take place during the year of cover give rise to claims which may not be reported for many years and then may take several years to be settled. The main reason for these delays is that the London Market tends to deal in reinsurance where the information is 'second-hand' in the sense that it comes from a primary insurer who may himself be subject to delays of information. For example the primary insurer may have a portfolio of risks covering compensation for personal injury—a class of happenings which notoriously take years to settle. The primary insurer may have taken reinsurance in the London Market to cover the situation where his total claims exceed a certain named limit, i.e. a 'stop-loss'. The reinsurer may not hear anything until the primary insurer's claims reach the agreed limit. The final outcome for the reinsurers in the London Market may then take a long time to become fully known.

4.2 The insurance data available in the market is also affected by its constitution. The two key figures in the market are the underwriter who uses his judgement formed from many years of practical experience to select and rate the risk, and the broker who places the risk for the insured. The risk will often (always at Lloyd's unless it is United Kingdom Motor) be placed on a coinsurance basis, often with 20 or 30 different underwriters. Detailed data may be available to the leading underwriter, but that detailed information may not be available to others on the risk and will not be recorded centrally. In view of the necessity to draw up correct Accounts, it is accounting data only which is commonly available. This of itself severely limits the choice of an actuarial method of 'looking' at the data, a point to which we return later.

4.3 In the case of Lloyd's the data, which is collected centrally, consist of premiums received and claims paid, both net of reinsurance. If we consider a particular 'year of account', i.e. all the risks written in a particular calendar year, we can see that until all the premiums are received and all the claims are known and paid, it is not possible to know the profit on that year of account. However, it is important to estimate each year the amount of the future liability on unpaid claims, both reported and unreported, in order to set aside enough money to meet that liability, and to release any balance of moneys in hand as profit.

4.4 Tables 1 and 2 show two typical sets of figures relating to a 'short-tail' account and a 'long-tail' account—the terms are self descriptive. Following tradition we describe the calendar year in which the risks were underwritten, i.e. the year of account, as Year of Development 1, the following calendar years as Years of Development 2, 3, etc. (For clarity we have not netted off premiums received against claims paid after Year of Development 3 which is common practice in Lloyd's.)

4.5 We give the cumulative figures from the start of each Year of Development. Also we show the 'Paid Loss Ratio' which is defined as:

$$\text{Paid Loss Ratio} = \frac{\text{Claims paid to date}}{\text{Premiums received to date}}$$

Table 1. *Short-tail cumulative*

| | Year of Development | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| Premiums received (£000) | 572 | 1119 | 1153 | 1153 | | |
| Claims paid (£000) | 291 | 793 | 999 | 1074 | | |
| Paid Loss Ratio (%) | 50 | 71 | 87 | 93 | | |

Table 2. *Long-tail cumulative*

| | Year of Development | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Premiums received (£000) | 584 | 699 | 740 | 743 | 761 | 771 | 783 |
| Claims paid (£000) | 4 | 78 | 190 | 255 | 548 | 670 | 734 |
| Paid Loss Ratio (%) | 0 | 11 | 26 | 34 | 72 | 87 | 94 |

| | Year of Development | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 8 | 9 | 10 | 11 | 12 | 13 | 14 | ..... |
| Premiums received (£000) | 787 | 789 | 810 | 813 | 814 | | | |
| Claims paid (£000) | 774 | 852 | 922 | 1014 | 1046 | | | |
| Paid Loss Ratio (%) | 98 | 108 | 114 | 125 | 129 | | | |

5. *Minimum reserves*

5.1 In order to maintain prudent discipline in the Lloyd's market, Lloyd's issues a list of minimum reserving percentages. The data which is collected centrally each year from the syndicates is grouped into about twenty audit categories e.g. Aviation Short-Tail. This data is used to determine and publish for each year of account, within each audit category, a certain percentage. That percentage is to be used by each syndicate. The premium income received for that year of account is multiplied by the audit percentage and the resulting amount is treated as a minimum figure for the reserve to be set up by the syndicate.

5.2 The list of minimum reserving percentages covers all past years of account. Hence, the syndicate data must be retained internally by year of account, even though in Lloyd's the portfolio is reinsured at the end of the third year of development, and each year thereafter.

5.3 The procedures for determining the minimum percentages from the central data are somewhat informal and once the percentages are set they are inflexible in application. The experience of a particular syndicate might differ quite considerably from the overall Lloyd's experience, but the minimum percentage remains the same. The terms of reference which started the research amounted to finding a more flexible method of setting the minimum reserves.

5.4 At the moment of writing, the proposed new method which is described below. has been tried on a limited experimental basis for three years. The evidence so far is favourable and the experiment is currently being widened to cover the whole market.

5.5 The method does not solve all problems, and is not a substitute for a proper assessment of reserves in individual cases, but it is a long step forward, and in our opinion the method has 'spin-offs' which are at least as valuable as the method itself. The potential further advantages are discussed later.

6. *The pattern of run-off*

6.1 Figure 1 shows the cumulative claims paid to date for years of account 1960 to 1971, it also shows two other curves indicated by dotted lines. These curves illustrate the fact that a very simple pattern can be generated which has a similar shape to the run-off pattern of the claims. They were calculated by thinking of the claims as births and their payments as deaths and applying the simple idea that the deaths in any year are always a constant proportion of those living at the beginning of the year. The numbers were calculated as in Tables 3 and 4.

6.2 Of course, in Tables 3 and 4 we started by knowing the total outstanding whereas in practice that is the figure we want to know. However it is clear that if we could fit the curve to data which had not fully run off it would indicate what the final run-off was going to be. The fact that a computer program might be needed to calculate the fitted curve is nowadays unimportant.

6.3 The rule of constant proportion leads straight to the fitting of a Simple Modified Exponential curve. Unfortunately, experience of fitting a variety of data sets has shown that this curve is a little too simple. (It is convergent only under fairly limited conditions which do not apply to much London Market data.) However its importance is that it provides a first level justification for fitting more complicated exponential curves to estimate ultimate claims. This is discussed later.

6.4 Another approach which the present authors have used successfully before[6] but have not explored with Lloyd's type data is as follows: Treat the

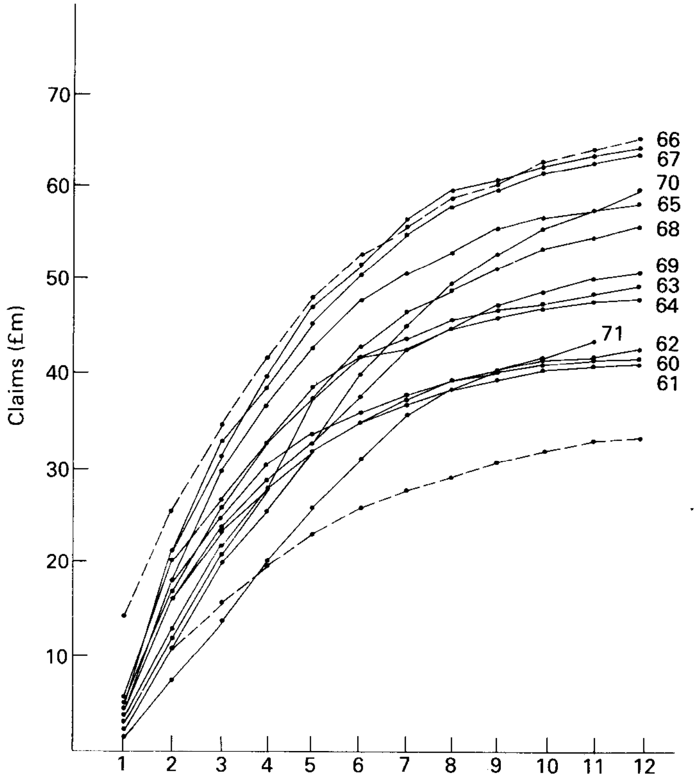


Figure 1. Non-Marine All Other cumulative claims for 1960 to 1971 years of account

Table 3. *£m*

| Year (1) | Unpaid (i.e o s + IBNR) at start of year 'living' (2) | 20% of (2) paid in year 'deaths' (3) | Claims paid to date i.e. (3) cumulative (4) |
|---|---|---|---|
| 1 | 70 | 14 | 14 |
| 2 | 56 | 11·2 | 25·2 |
| 3 | 44·8 | 9·0 | 34·2 |
| 4 | 35·8 | 7·2 | 41·1 |
| 5 | 28·6 | 5·7 | 47·1 |
| 6 | 22·9 | 4·6 | 51·7 |
| 7 | 18·3 | 3·7 | 55·4 |
| 8 | 14·6 | 2·9 | 58·3 |
| 9 | 11·7 | 2·3 | 60·6 |
| 10 | 9·4 | 1·9 | 62·4 |
| 11 | 7·5 | 1·5 | 63·9 |
| 12 | 6·0 | 1·2 | 65·1 |
| 13 | | | |

Table 4. £m

| Year | Unpaid (i.e. o/s+IBNR) at start of year 'living' | 15% of (2) paid in year 'deaths' | Claims paid to date i.e. (3) cumulative |
|---|---|---|---|
| (1) | (2) | (3) | (4) |
| 1 | 40 | 6·0 | 6·0 |
| 2 | 34 | 5·1 | 11·1 |
| 3 | 28·9 | 4·3 | 15·4 |
| 4 | 24·6 | 3·7 | 19·1 |
| 5 | 20·9 | 3·1 | 22·3 |
| 6 | 17·7 | 2·7 | 24·9 |
| 7 | 15·1 | 2·3 | 27·2 |
| 8 | 12·8 | 1·9 | 29·1 |
| 9 | 10·9 | 1·6 | 30·7 |
| 10 | 9·3 | 1·4 | 32·1 |
| 11 | 7·9 | 1·2 | 33·3 |

incident causing the claim as a birth (of a claim). It remains living and (inappropriately) 'healthy' whilst it is IBNR. When it is notified it becomes 'sick' and remains so whilst it is 'known and outstanding'. When it is finally settled it dies. Simple proportional rules for becoming sick and then dying seem to give an appropriate pattern.

7. *Recent years of account*

7.1 It is obvious that to fit a curve several points are needed on the graph, i.e. to fit a curve to the run-off data from a given year of account we need the run-off to have progressed for several years of development. The major financial problem is to deal with those relatively recent years of account, where the run-off has not yet progressed very far, and the outstanding liabilities are of financial significance. To deal with this problem we build on what we have so far by using a trick. It is a visual or graphical treatment designed to show if there is a pattern which can be used.

7.2 Let us take one category of business and look at several past years of account where we are fairly certain of the ultimate loss-ratio either because the year is early enough to have run-off almost fully or because it is early enough to allow the curve fit to take place as described above and hence to allow the curve to tell us the ultimate loss ratio.

7.3 For these years of account we can make a graph. We can make a graph for each year of development. Let us consider the graph for year of development 1. We can mark the paid loss ratio to year of development 1 along the horizontal axis and the corresponding ultimate loss ratio along the vertical axis. We can thus plot a point for each year of account. Figure 2 for the 'Marine Time' Account shows this.

7.4 We notice that the points lie in a straight path. We can 'formulize' this pattern by drawing a line through the points by eye, or by using the built-in

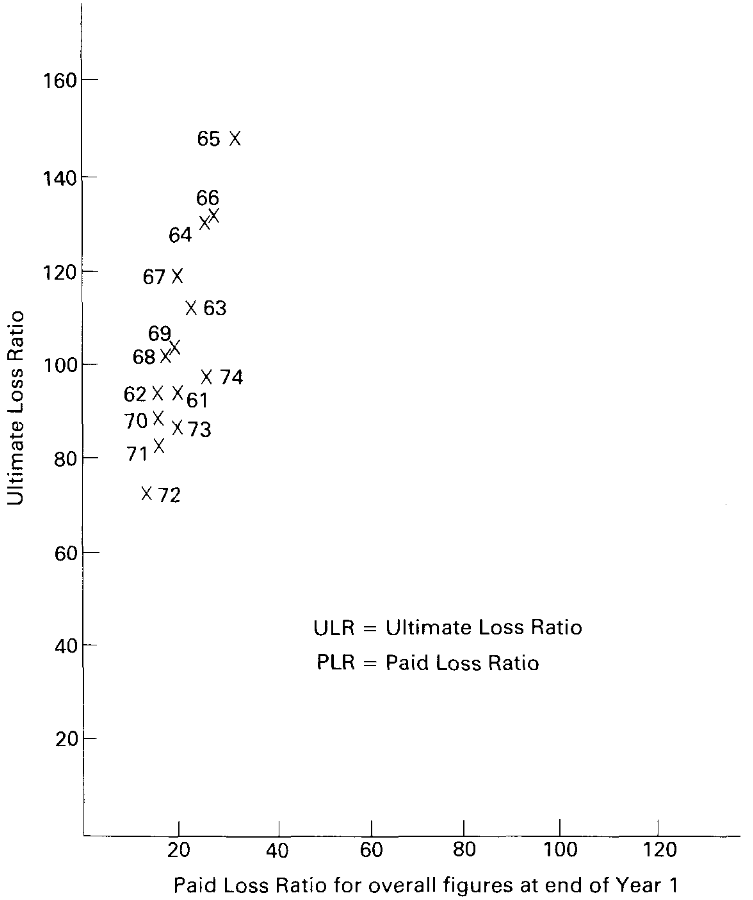


Figure 2. Marine Time Account.

program of many common modern cheap hand calculators, to find the line of 'best fit'. That line, which approximates to all the historical points turns out to be:

ultimate loss ratio = 3·4 × paid loss ratio + 33%
or ULR = 3·4 × PLR + 33%

and 3·4 represents the slope of the line, i.e. '3·4 in 1', very steep and 33% represents the intercept on the vertical axis. It is shown in Figure 3.

7.5 In fact, somewhat surprisingly, we can display the present minimum

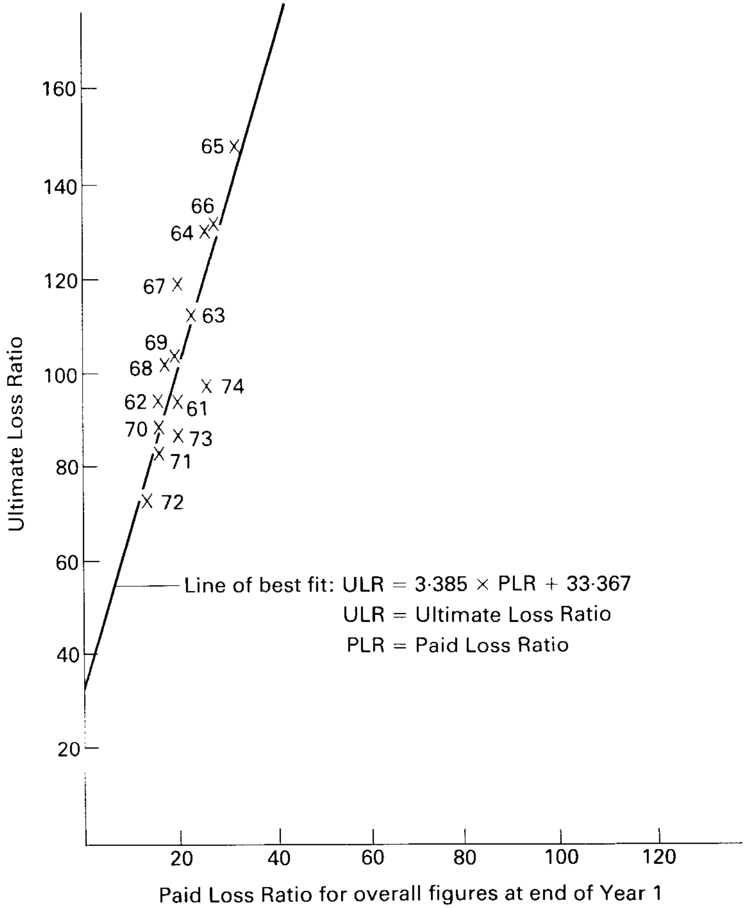


Figure 3. Marine Time Account.

reserving method on the same graph. The actual minimum reserving percentage in the above example was 78%. Now this implies that

$$\text{(implied) ULR} = \text{PLR} + 78\%$$
$$\text{i.e.} \quad \text{ULR} = 1 \times \text{PLR} + 78\%$$

which may be interpreted on the graph as a line with a slope of 1, i.e. '1 in 1' and an intercept on the vertical axis at 78%.

If we place the line used by—or implied by—the present method on the same graph as the historical line we obtain Figure 4.

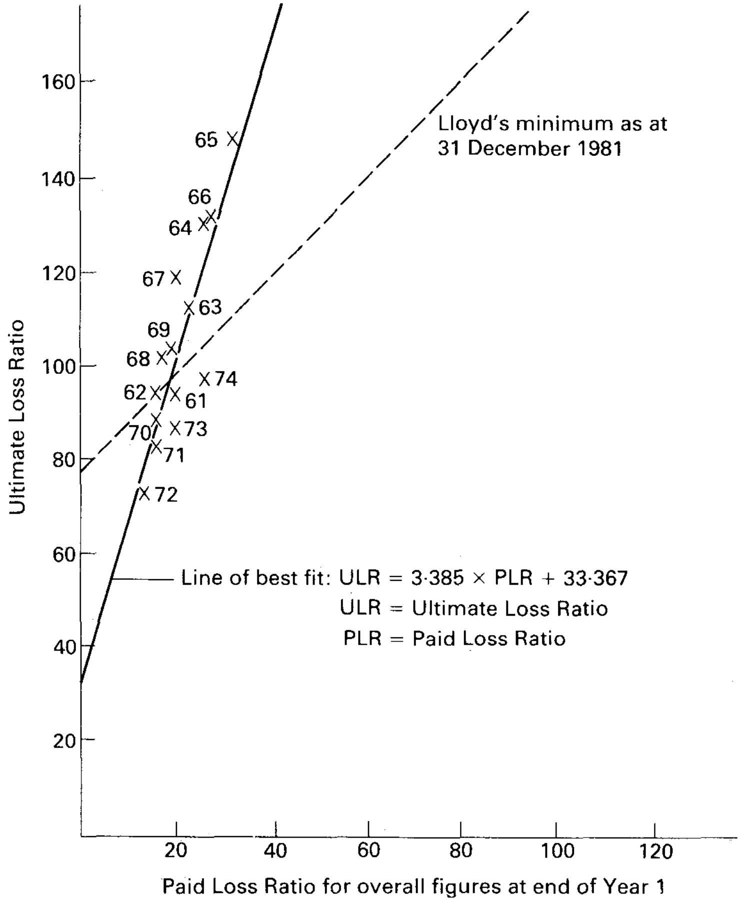


Figure 4. Marine Time Account.

In this chosen actual example the two lines are by no means the same, and it would seem a step forward to use the line which represented the history of what has actually happened rather than a line which seems to represent nothing in particular. The argument is not really affected by the fact that we do not normally think of the present method as a line on a graph.

*Variability*

8.1 Plotting the historical points on the graph allows us to introduce the idea of variability and the idea of a measure of variability. The concept of a measure of variability is important and useful because it allows us to make statements as to whether one value is close to another or far away. Under the present method the concept does not exist and no use can be made of it.

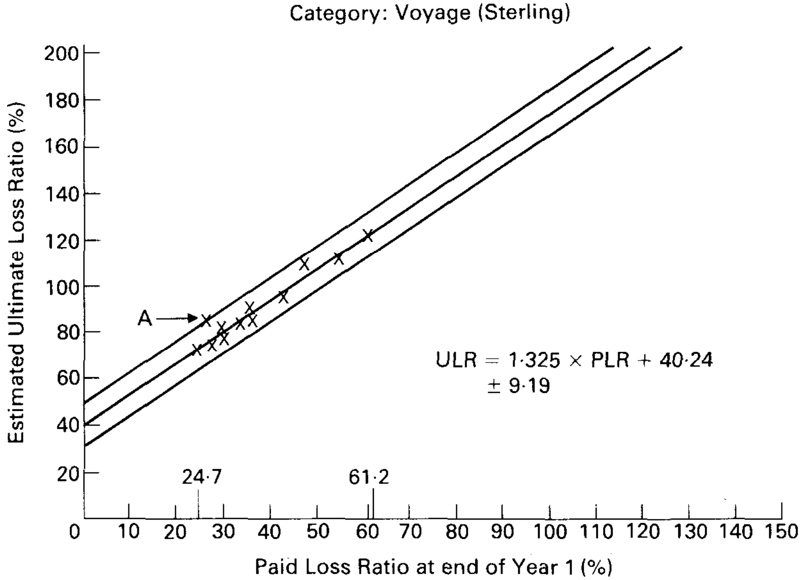


Figure 5

8.2 We recognize that the points do not actually lie on the line of best fit. We can draw a path about that line. A simple and useful path can be obtained by making it parallel to the line of best fit with the line of best fit running down the centre of the path. We can also arrange that one edge passes through the point which happens to be furthest away from the centre line. So the path is symmetrical about the central line of best fit and exactly encloses all the points. See Figure 5 where the point marked A happens to be the furthest from the centre line.

8.3 The width of the path is obviously a measure of the variability. If a new point is plotted from a new year or from, say, the data of a particular syndicate, and it lies inside the path then there is no point in chasing the reason why it is different from the other points. It is 'close' and there is no point in wasting time trying to explain any differences. On the other hand a new point which lies outside the path is interesting. It may be a warning of a change in the underlying experience. We return to this theme later.

9. *Comparison of present line with historical path*

9.1 In the earlier example where the line representing the present method is shown to look very different from the actual historical line the diagram is a little harsh. If we restrict ourselves to the range of actual historical paid loss ratios on the horizontal axis we can often see that within that range the two lines cross.

Furthermore, if within that range the line from the present method lies inside the historical path it could be said that the present method is close to the historical facts within the range of those facts.

9.2 This comparison was made for all the audit categories and all the years of account; the results varied. In many cases the present method's line was inside the historical path; in other cases it was not. However the historical line and its path appear to have all the advantages:

(i) By definition the line of best fit will always be closer to history than the present method.
(ii) If we needed to estimate an ultimate loss ratio when the given paid loss ratio lies outside the historical range experienced so far, then we would have more confidence in using the historical line than the line from the present method.
(iii) The path gives us a measure of closeness. The present method does not even contain the concept.

10. *The development of the path*

10.1 When the graph is drawn for the paid loss ratio at year of development 1 we would expect a fairly wide path. When we reach the year of development where the 'tail' of claims has effectively run off, the paid loss ratio will equal the ultimate loss ratio; the line of best fit will pass through the origin of the graph and

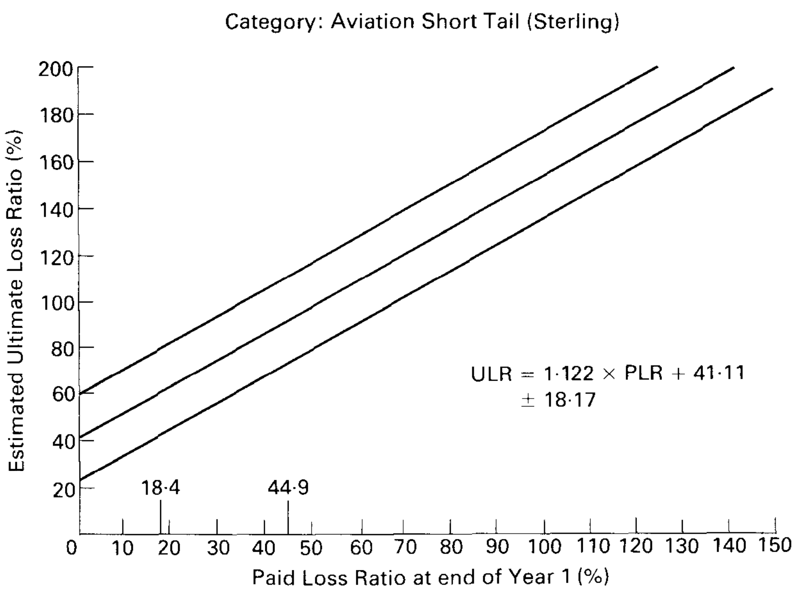


Figure 6

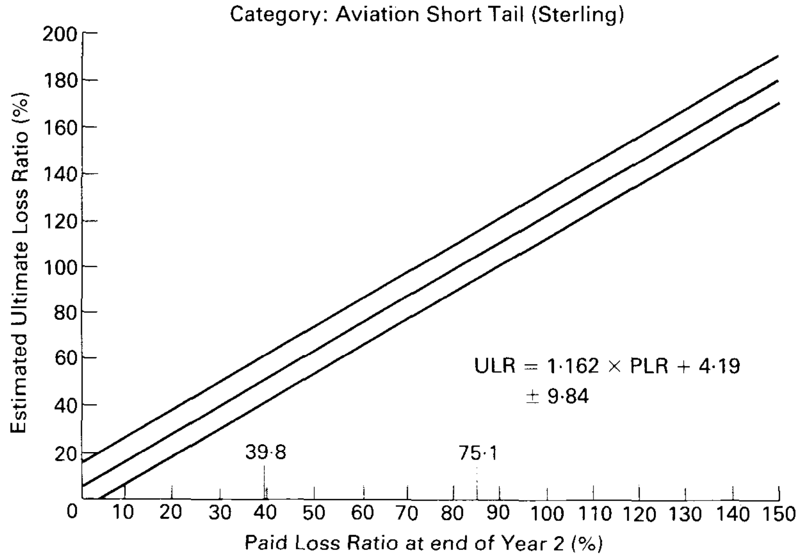


Figure 7

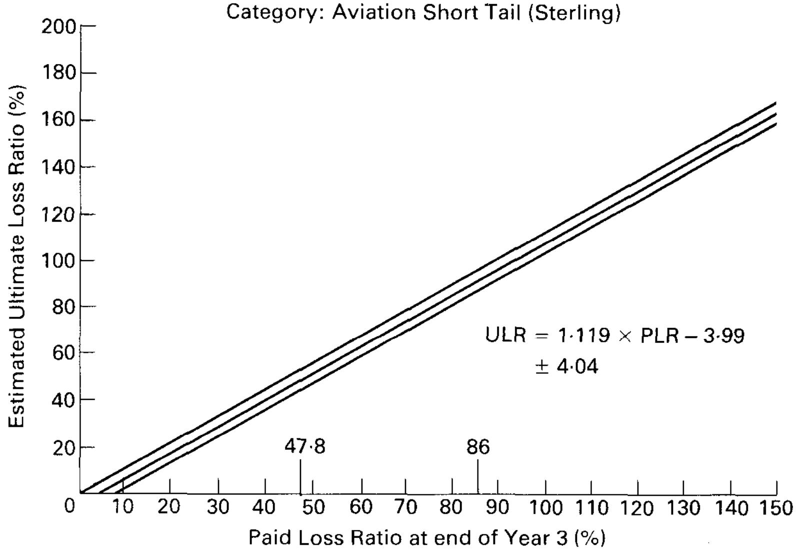


Figure 8

the slope of the line will be '1 in 1', i.e. 45°. The path will 'collapse' to a zero width because there will be no fluctuation left. For intermediate years of development we find an intermediate position. As the years of development progress we find the path gets narrower. See figures 6, 7, 8 and 9 which give the paths for years 1, 2, 3 and 4 on a short tail account.

10.2 On a very few occasions this does not happen as we move from one year to the next because the width of the path depends on a single point—the furthest. (The use of a confidence interval would be better; we use it in most of our work but for the market as a whole the extra sophistication is probably counter-productive.)

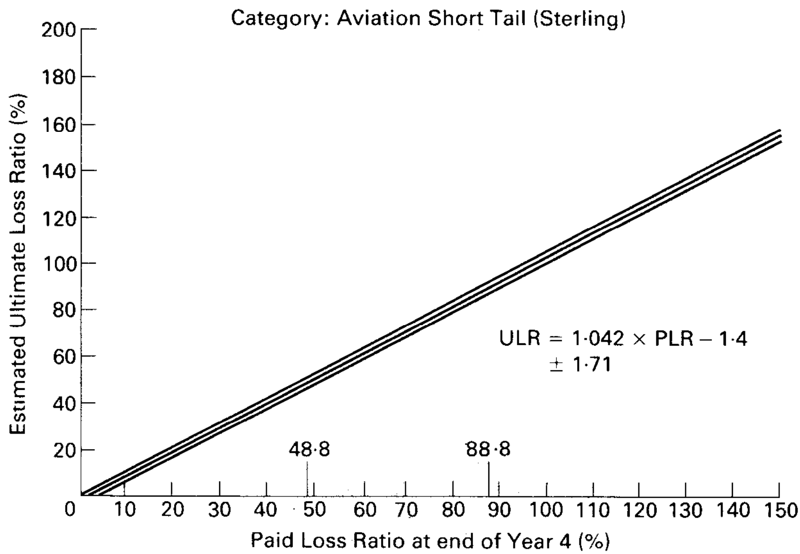


Figure 9

## 11. *Type of path*

It is useful to discuss the type of path displayed by the different categories of business under the headings of:

(a) slope: (i) positive, (ii) negative or (iii) zero (i.e. upwards, downwards or horizontal)

and

(b) width: (i) narrow or (ii) wide.

(a) (i) A positive slope shown in Figure 10 is the 'natural' case. It means that if the paid loss ratio for that year of development is higher than usual then the ultimate loss ratio is likely to be higher than usual.

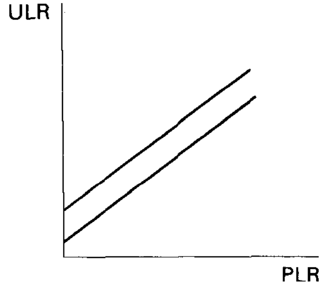


Figure 10

(a) (ii) A downward slope shown in Figure 11 is 'unnatural'. It has been suggested that it could happen through reinsurance recoveries and we have heard other explanations. Luckily the working party did not actually find any such cases because they imply an ultimate loss ratio being lower than the loss ratio paid to date—awkward for setting a minimum; a nil minimum reserve is probably best under such circumstances, i.e. you are not allowed to take a credit. An alternative which we would prefer is to suspect the data.

(a) (iii) A horizontal path as in Figure 12: this happens in the early years of development of many of the categories.

We do in fact test to see whether the calculated slope is statistically significantly different from zero and use a horizontal path where it is not. This situation implies that the paid loss ratio to date gives no useful information about the ultimate loss ratio. The best estimate is then the average of past ultimate loss ratios irrespective of the paid loss ratio to date.

(b) (i) A narrow path indicates that the ultimate loss ratio can be estimated very closely from the paid loss ratio to date. In one or two categories this was true even at the first year of development.

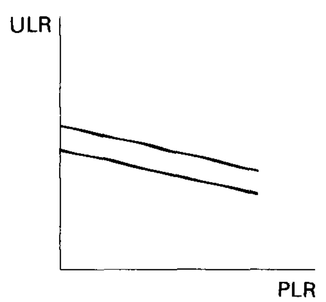


Figure 11

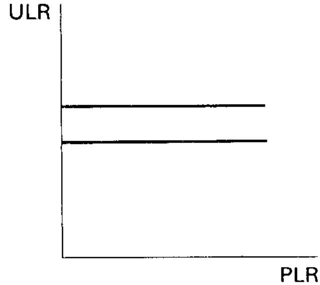


Figure 12

A horizontal narrow path means that the ultimate loss ratio is closely known—but it is irrespective of the paid loss ratio to date.

(b) (ii) A wide path indicates that the estimate of the ultimate loss ratio based on the paid loss ratio has a wide margin of error—*and that has been the historical nature of the business in that audit category*. Usually, but not necessarily, a wide path is also horizontal.

12. *Recommended minimum*

The first large scale experiment with the new method took place under the control of a small working party which reported to the Audit Committee. There was considerable discussion inside the working party before we decided to recommend the 'obvious' choice of the line of best fit as the line to use for the minimum reserve. The upper edge of the path seemed too high, the lower too low. The use of the line of best fit as minimum allowed one to say that the total reserves set up in Lloyd's were at least as great as the average indicated by past experience; that seemed to be a useful statement to be able to make.

13. *Simplicity*

There are three important features of the proposed method:

(a) It uses the same central data. It does not require collecting different data.
(b) The current annual booklet of minimum reserving percentages contains one figure for each year of account within each audit category. Under the proposed method all that happens is that each figure is replaced by two figures.
(c) The extra amount of calculation required to obtain the amount of the minimum reserve is trivial.

14. *The graphs*

14.1 However it is hoped that syndicates will not limit their thought horizons to the simplest features of the proposed method and graphs are being prepared centrally and issued each year for general use. There is one for each year of account within each audit category, showing for the all-Lloyd's data the line of best fit, the path, the end points of the historic range of paid loss ratios on the horizontal axis, and the equation of the line of best fit and the path, in the form

$$\text{ULR} = \text{A} \times \text{PLR} + \text{B} \pm \text{half 'width' of path.}$$

$$\text{e.g.} \quad \text{ULR} = 1{\cdot}12 \times \text{PLR} + 41{\cdot}1 \pm 18{\cdot}2\ (\%)$$

By plotting on the same graph the points from their own experience we hope that the underwriters will obtain useful information about their own experience as well as leading to a useful dialogue with the auditors.

14.2 Much of the time of the working party was spent examining the picture produced by plotting the results of individual syndicates on the all-Lloyd's graphs. Examples were found of all situations. The syndicate points could lie all within the all-Lloyd's path, often even when the paid loss ratios lay outside the range of the historic all-Lloyd's paid loss ratios. At the other extreme a syndicate's points could show an almost random scatter; that should lead to an interesting discussion with the auditor on the (ir)relevance of the audit minimum reserve in those cases.

14.3 The most interesting cases were those which showed a syndicate's own path to be narrow and different from the all-Lloyd's path. One syndicate found this to be the case in the category 'Non-Marine All Other'. The all-Lloyd's path was horizontal and very wide. The syndicate's own path was quite steep and very narrow. Their comment was that they had thought they were writing a consistent portfolio from year to year and that it was very different from the average in Lloyd's but this was the first proof they had. In fact one point, i.e. one year of account, was very different from the others as judged by the path, and the underwriter said he would look at that year of account but he would check first for any data errors!

## 15. *Standard background data*

An advantage of the new method, which is completely lacking in the present method, is that it allows an individual syndicate to look at its own data against the background of similar data gathered from the Lloyd's market as a whole. No actuary needs to be convinced of the value of a base set of data from which he can measure his own company's experience. The fact that a standard mortality table may fit his own experience only approximately does not prevent him from making very good use of the standard table. The situation may be shown graphically. Figure 13 represents typical data. One has no idea how to interpret it. Figure 14 shows the same data against a particular background which represents a large body of similar data. Figure 15 shows the same but with very different background data.

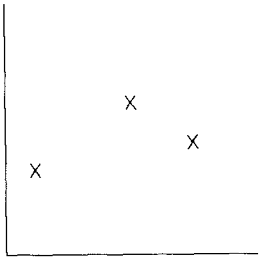

Figure 13

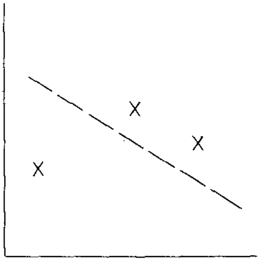

Figure 14

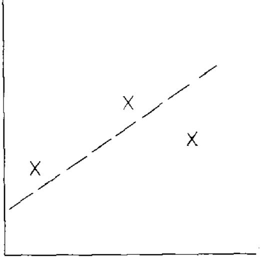

Figure 15

One's interpretation of one's own data would be different in the case of Figure 14 from Figure 15, e.g. one's expectation of how future 'points' might develop.

16. *Second method*

16.1 There is a second method which compares an individual syndicate's data more directly with all-Lloyd's data—or, for that matter, with any other set of data—in order to see if there is a useable similarity.

16.2 In this method the graph is drawn for a year of account rather than for a year of development. Along the horizontal axis are marked the paid loss ratios for all-Lloyd's data as the years of development progress until the ultimate position. Along the vertical axis are marked the corresponding paid loss ratios for the syndicate. If the points thus produced lie reasonably on a straight line then we can use that line to estimate the syndicate's ultimate loss ratio from the

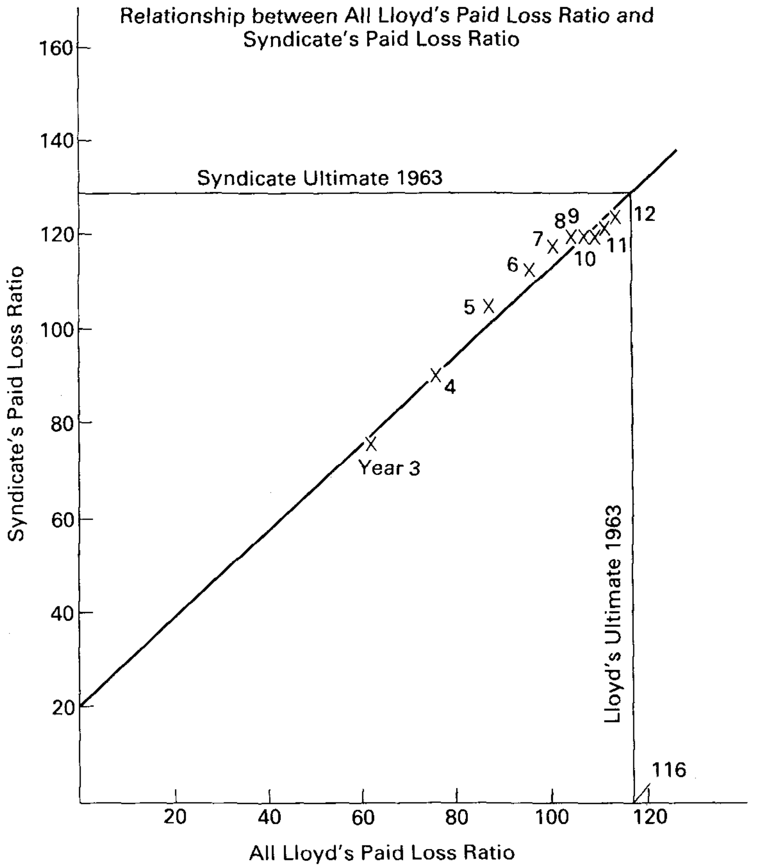


Figure 16. Non-Marine All Other account—combined currencies.

position where it cuts the all-Lloyd's ultimate loss ratio as in Figures 16 and 17.

This method is close to the actuarial use of a standard table. If the horizontal axis is marked up from any stable standard claims run-off which is likely to be of a similar shape to the experience under examination, then this method will show how the standard run-off can be amended to represent the experience being examined. It should be noted that a useful relationship can be shown which is not restricted merely to a constant proportion of the standard.

17. *Further central data*

17.1 The use of standard run-off tables and allowance for variability may be two steps forward in the techniques used by the Lloyd's market if the co-

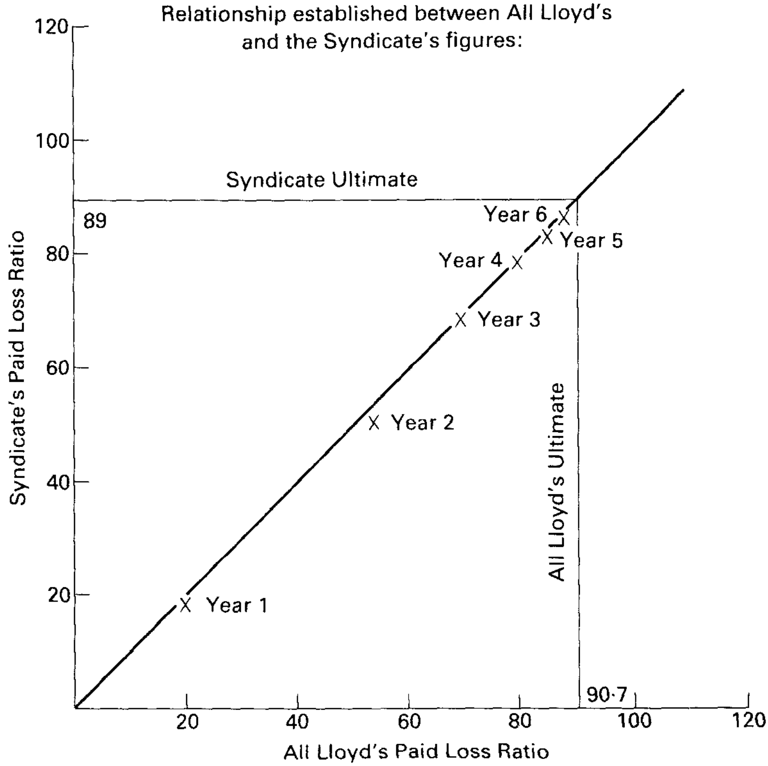


Figure 17. Marine Time All Risks 1975 year of account.

operation required for central gathering of data can be achieved. The technology is now available, and the use of the new method may help to dispel the suspicion of revealing any data even for central purposes which exists in the 'non-life' world.

17.2 The techniques presented here can be applied to gross data, net data, paid data, paid plus outstanding data. That is why we have not attempted to define closely the basis of the data used in this paper.

17.3 The methods are designed to see if there is a useable pattern in past data and to see if new data are following that pattern or varying from it. Hence new applications require the existence of a past body of data. In practice that may mean waiting until such data is gathered for a sufficiently long period of time. The alternative is to sample historical data, e.g. from the records of some of the larger syndicates.

17.4 Two further bases of data spring to mind. The first is 'paid plus outstanding' because if past judgement has been reliable then it should have shown as a narrower path i.e. as a better predictor than 'paid' alone in some categories. The second is the subdivision of categories which are heterogeneous, e.g. 'Non-Marine All Other'; that is shown by the appearance of a wide horizontal path.

18. *Solvency and year of development 0*

18.1 Following custom we have called the year in which the business is written, the year of development 1. Let us look at 'year of development 0', i.e. before the business is written. Can we see a pattern?

18.2 We cannot now use the 'paid' but we can look at the history of ultimate loss ratios themselves, using past years of account which are fully developed or nearly so, or where we are reasonably confident of using our curve fit to estimate the ultimate.

18.3 Figure 18 plots the ultimate loss ratios for one category for years of account 1961 to 1975. The data are in Table 5. Table 5 shows how the data are reordered in order to plot them on lognormal graph paper. The arithmetic is trivial; the work is done on special graph paper which can be purchased, preprinted, ready to use.

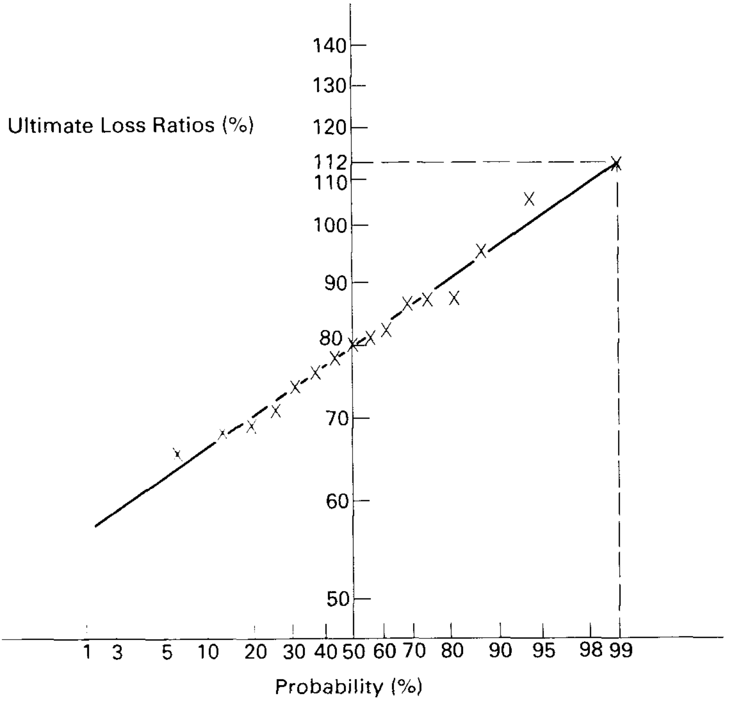


Figure 18

Table 5

| Year of A/C | ULR (%) | Position | *ULR in* ascending order (%) |
|---|---|---|---|
| 1961 | 74 | 1/16 = 6% | 65 |
| 1962 | 76 | 2/16 = 13% | 68 |
| 1963 | 82 | 3/16 = 19% | 69 |
| 1964 | 87 | 4/16 = 25% | 71 |
| 1965 | 96 | 5/16 = 31% | 74 |
| 1966 | 88 | 6/16 = 38% | 76 |
| 1967 | 86 | 7/16 = 44% | 78 |
| 1968 | 80 | 8/16 = 50% | 80 |
| 1969 | 81 | 9/16 = 56% | 81 |
| 1970 | 71 | 10/16 = 63% | 82 |
| 1971 | 68 | 11/16 = 69% | 86 |
| 1972 | 65 | 12/16 = 75% | 87 |
| 1973 | 69 | 13/16 = 81% | 88 |
| 1974 | 107 | 14/16 = 88% | 96 |
| 1975 | 78 | 15/16 = 94% | 107 |

18.4 The straight line pattern is unmistakeable. For some purposes, e.g. stop-loss at very high layers, the pattern may not be good enough, but compared with looking at the figures as presented 'naturally' in Table 5 the picture is quite startling.

18.5 On the graph, the horizontal axis represents probability. Hence if we wish to find the implied ultimate loss ratio which would be exceeded only once in a hundred times we can draw a vertical line at the 99% point, see where it cuts our historical pattern and read the corresponding ultimate loss ratio of the vertical axis—in this case 112%, as in Figure 18.

18.6 It could be argued that an insurer should not write that category of business unless he can show opening capital of at least 112% − 100% = 12% of premium income, i.e. his written premium should not be more than eight times (100/12) his opening capital. It is not the purpose of this paper to argue for a probability of 99%. The graphs show that a probability of 99·9% would require capital of 25% of premium income for the same category.

18.7 In traditional 'non-life' accounting that capital would be 'off balance sheet' capital—or perhaps better described as 'off revenue account' or outside the technical reserves (provisions). We can also see from the graph or from the figures themselves that the average historical ultimate loss ratio for this category has been 80%. From a historical point of view an ultimate profit of 20% of premium can be achieved on average in return for opening capital of 12% of premium.

19. *Return on capital*

19.1 Having introduced the measurement of capital required via year of development 0, can we comment on the 'off balance sheet' capital required at later years of development and, hence, how the initial capital can be released as the original year of account develops?

19.2 The answer seems to lie in the new graphs; the upper edge of the path for that year of development in that category, can be used to estimate a cautious value of the ultimate loss ratio instead of the line of 'best fit'.

19.3 Following the previous example the path for year of development 1 was in fact

$$\text{ULR} = 1{\cdot}1 \times \text{PLR} + 41\% \pm 18\%$$

The average paid loss ratio for all-Lloyd's was 34%. Hence before the business is written we might expect on average to find

| | | | |
|---|---|---|---|
| ULR on upper edge of path | = | 1·1 × 34 + 41 + 18 | = 96% |
| ULR on middle path | = | 1·1 × 34 + 41 | = 78% |
| Extra capital required | = | | 18% |

As with any investment the out-turn will be different from initial estimates, and the actual paid loss ratio will determine the capital requirement remaining at year of development 1, and similarly for other years of development.

The 'cash flow' over the first year, greatly simplified will look like this:

| | | |
|---|---|---|
| premium from policyholder | 100 | |
| solvency capital from insurer | 12 | |
| | | 112 |
| interest earned, say nil | | |
| claims paid, say nil | | |
| expenses and tax, say nil | | |
| reserve | 78 | |
| solvency capital from insurer | 18 | |
| | | 96 |
| surplus for the year | | 16 |

19.4 The above treatment makes no allowance for the fact that premiums are not fully received in the first year and that the first year of paid loss ratio is based on premiums received in the first year. Indeed it is too oversimplified to be useful other than to indicate the line of approach. That approach can be used for further years of development until the tail is fully run off.

19.5 'That approach' is in fact the modern actuarial method of analysis of premium rates as a 'return on capital'. The authors intend to show the practical detail in a further paper.

## PART II

20. The work for Lloyd's we have been describing is one aspect of our work on London Market reserves. This part of the paper considers the more general work in some detail, and in particular the computer system we have designed.

21. *System requirements*

21.1 The need to develop this system arose from a demand, which has been increasing slowly over the past ten years, for us to comment on reserves set up by companies writing marine, aviation and reinsurance accounts, or alternatively to advise on such reserves. Statistics have in fact necessarily tended to be subordinate to the Accounting function though we suspect this is also true of other markets. Hence there can sometimes be the problem that if an error is discovered in the statistics, e.g. premiums have been paid in Italian lire rather than U.S. dollars(!) it will be corrected from discovery, but the history will be left unchanged so that the statistics still reconcile with published Accounts. This of course distorts the run-off pattern. Further problems arise from the use of very broad risk categories which cannot be assumed to be homogeneous over time—the classic example of this is probably Non-Marine All Other at Lloyd's—and from the fact that concepts we take for granted elsewhere can be meaningless in this market—thus number of claims is not a useful concept if you are writing a catastrophe excess of loss treaty covering property damage exceeding $10 million in aggregate any one incident for a California company. Hence most of the reserving methods commonly in use break down. We needed a set of methods which

(1) Were able to cope with long tail business.
(2) Used only information on premiums, paid claims and claims outstanding as notified.
(3) Could provide estimates where there were missing items of information from the run-off triangle.
(4) Could handle multi-currency portfolios. Most of the companies whose reserving we examine write substantial U.S. dollar business even though they report in U.K. pounds, and this is of course also true of Lloyd's.
(5) Would enable us to set a range of values within which reserves would be acceptable. After all no single estimate can be correct unless we have business which has completely run off. We would expect in the early years of development of a year of account (which can either be a policy year or an accident year) that the range would be relatively wide and should reduce as development increases.
(6) Would enable the use of market information or information from other similar business to establish reserves for a particular insurer (or syndicate), where the data that insurers could supply was insufficient.

21.2 It was vital that the system should be able to cope with all the preliminary data handling, and would be flexible enough to allow the data to be looked at in a variety of ways. Data can be accepted in a variety of formats. The data can be either cumulative or incremental. Claims data can show paid claims and claims outstanding either separately or summed, and can be expressed either as loss ratios or cash. A number of time intervals are allowed from quarterly to annual. The system can accommodate several currencies, which can be combined or not at the user's discretion. When currencies are combined, uniform exchange rates are assumed to apply for all periods of origin and development. Data from up to 99 separate long and short tail categories can be accepted in any of the currencies, and again at user option any or all categories can be combined.

21.3 A major consideration underlying our whole approach is that for the classes of business we are considering standard assumptions, e.g. homogeneous account from year to year, standard pay out pattern from year to year, no change in speed of advice etc., would almost certainly all be violated. This suggested as a basic point of departure that we would examine the run-off of each year of origin separately. It also suggested looking at the development of loss ratios rather than losses. Empirical considerations suggested that if we were seeking a smooth curve to fit the shape of loss ratio at development time $t$, plotted against $t$, that curve would have a negative exponential shape.

22. *An outline of the method*

Ultimate loss ratios are estimated by a three stage process (it should be noted that Stage 1 whilst desirable is not essential—in fact it has to be omitted in working with the central Lloyd's data):

(1) We first estimate ultimate premiums. In practice we use the 'Chain Ladder' method. There is no reason why other methods could not be used in appropriate circumstances.
(2) We then fit a curve, for each account year separately to the run-off of loss ratios as a percentage of estimated ultimate premium.
(3) The run-off patterns derived in (2) are combined by applying a regression technique and we can then obtain estimates of ultimate loss ratios together with measures of the accuracy of those estimates.

This approach is explained below by means of an example based on typical Non-Marine Short Tail Data.

Appendix 1 contains computer produced tables and graphs relating to this example which are representative of the output produced by the system.

22.1 Estimating ultimate premiums

22.1.1 In this example, no premiums are received after development year four. The cumulative premium development for accounting years 1981 to 1984 is as follows:

| Development | Account year | | | |
|---|---|---|---|---|
| year | 1981 | 1982 | 1983 | 1984 |
| 1 | 1,350 | 1,480 | 1,550 | 1,600 |
| 2 | 3,280 | 2,750 | 3,440 | |
| 3 | 3,650 | 3,000 | | |
| 4 | 3,730 | | | |

The Chain Ladder method gives the following development factors:

$$\text{Development year 2: Development year 1} = \frac{3280+2750+3440}{1350+1480+1550} = 2{\cdot}162$$

$$\text{Development year 3: Development year 2} = \frac{3650+3000}{3280+2750} = 1{\cdot}103$$

$$\text{Development year 4: Development year 3} = \frac{3730}{3650} = 1{\cdot}022$$

These are then applied to produce estimates of ultimate premium. Since premium is being used as a scalar the accuracy of premium estimates is not too critical.

22.2 Estimation of loss ratios by curve fitting

22.2.1 In this example, there is little claim development after year 7 (see Appendix 1 for run-off table).

Account years are first made comparable with each other by dividing the cumulative claim figures by the estimated ultimate premiums to give a table of loss ratios. The method can be applied equally well to either paid loss ratios or incurred loss ratios, but in the example we consider paid.

Thus account year 1979 gives rise to the following ratios:

| Development year | Paid Loss Ratio (%) |
|---|---|
| 1 | 19·3 |
| 2 | 60·0 |
| 3 | 79·4 |
| 4 | 84·4 |
| 5 | 87·0 |
| 6 | 87·1 |

These are then plotted on a graph. See Figure 19.

22.2.2 The aim of the method is to find a family of curves which when fitted tc the above development graphs will satisfy two criteria:

(a) For the account years where the ultimate loss ratio is already known with a

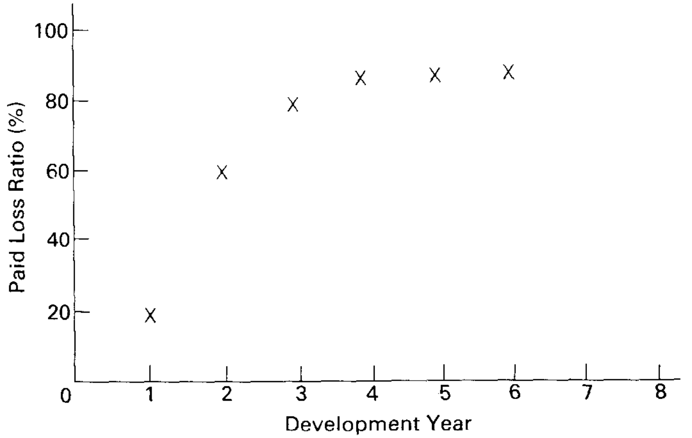


Figure 19

fair degree of certainty (in the example 1973 to 1978), the curve must level out at a value near that loss ratio.

(b) For later account years (in the example 1978 to 1981), the curve must fit the known data well and also allow for a reasonable amount of future development. (In most cases this will mean a development period similar to the more fully developed years.)

22.2.3 Thus the family of curves is used to give estimates of ultimate loss ratios for account years 1973 to 1981. For later years, not enough development has yet taken place for a satisfactory curve to be fitted.

22.2.4 Appendix 1 contains a selection from the graphs of the curves fitted in this example together with the developing loss ratios. Each loss ratio is represented by a vertical line. The general mathematical formula for the family of curves used in the example (which is the family of curves we use most often) was suggested by Mr David Craighead[4][5] in his papers to the Institute of Actuaries. The formula for the curve is shown in Appendix 1. As you will see it has a typical negative exponential shape which we have earlier shown by general reasoning to be acceptable. There are 3 parameters, A determines the ultimate loss ratio, B and C the length of tail. Fitting is by numerical methods using a more powerful variant of the Golden Section search. We have found other curves useful (the system has been designed to fit any curve)—on occasion the Simple Modified Exponential itself gives good results but the data has to be very smooth with the loss ratios monotonically increasing and their first differences monotonically decreasing. In practice we may need to place constraints on the curve fitting, for example it may be necessary to assume that the loss ratio reaches its ultimate value after a fixed period.

22.2.5 On occasions we have found that the graph produced by the computer does not suggest a smooth curve. Particularly when looking at incurred loss ratios we have found the development oscillates violently. An advantage of the system is that since it presents this in visual form it can be discussed with the underwriter, and the most common explanations we have found for the aberrant patterns are:

(i) Miscoding of data either by currency or category,

or

(ii) Data corrections which have not been carried back to the origin of the year of account,

so that the system is acting as a powerful check on the data.

22.3 Estimation of Ultimate Loss Ratios by 'Line of Best Fit'

22.3.1 We have now analysed the run-off one account year at a time. This section analyses the run-off by examining one development year at a time for all account years together. Thus we use all the information in the run-off triangle.

For example, at development year 2, we have the data of Table 6.

If these points are plotted we obtain Figure 20.

Table 6

| Account year | Loss ratio at development year 2 (%) | Estimated ultimate loss ratio from previous section (%) |
|---|---|---|
| 1973 | 55·2 | 78·9 |
| 1974 | 60·7 | 89·7 |
| 1975 | 72·4 | 95·8 |
| 1976 | 63·3 | 87·6 |
| 1977 | 58·8 | 78·6 |
| 1978 | 78·0 | 103·4 |
| 1979 | 60·0 | 88·1 |
| 1980 | 68·8 | 99·2 |
| 1981 | 70·6 | 102·9 |

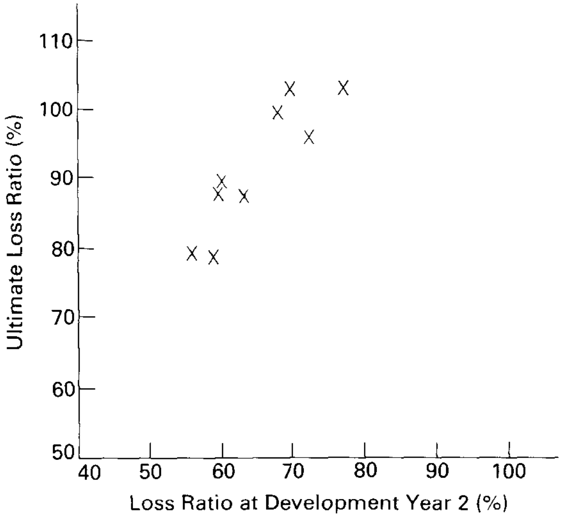


Figure 20

The plot is then examined to see if there is a statistically significant relation between the loss ratio at year 2 and the ultimate loss ratio. In practice we fit a regression line and test whether the gradient is significantly different from zero.

22.3.2 If we accept the hypothesis that the gradient is non zero, we can then use the regression line to estimate the ultimate loss ratio, given we know the loss ratio at year 2. Further we can construct a confidence interval around the line. We have found a 90% confidence interval does the right job for our analyses of individual portfolios. We also construct the path based on the historical point furthest from the regression line.

22.3.3 Rejection of the hypothesis implies no correlation between the loss ratio at year 2 (say) and the ultimate loss ratio. In this case we would estimate the ultimate as the average of historic ultimate loss ratios.

In the above example, the regression line fitted is:

Estimated ULR = 1·141 × Year 2 LR + 17·06% and is shown in Figure 21.

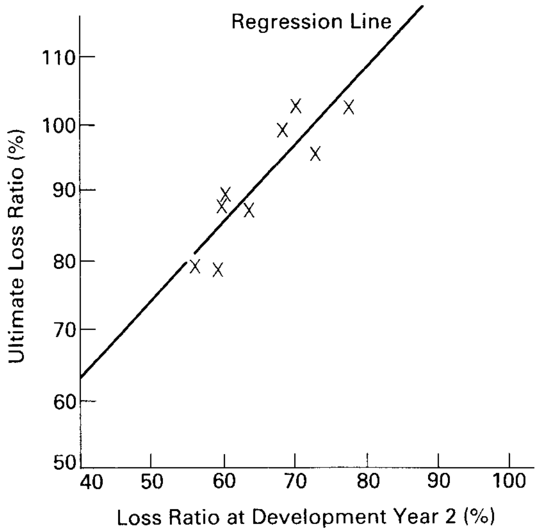


Figure 21

We can now estimate the ultimate loss ratio for 1983 (where year 2 is the latest known development ratio):

$$\begin{aligned}\text{1983 Ultimate Loss Ratio} &= 1{\cdot}141 \times 70{\cdot}03 + 17{\cdot}06 \\ &= 97{\cdot}0\%\end{aligned}$$

We have also calculated a 90% confidence interval which is + or − 8·3%. Thus we have a range of values within which we would expect the ultimate loss ratio to lie.

22.3.5 A similar approach may be applied to development year 3 giving an estimate of the ultimate loss ratio for 1982 of 101·7%. The full results produced using this method are as in Table 7.

The confidence intervals shown are those which apply within the range of historic values.

22.3.6 Sometimes special features of a particular account year, e.g. a rapidly changing account or the fact that very few years have reached the same point of development, may mean that we cannot use the regression line, so that we have to fall back on the estimates from curve fitting alone. We try to avoid this where possible since then no range can be quoted, even though we know there is a range of possible values.

22.3.7 The method we have adopted to avoid making estimates from curve fits

Table 7

| Account year | Latest known development year | Estimated ultimate loss ratio using regression technique (%) | Confidence interval (+ or −) (%) |
|---|---|---|---|
| 1978 | 7 | 103·3 | ·6 |
| 1979 | 6 | 87·7 | ·5 |
| 1980 | 5 | 98·0 | 1·2 |
| 1981 | 4 | 100·5 | 2·5 |
| 1982 | 3 | 101·7 | 2·8 |
| 1983 | 2 | 97·0 | 8·3 |
| 1984 | 1 | 80·5 | 16·8 |

alone, where very few account years have advanced to a particular development year, is to use the curve fit to provide estimates not only of ultimate loss ratios but also the expected paid or incurred loss ratios to each development year. These expected loss ratios for any development year can then be plotted against estimated ultimate loss ratios in exactly the same way as actual loss ratios. We can then derive a regression line and confidence interval. For some recent account years we have also had to adopt the approach where there is a large amount of fluctuation in the data for the early development years generally.

23. *Analysis of Experience*

One of the most powerful tools available to us is the comparison of actual with expected. In the case of a claims reserving system it is important to monitor how well the system is performing from period to period and the extent of adverse deviations. Further we can see if trends are developing which are likely to render the experience of current years of account different from that of older years. In our system we have a standard analysis printed out which compares the development of claims, premiums and estimates of ultimate loss ratio and IBNR over successive periods. Appendix 2 lists the output items monitored in the form of a glossary. It will be seen that all important items are monitored.

24. *Conclusion*

It will thus be seen that our work for London Market companies and Lloyd's has led to the development of a powerful system which is itself generating new approaches to analysis of reserves.

25. *Acknowledgement*

We wish to thank Mr Andrew English, M.A., A.I.A., for the application of the curve fitting algorithm and much programming.

## REFERENCES

(1) Unpublished summary presented to the Chairman of the Audit Committee also requesting a working party to investigate the methods.
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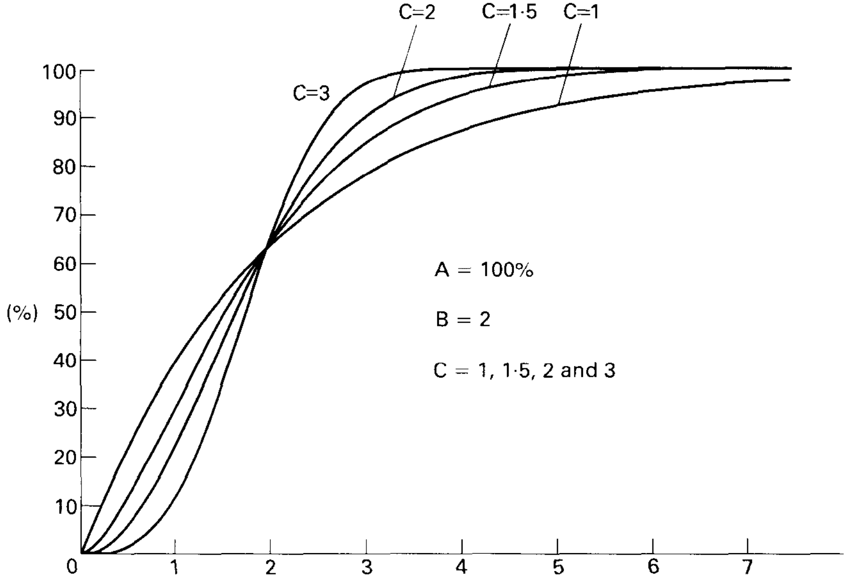
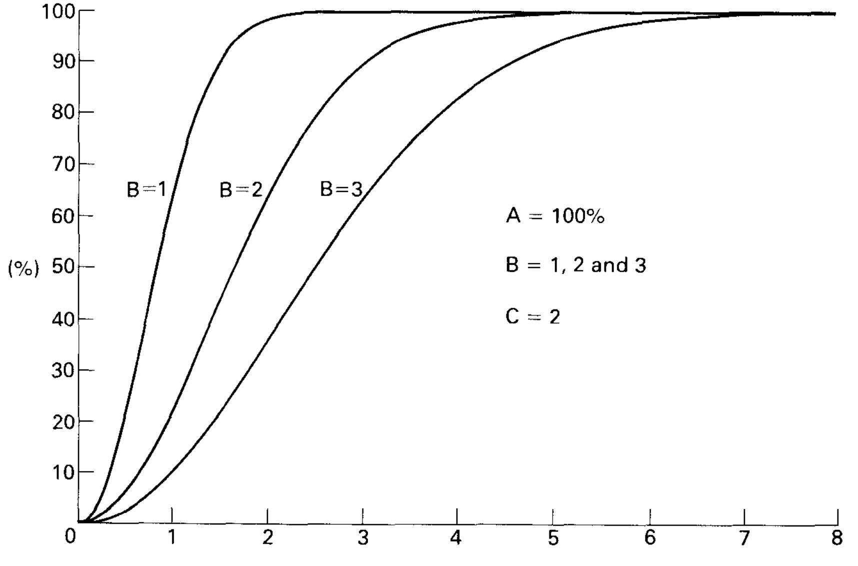
(3) Unpublished draft 'Report No. 2. Return on Capital and the Reserve to Close' December 1984.
(4) CRAIGHEAD D. H. (1979). Some aspects of the London reinsurance market in worldwide short-term business, *J.I.A.* **106,** 286.
(5) CRAIGHEAD D. H. (1981). Analysis of a London Market Reinsurance Offices Accounts, *J.S.S.* **26,** 1.
(6) EAGLES L. M. (1968). The actuarial assessment of a newly established motor insurance business, 18th *I.C.A.*, 415.

Example graphs to show how shape of curve fitted varies with changing parameters.

Formula for Curve: (as suggested by Mr D. H. Craighead)

$$L = A \times [1 - \text{EXP}\,(-[t/B]^c)]$$



RUN NO. 25 EXAMPLE RUN

TITANIC INSURANCE GROUP 13
SHORT TAIL DATA
PREMIUMS
AS RECORDED
CURRENCY GROUPS INCLUDED
£

ALL GROUPS EXPRESSED IN £

| Year | Quarter | 1973 | 1974 | 1975 | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4 | 600 | 650 | 700 | 730 | 800 | 950 | 1030 | 1200 | 1350 | 1480 | 1550 | 1600 |
| 2 | 4 | 1460 | 1340 | 1620 | 1540 | 1550 | 2060 | 2240 | 2380 | 3280 | 2750 | 3440 | |
| 3 | 4 | 1590 | 1490 | 1730 | 1740 | 1720 | 2300 | 2480 | 2600 | 3650 | 3000 | | |
| 4 | 4 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | 2540 | 2660 | 3730 | | | |
| 5 | 4 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | 2540 | 2660 | | | | |
| 6 | 4 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | 2540 | | | | | |
| 7 | 4 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | | | | | | |
| 8 | 4 | 1620 | 1520 | 1760 | 1780 | | | | | | | | |
| 9 | 4 | 1620 | 1520 | 1760 | 1780 | | | | | | | | |
| 10 | 4 | | | | | | | | | | | | |
| 11 | 4 | | | | | | | | | | | | |
| 12 | 4 | | | | | | | | | | | | |

RUN NO. 25 EXAMPLE RUN

TITANIC INSURANCE GROUP 13
SHORT TAIL DATA
PREMIUMS
CHAIN LADDER PREDICTION (WITHOUT INFLATION)
CURRENCY GROUPS INCLUDED
£
ALL GROUPS EXPRESSED IN £

DEVELOPMENT QUARTER 4

| Year | 1973 | 1974 | 1975 | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 600 | 650 | 700 | 730 | 800 | 950 | 1030 | 1200 | 1350 | 1480 | 1550 | 1600 |
| 2 | 1460 | 1340 | 1620 | 1540 | 1550 | 2060 | 2240 | 2380 | 3280 | 2750 | 3440 | 3459 |
| 3 | 1590 | 1490 | 1730 | 1740 | 1720 | 2300 | 2480 | 2600 | 3650 | 3000 | 3794 | 3815 |
| 4 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | 2540 | 2660 | 3730 | 3066 | 3877 | 3899 |
| 5 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | 2540 | 2660 | | | | |
| 6 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | 2540 | | | | | |
| 7 | 1620 | 1520 | 1760 | 1780 | 1760 | 2350 | | | | | | |
| 8 | 1620 | 1520 | 1760 | 1780 | 1760 | | | | | | | |
| 9 | 1620 | 1520 | 1760 | 1780 | | | | | | | | |
| 10 | | | | | | | | | | | | |
| 11 | | | | | | | | | | | | |
| 12 | | | | | | | | | | | | |

RUN NO. 25 EXAMPLE RUN

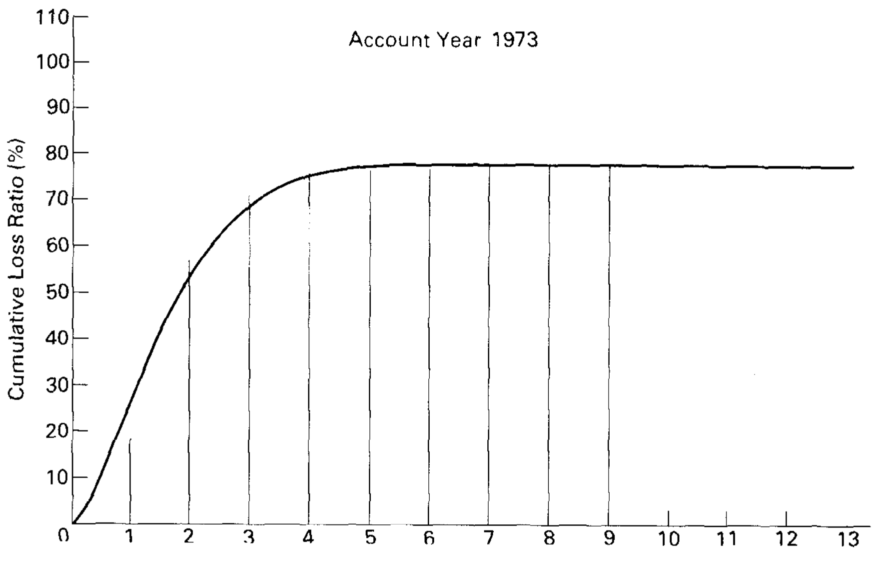
TITANIC INSURANCE GROUP 13
SHORT TAIL DATA
CLAIMS
AS RECORDED
CURRENCY GROUPS INCLUDED
£
ALL GROUPS EXPRESSED IN £

| Year | Quarter | 1973 | 1974 | 1975 | 1976 | 1977 | 1978 | 1979 | 1980 | 1981 | 1982 | 1983 | 1984 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4 | 280 | 296 | 502 | 398 | 363 | 719 | 489 | 546 | 892 | 751 | 1097 | 580 |
| 2 | 4 | 894 | 922 | 1274 | 1127 | 1034 | 1834 | 1525 | 1831 | 2635 | 2247 | 2715 | |
| 3 | 4 | 1147 | 1254 | 1554 | 1426 | 1248 | 2278 | 2018 | 2404 | 3437 | 2865 | | |
| 4 | 4 | 1236 | 1320 | 1637 | 1502 | 1335 | 2378 | 2143 | 2539 | 3611 | | | |
| 5 | 4 | 1259 | 1348 | 1670 | 1539 | 1362 | 2405 | 2209 | 2569 | | | | |
| 6 | 4 | 1267 | 1355 | 1675 | 1552 | 1378 | 2418 | 2213 | | | | | |
| 7 | 4 | 1275 | 1357 | 1688 | 1560 | 1382 | 2424 | | | | | | |
| 8 | 4 | 1278 | 1361 | 1689 | 1561 | 1383 | | | | | | | |
| 9 | 4 | 1278 | 1362 | 1690 | 1561 | | | | | | | | |
| 10 | 4 | | | | | | | | | | | | |
| 11 | 4 | | | | | | | | | | | | |
| 12 | 4 | | | | | | | | | | | | |

EXAMPLE RUN

TITANIC INSURANCE GROUP 13
SHORT TAIL DATA
PAID LOSS RATIO

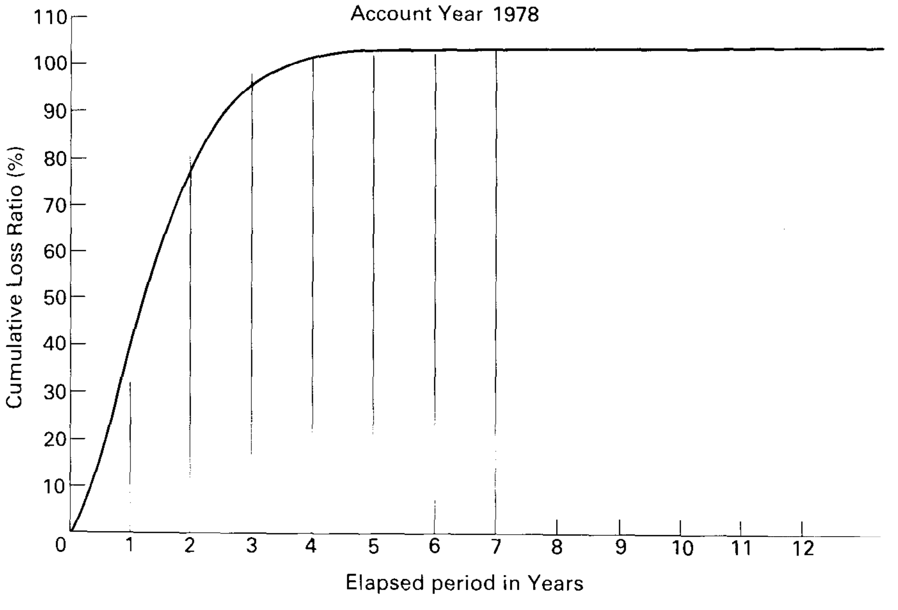
CURRENCY GROUPS INCLUDED
£
ALL GROUPS EXPRESSED IN £



EXAMPLE RUN

TITANIC INSURANCE GROUP 13
SHORT TAIL DATA
PAID LOSS RATIO

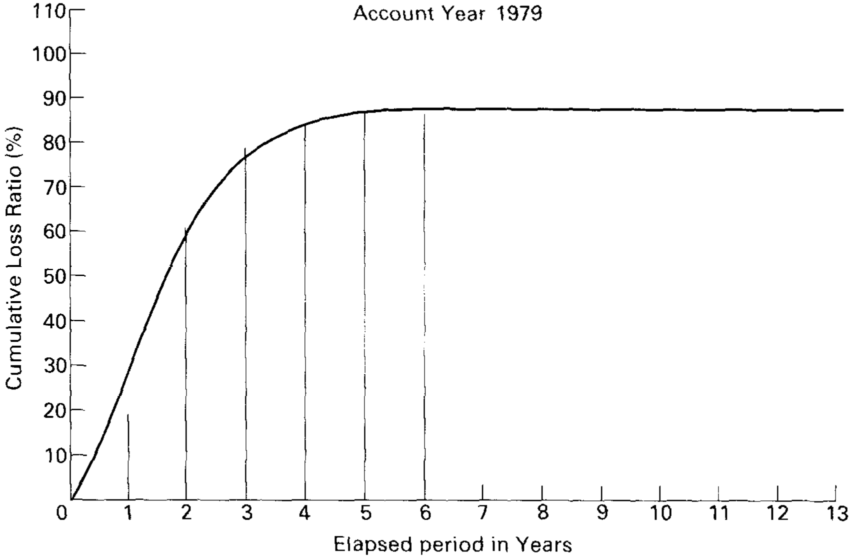
CURRENCY GROUPS INCLUDED
£
ALL GROUPS EXPRESSED IN £



EXAMPLE RUN

TITANIC INSURANCE GROUP 13
SHORT TAIL DATA
PAID LOSS RATIO

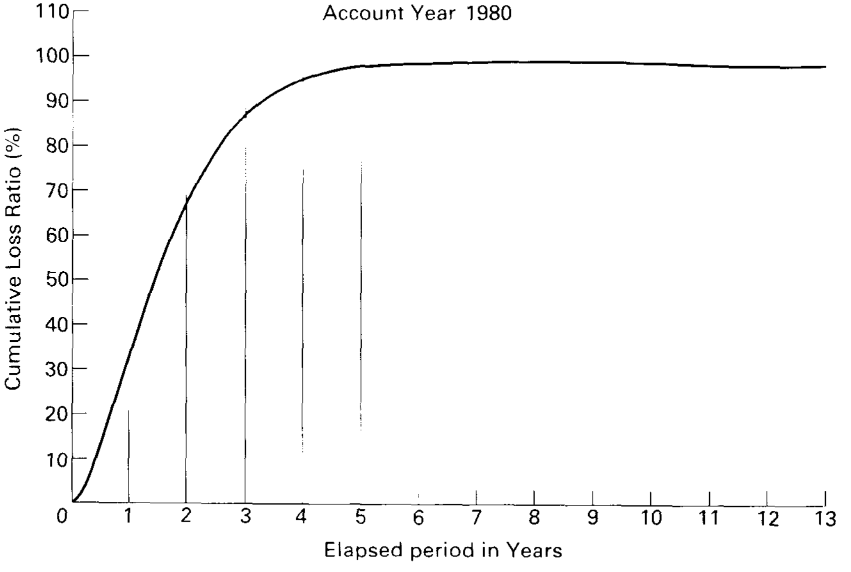
CURRENCY GROUPS INCLUDED
£
ALL GROUPS EXPRESSED IN £



EXAMPLE RUN

TITANIC INSURANCE GROUP 13
SHORT TAIL DATA
PAID LOSS RATIO

CURRENCY GROUPS INCLUDED
£
ALL GROUPS EXPRESSED IN £



EXAMPLE RUN

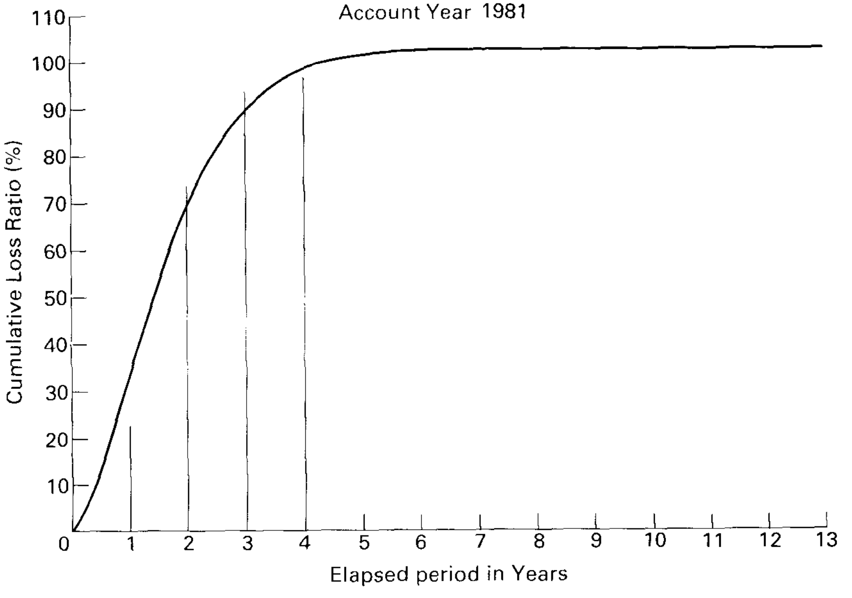
TITANIC INSURANCE GROUP 13

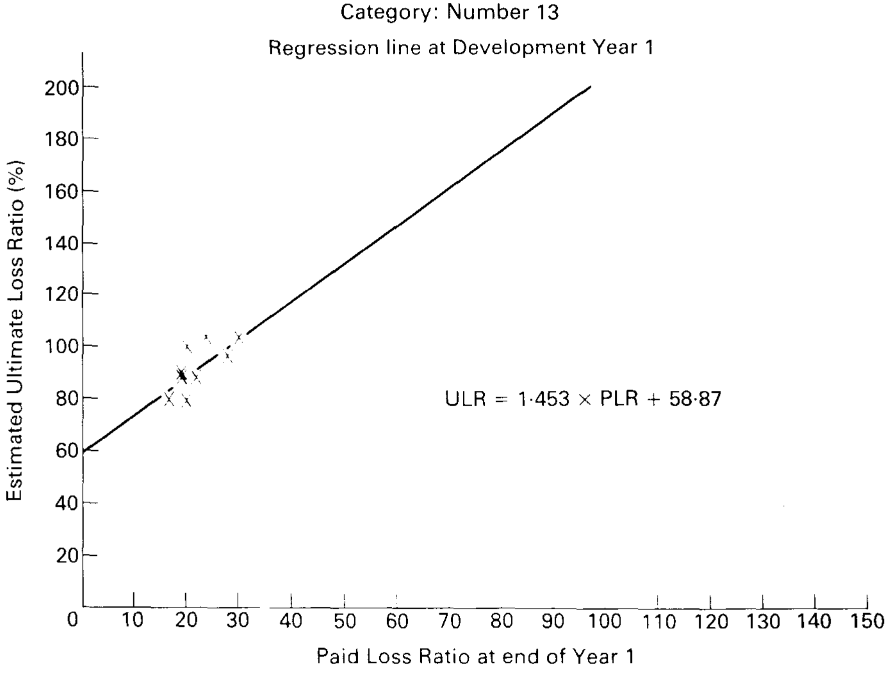
SHORT TAIL DATA

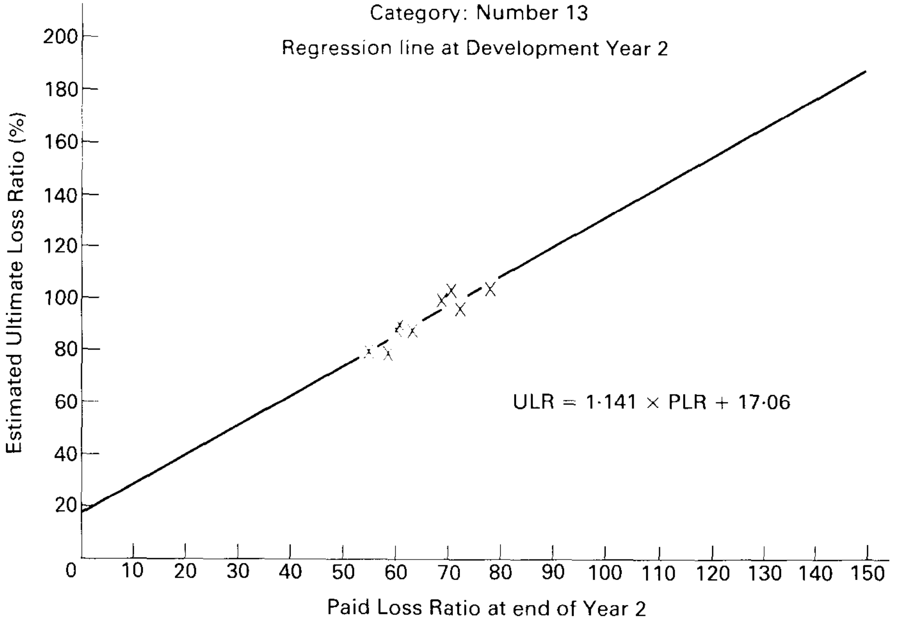
PAID LOSS RATIO

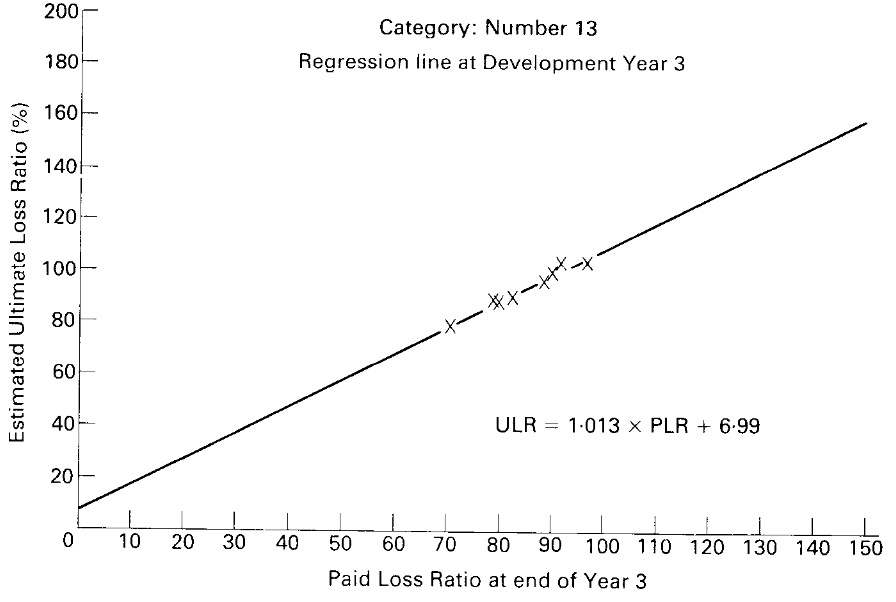
CURRENCY GROUPS INCLUDED
£

ALL GROUPS EXPRESSED IN £




Category: Number 13
Regression line at Development Year 1
Estimated Ultimate Loss Ratio (%)
0
20
40
60
80
100
120
140
160
180
200
ULR = 1·453 × PLR + 58·87
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
Paid Loss Ratio at end of Year 1


Category: Number 13
Regression line at Development Year 2
Estimated Ultimate Loss Ratio (%)
0
20
40
60
80
100
120
140
160
180
200
ULR = 1·141 × PLR + 17·06
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
Paid Loss Ratio at end of Year 2


Category: Number 13
Regression line at Development Year 3
Estimated Ultimate Loss Ratio (%)
200
180
160
140
120
100
80
60
40
20
0
ULR = 1·013 × PLR + 6·99
10 20 30 40 50 60 70 80 90 100 110 120 130 140 150
Paid Loss Ratio at end of Year 3

# APPENDIX 2

## ANALYSIS OF MOVEMENTS BETWEEN SUCCESSIVE RESERVING INVESTIGATIONS

### *Glossary of major summary details*

ESTIMATED TOTAL PREMIUMS FROM PREVIOUS ANALYSIS

The estimate of total premiums receivable made at the immediately preceding update with currencies converted at the rates then applying.

ESTIMATED TOTAL CLAIMS FROM PREVIOUS ANALYSIS

The estimate of total claims paid, outstanding and IBNR made at the immediately preceding update with currencies converted at the rates then applying.

ESTIMATED ULR FROM PREVIOUS ANALYSIS

Estimated Total Claims from Previous Analysis expressed as a percentage of Estimated Total Premiums from Previous Analysis

PREMIUMS BOOKED

Premiums booked to date shown with currencies converted at the rates then applying.

CLAIMS PAID OVER THE PERIOD

CLAIMS PAID & OUTSTANDING

Claims paid and outstanding to date shown, excluding IBNR, with currencies converted at the rates then applying.

CLAIMS PROVISION FROM PREVIOUS ANALYSIS

Estimated Total Claims from Previous Analysis less claims paid as recorded at that update—i.e. claims outstanding plus claims IBNR as estimated in the previous analysis.

ADJUSTMENT DUE TO CURRENCY FLUCTUATION

The increase in paid claims to previous update with currencies converted at current exchange rates, over paid claims to previous update with currencies converted at the then applying exchange rates, expressed as a percentage of the latter.

REMAINING CLAIM PROVISION FROM PREVIOUS ANALYSIS

Claim Provision from Previous Analysis increased by the Adjustment due to Currency Fluctuation less the amount of claims paid over the period, converted at current exchange rates.

ESTIMATE OF TOTAL PREMIUMS

The estimate of total premium receivable made at this update with currencies converted at present exchange rates.

MOVEMENT IN ESTIMATED TOTAL PREMIUMS

The increase in Estimate of Total Premium over Estimated Total Premiums from Previous Analysis, expressed as a percentage of the latter.

ESTIMATE OF PREMIUMS OUTSTANDING
Estimate of Total Premiums less Premiums Booked at this update with currencies converted at present exchange rates.

ESTIMATE OF TOTAL CLAIMS
The estimate of total claims paid, outstanding and IBNR made at this update with currencies converted at present exchange rates.

MOVEMENT IN ESTIMATED TOTAL CLAIMS
The increase in Estimate of Total Claims over Estimated Total Claims from Previous Analysis, expressed as a percentage of the latter.

ESTIMATE OF ULTIMATE LOSS RATIO
Estimate of Total Claims expressed as a percentage of Estimate of Total Premiums.

STANDARD ERROR
Standard error of Estimate of Ultimate Loss Ratio if computed as the average of preceding underwriting years.

MOVEMENT IN ESTIMATED ULTIMATE LOSS RATIO
The increase in Estimate of Ultimate Loss Ratio over Estimated ULR from Previous Analysis, expressed as a percentage of the latter.

ESTIMATE OF CLAIMS IBNR
Estimate of Total Claims less Claims Paid & Outstanding.

ESTIMATE OF CLAIM PROVISION
Estimate of Total Claims less Claims Paid, i.e. the estimate of claims outstanding plus claims IBNR.

MOVEMENT IN CLAIM PROVISION
The increase in Estimate of Claim Provision over Remaining Claim Provision from Previous Analysis, expressed as a percentage of the latter.

## ABSTRACT OF THE DISCUSSION

**Mr N. R. Gillott** (opening the discussion): The very fast increase in the number of actuaries employed in the London market is a sign that the financial complexity of the business is now being recognized. New techniques to simplify that complexity, so that non-financially trained insurance men can more fully understand their data, must be welcome. However, in welcoming such techniques, it would be wrong to imply that they could give a reasonable answer in the hands of an untrained user. The skill of the actuary is to understand his data and make appropriate adjustments, within the method he chooses, to allow for elements which cannot be taken into account purely mechanically. Thus the proposed technique is not an answer to all problems, but is probably a step forward in trying to answer those problems.

The authors are effectively advocating two uses for their method. One is to give a mechanical reserving method for all Lloyd's syndicates, and the other is to help an actuary to understand the data he has and to set an appropriate reserve.

The beauty of the new method for an actuary or other similarly qualified person is that, despite its simplicity, it gives him a new and potentially powerful method of looking at data. Let me illustrate this point and a number of others by highlighting the results that I obtained when I tried out some of the proposed methods, working on the data of my own company. My area of involvement is principally in directly written business and the fact that my data are on the face of it, rather less complicated than London market data is a positive advantage in stripping out unnecessary complications so as to see exactly what is happening in the new method. It also gave me a chance to test the authors' hypothesis that their method and its spin-offs should be useful in a much wider context than they describe.

Starting with some short-tail data I obtained satisfactory results. The *estimated outstanding* was close to the actual out-turn and certainly gave a better answer than the chain ladder method and some of its derivatives. Perhaps this was to be expected, as the proposed method does overcome what is perhaps the most serious difficulty of the chain ladder method—that is that the final reserve is so dependent on a very small figure base in the latest years of origin. In applied mathematics terms, we can think of the line in the paper as $y = mx + c$. A chain ladder is basically of the form $y = mx$, whereas the current Lloyd's percentages have the form $y = x + c$. Perhaps the new method might be thought of as a credibility formula between the chain ladder, which makes the historical data for a particular year of origin totally credible, and a formula, $y = c$, which gives no credibility to past data. On the face of it the new line of $y = mx + c$ would be expected to be the most flexible for fitting historical data.

When using the method on a longer-tail liability account, things did not go quite so well. The scatter of points at development year 1 looked fairly random, and the line of best fit had a negative slope. Similar graphs at years of development 2, 3, 4 and 5 did little to ease my anxiety. The points were certainly becoming less random, but the line of best fit still had a negative slope. In § 11, this is called an 'unnatural' line and it implies that the ultimate loss ratio is lower than the paid loss ratio. I believe this to be incorrect; all that is implied is that the lower the paid loss ratio, the higher the ultimate loss ratio is likely to be and *vice-versa*. Thus, there is a negative correlation between them. In my case this was entirely logical as the paid loss ratio at a given development year was decreasing as the speed of settlement was decreasing; and the ultimate loss ratio was increasing as the market was generally becoming less profitable over the period.

I then ran my data through the computer programmes described in Part II. The output was very helpful in understanding the data. Certainly the decreasing speed of settlement was shown very clearly by an increasing value of B within the curve formula given in Appendix 1. I feel that the description of the computer output within the paper does not do full justice to its real worth.

I was left with a problem when using my historical data to project my ultimate run-off ratios for later years. Of course, in practice, an actuary would make suitable adjustments to allow for the underlying changes in the data. While the reason for the underlying changes was clear to me in this particular case, it may not be so obvious in the Lloyd's or the London market. While no method can adequately deal with changes in underlying data, I feel that, at least the statement in § 8.3, that there is no point in wasting time on a point which lies inside the path, should be changed. Perhaps any new observation lying inside the path, but outside the range of past data should have some time wasted on it, or alternatively, perhaps the path should narrow as it runs away from the historical data; this would be equivalent to an expanding funnel of doubt. In § 14.2, it is stated that cases were found where the syndicate's data lay within the all-Lloyd's path despite the paid loss ratio lying outside the range of all-Lloyd's paid loss ratios. I feel sure that there must have been many cases rather like mine where the ultimate loss ratio could not be predicted from a paid loss ratio that lay within the all-Lloyd's path.

This leads to the consideration of using the all-Lloyd's path as a standard table. The authors state that no actuary needs to be convinced of the value of the standard table, but this one does need convincing. On the life side, a standard mortality table is undoubtedly an asset; any deviation from it may take the form of a slightly different overall mortality level or even a slightly different slope, but either way the difference can be measured. In general insurance, however, the position is rather different; sometimes a standard table can be helpful, whereas at other times it can be very confusing. As the authors point out in § 14.3, they found cases where the standard table of market data was totally different from the data of an individual syndicate. Indeed, I found such a case within my own sphere. In such circumstances, giving the standard table to an insurer with only limited historical data could be very confusing. Indeed, could we not actually be leading him away from the right answer? The use of a credibility type formula weighting the syndicate's own data line with that of the total Lloyd's population does seem to have some appeal.

When using the proposed method on other data, I had varying degrees of success. However, as with any statistical method for estimating outstanding claims, a straightforward application of the method will virtually never give the right answer. The skill of the person doing the analysis is essential in detecting homogeneity of data, for whatever reason, and making suitable adjustments. This is certainly the case with the current method and thus the worthy aims set out in § 21 have inevitably only been realized to a certain extent. However, while the skill of the person setting the reserves remains paramount, I believe that the general method set out in Part II is very helpful to that person in understanding the data. The various methods used, and in particular the structured way of looking at the data by account and by development year, can extract no more information than is contained in the base figures; however, the methods probably do allow all the information contained therein to become apparent.

Having said that the proposed method is useful in the hands of an appropriately competent person, where does this leave its use for setting minimum Lloyd's percentages? To use the method mechanically for such work will inevitably mean that it will be used in many cases where it is not appropriate, because the all-Lloyd's line will not be applicable to individual syndicates. However, it is almost certainly a better method than the current Lloyd's minimum percentages which make no allowance for differing gradients in the line relating paid to ultimate loss ratios. Thus, the new method is a practical alternative to the current Lloyd's percentages, which are seen as distinctly arbitrary and not appropriate in a number of real circumstances. Indeed, it is probably the powerful, yet simple, nature of the new method that will ensure its future use for the setting of Lloyd's minimum percentages. Paradoxically, it is this simplicity allied to the apparent power of the method which, while ensuring its use in the near future, may lead to its ultimate downfall. There is the real danger that, unlike the current percentages which are seen to be arbitrary, the new method will have an air of authority. It uses the syndicate's own past data. It will appear to be mathematically sound and, perhaps worst of all, it will be seen to have actuarial backing. The danger is that among less sophisticated syndicates the method may be assumed to possess magical powers and be capable of giving the answer—rather than as a starting point for discussion. It will be important for actuaries to make it clear that the method is unable to foretell the future, although it is able to give a useful insight into it. Hence I believe it to be a reasonable, although not infallible, way to set Lloyd's minimum percentages.

In §§ 18 and 19 the authors whet our appetites by briefly commenting on the analysis of premium rates as a return on capital and I wonder whether they have some answers to the problems that many of us are grappling with.

**Mr J. M. Taylor:** The simplistic approach used throughout the paper has been essentially practical, and by so being this will probably do much more to convince the Lloyd's market that actuaries have a part to play in this intricate area of insurance than any more theoretical presentation might have done. The point is that the actuary is not automatically accepted in general insurance at this time. The profession must therefore be very careful of its image, and it is vital that we should project ourselves as having something practical to contribute if we are to have any significant presence in this field. The increasing numbers of actuaries becoming involved in the Lloyd's and London markets should therefore have reason to thank the authors for demonstrating these practical qualities and the ability, above all, for actuaries to use these particular skills in the use of different time cohorts and their development.

I was pleased to see that the authors made the concept of variability an important part of the paper. In many ways this should be as important a part of the work when assessing reserves in general insurance as determination of the mean or expected result. This is particularly true of Lloyd's business, where the data tend to be very sparse in detail and long-term in developing to the completed result. Equally, however, for these reasons the measurement of variability and use of confidence intervals has always been very difficult to achieve for Lloyd's business. Either the completed data are too old to apply to current conditions, or are insufficiently developed to measure the true variability of development towards ultimate for the more recent cohorts. I was therefore interested to see the approach adopted in the paper. I did wonder, however, whether the method is not a little incestuous in this respect, particularly where, as in Part II, it is necessary to make use of projected ultimate loss ratios for the incompleted years of account, which in many cases may be most of the years available to the actuary. Since the projection of such years will tend to be forecast by a common family of curves, or by some other actuarial method which assumes the same relative patterns of development, it seems that this will necessarily constrain the relation of *paid* to *ultimate* loss ratios towards a linear path. This being so, the true confidence limits applying to these forecasts from the graphical linear fits are probably wider than the method suggests in such cases.

Thus, whilst being very much in favour of indicating a measure of variability to such forecasts, I feel that we should be very careful how we express the levels of confidence obtained in this way. Should we really be saying in this example that we are 90% certain that the result will lie between $x$ and $y$? We might therefore question whether the method used for this purpose is essentially any different in principle to, say, the chain ladder family of methods extended so as to make use of the variability which can be measured between the observed development ratios in such methods. Even if it does no more than this, the method does provide a readily understood, practical calculator for the use of Lloyd's syndicates as had been demonstrated in Part I.

**Dr S. M. Coutts:** As an actuary who has tried to set claim reserves in the London market for the past 4 years, I welcome this paper as a major step forward in establishing methodology where, 6 years ago, most actuaries were saying that reserving for this type of business was almost impossible. It gives the structure of an actuarial report which is becoming ever necessary to prepare in general insurance. The report itself would show source of data, assumptions, model use, results, and professional interpretation in the way the paper describes.

Considering §21.1, I agree that obtaining data which are reasonably homogeneous is one of the major problems encountered in the London market. Until recently in my own company the only triangulation data we had showed by two separate currencies, casualty, fire and miscellaneous. It took an enormous amount of resource and time to redefine triangulation data net of reinsurance by separate currencies, territories and sub-divisions of major lines of business—for example, subdividing the old combined casualty lines into casualty proportional, casualty non-proportional, personal accident and health.

Whatever formula is devised it is necessary to have a rationale so that it can be objectively critiqued. This paper shows how such a formula can be devised.

There may be some people who criticized this paper because the method is too simple, but let me remind life and pension actuaries that the reserving of life and pension business involves a trivial calculation, hence the simplicity of a method is not the real criterion, but the criterion is: does it work?

I have a fear that the people reading this paper who are involved with the Lloyd's business may be inclined to say "we now have an alternative basis to our minimum reserves. It has been done by an actuary so it must be right and let's substitute this for the minimum basis". Others outside Lloyd's could further add that "formula estimates should be written into legislation". My warning is that this is not the way I believe the authors want us to interpret the results. I sense (see example § 14.2) that the authors would like each syndicate to reserve on its own data. I know that at present the minimum Lloyd's percentages can benefit the syndicates when the minimum is way below what should be prudently reserved and handicaps other syndicates in that it is far too severe. In addition, I think that any formula base which may fit one situation may not fit another, hence I go back to my original theme that it is necessary to have a professional report showing the basis of the reserving.

I appreciate that I am implying that each syndicate should set up its reserves based on its own experience. The cry will them come "the data base is too small for any meaningful analysis". I would say that availability of reports from other syndicates can give an objective overview of bases and can be the starting point to assist small syndicates; and the Institute should do research on small data bases, and to do this, we need a data base to analyse. With this in mind, I would make one more plea and it seems that Lloyd's have now allowed a consultant to view their data objectively. Perhaps they could show themselves able to give, under the auspices of the Institute to aid further research, non-political data—that is data that are out-of-date and not sensitive—and I would like to include in this, non-Lloyd's companies, so that a general market database could be set up.

**Mr F. E. Guaschi:** The method used by the authors has one thing in common with all the other methods currently in use; namely, that the first and most essential thing is for information to be kept for each underwriting year separately. This means that each claim must be correctly allocated to its underwriting year. The same procedure applies to premiums, since unfortunately in this business of general insurance not all premiums which are due in 1986 will be paid in 1986. Therefore, any premiums and claims which emerge in 1987, 1988 and so on, and which relate to business written in 1986, must be correctly allocated to that year. This vitally important concept is, of course, what is enshrined in the basic principles of exposed-to-risk.

In the past, actuaries tended to look upon general business as one-year business, and so it is in many ways. The trouble, as the paper illustrates all too clearly, is that the claims which arise in that one year sometimes take a long while to settle. Most of the problems which have arisen in recent years have come from this so called long-tail business. A typical example of this is accident third party liability insurance, where claims arising from an event occurring in the inception or underwriting year may not be settled for many years, 25 or more in some cases, and generally at much inflated values, too. So, as the claims information accumulates for a particular underwriting year, the final claims total tends towards a definite limit and follows the typical Ogive curve at the beginning of Appendix I. There are many methods which have been developed to solve the problem of fitting a suitable mathematical formula to this basic curve, and they will be found scattered in the papers of the various actuarial journals around the world. Some of these are rather fearsome, and as used to be said of Einstein's theory, can only be understood by one or two people.

I should like to draw the attention of those who are perhaps unfamiliar with this area of actuarial activity to the curious feature of general insurance reserving. It is that all the methods are based on the year of entry. Actuaries in more traditional fields are used to basing their valuation methods on either year of maturity or age attained. I would suggest that the profitability of, say, term assurance business or risk premium business of life reinsurance companies might with advantage be tested on the basis of examination by year of entry.

In the last six months I have had the valuable opportunity of seeing a practical demonstration of the method to non-actuaries on several different occasions. It is so simple that it is easily followed and understood by non-actuaries and actuaries alike—the graphical method which other people have pointed to is a very powerful way of illustrating what is happening to the business, and in particular, as Dr Coutts and Mr Taylor have mentioned, I would like to draw your attention to the concept of the 'path' in § 8. This is of fundamental importance in showing the inexactness of any method of reserving. It attempts in a commonsense way to set reasonable upper and lower bounds to the estimates of ultimate loss ratios, and the interpretation of its width is easily understood. It does not mean that the ultimate loss ratio might not fall outside it, but I believe the people to whom I have seen this method described fully understand that. It is a completely new concept in the business of general insurance reserving and I have not seen it anywhere else.

The word 'practical' is what there is to guide us in this tricky area. We have to make ourselves understood or become just another learned body whose members merely talk to each other. Increasingly our traditional methods are rightly being questioned. We are also being called upon to justify our practices and our assumptions. There is an increasing control and supervision of insurance, especially by the D.T.I. and with good reason, and greater scrutiny by the mass media and consumer groups. We have nothing to hide and we have a lot to give. Our only problem has been that we have not been very good at explaining ourselves.

In life assurance we have invented our own language, but in general insurance the actuary has to learn the language of the underwriters and the claims managers. The amazing thing, although not really amazing when you come to think of it, is that once the actuary sets out to explain his methods not in his own language but in this newly-acquired language, he can produce the kind of reserving methods in this paper.

As mathematicians we are trained to seek optimum solutions, but as actuaries we will sometimes, and increasingly often, find that a good, feasible solution will make our meaning far clearer to non-actuaries.

**Mr D. H. Craighead:** Lloyd's is the oldest of our great insurance institutions within the City of London and it remains unique, fulfilling a very important function in the international insurance market. It has always surprised and disappointed me that actuaries have had so little contact with Lloyd's over the years. Perhaps underwriters have been wary of intervention by actuaries in a field where flair and intuition play such a vital part in the process, while actuaries have been loathe to attempt mathematical analysis in an area where imprecision seems to be endemic and exact evaluation is fraught with difficulties. It is really only the advent of computers that has brought a greater emphasis on precision. There are aspects of this analysis that can be criticized, but it is an imaginative leap into a field previously largely unexplored.

The Lloyd's audit percentages used as a minimum basis of reserves to close for the purposes of audit, have long been criticized for being dependent on premiums and not on claim development. They are qualified by a statement that the figures produced must not be less than the outstanding liabilities as at the date concerned, which must include an element to take care of unnoted and unknown liability. Furthermore, the percentages required are changed from year to year, as required by a type of adaptive control. Practical difficulties and the demand of appearances have limited the percentages to figures approaching 100%, which is clearly insufficient in long-tail business unless full account is taken of discounting, and are probably still insufficient for recent casualty business emanating from the United States. All prudent underwriters have tended to examine the percentages closely in terms of their special portfolio characteristics, and to increase them substantially in special classes.

This paper presents a method that is at least dependent on claim development, and as such must represent a major step forward. It also represents a method which appears reasonably commonsense and easy to apply in practice. If it tends to over-simplify the picture, then at least it presents a new jumping-off point. The real difficulty is that syndicates vary so widely, both in the portfolio of business they underwrite, and in the way they keep statistics. While the method set out in the paper will be useful for audit requirements by the Committee of Lloyd's, it must represent only a starting point for individual syndicates. The pattern of development varies too widely. Furthermore, the authors have had to base their analytical classifications on Lloyd's audit codes, and these are of very limited use in reserving. The worst of all, as the authors indicate, is the non-marine division into 'short-tail' and 'all others' where 'all others' can cover an enormous variety of business. Compare it, for example, with Kiln's published method of grouping business into five categories for the purposes of reserving, ranging from clearly short-tail to ultra long, such as products liability written on excess or loss basis. The Lloyd's system presents many difficulties, such as large treaties which cover all classes of business, marine aviation and non-marine, both property and casualty. Little distinction is made between different ways of writing business: proportional and non-proportional treaties, for example. Particular difficulty arises from covers and binders. Retrocession business can be a nightmare in reserving, as it can cover cessions of facultative business and both proportional and non-proportional treaties, perhaps the second, third or even fourth time around.

I am not sure about the confidence limits that are suggested, nor the wisdom of stating them, although I can see the appeal. Some indication of possible variability in ultimate results must be given, but the lines drawn appear to rest on shaky foundations. Why should they be parallel? Why not an expanding funnel? And to give such clear figures seems to indicate to non-actuaries and non-statisticians a clear limit of possibilities. Such concepts as 90% or 95% confidence limits are not generally understood outside our ranks, and it is dangerous to instil any sense of an absolute limit to potential liability.

It is useful to have a glossary of major summary details at the end of the paper, but I am not happy about the definition of 'adjustment due to currency fluctuation'. If the definition refers to the exchange of miscellaneous currencies into pounds, then it is acceptable, although needing fuller description. If it refers to statistics amalgamating pounds and dollars, then I must disagree entirely. This whole matter of currencies and the way they are dealt with in statistics, is a very complex one.

Part II deals with the application of the method to companies in the London market. It is clearly set out and will prove a valuable aid to actuaries working in that field, but in practice there are many complicating factors, and anyone working newly in that area must be aware of hidden dangers. The effect of excess loss catastrophe reinsurances outwards for both Lloyd's syndicates and companies is one such complicating factor. The reinsurances are mostly whole account protections placed in layers.

If gross and reinsurance figures are analysed separately, the development pattern is apt to be very rough indeed. If they are analysed on a net basis, then it is unlikely to be possible to attain a breakdown between audit categories, classes or even long- and short-tail business. It is also essential to ensure that the protections are not already almost burnt through. The strength of the security must be considered.

**Mr C. D. Daykin:** The business with which we are concerned is characterized by uncertainty, often very great uncertainty. Faced with such uncertainty, many in the London market may be tempted to argue that it does not matter what figure goes in for the provisions; neither auditor nor supervisor will know any better than they do. Sometimes future investment income will be prayed-in-aid if the reserves are too low; it is said, investment income will cover the additional provisions as and when they need to be put up.

The authors have shown that there is a better way forward than using uncertainty as an excuse for establishing provisions on a weak or arbitrary basis. Their strategy is simple: use what data you have and examine it in different ways to discern any patterns that may be helpful; use methods which help you to interpret the degree of uncertainty; even if the methods rely on mathematics, ensure that the results can be presented simply, preferably visually, to managers, underwriters and claims people; monitor the results and refine both methods and conclusions as more data becomes available.

I would like to consider § 18, relating to solvency, in my capacity as Chairman of the Solvency Working Party of the General Insurance Study Group. The authors focus on the free assets necessary before writing the business. Given that the ultimate loss ratio on the business to be written is uncertain, but given also that it could well be over 100%, it is clear that free assets are needed which are sufficient to cover the possible excess over 100%, not perhaps in all conceivable circumstances, but at least to ensure that in a high proportion of possible outcomes, adequate additional resources are available in excess of the premiums received. Of course it is appropriate that future investment income be taken into account in looking at the excess of the loss ratio over 100%, which the free assets may have to cover. Consideration also needs to be given to the basis on which provisions will be set up, since this may add to the initial strain, even if there can be releases later on the run-off. The phenomenon is akin to new business strains in life insurance.

However, this is only part of the story. Another aspect is implicit in the rest of the paper, although not drawn out. For the business that has already been written, provisions will have been set up, possibly using the authors' methods. These provisions may be 'best estimates' as given by the method. If they have not been discounted, they may contain an implicit margin, but most probably not an explicit one. With the high level of uncertainty about the eventual outcome, this could leave a significant possibility that they will prove to be inadequate. The factor must also be taken into account in looking at the adequacy of the free assets. Being able to place a measure on the uncertainty of the provisions as the authors do, is essential.

What it amounts to is the problem of managing the total uncertainty of a general insurance operation. The financial management of a general insurance company requires information about all the uncertainties in a measurable form—assets and reinsurance provide other dimensions of uncertainty which are not touched on in the paper but which are important in practice. The manager needs to be able to see the results of a simulation of his business which illustrates the range of possible outcomes and provides him with an understanding of their respective probabilities. He can then explore the effect of different business strategies, in terms of rates of growth, riskiness of business written, reinsurance, investment strategy, etc., against his own laid-down safety criterion

Clearly there is a trade-off between high capital backing, with resulting low probability of insolvency, and lower capital but a higher probability that insolvency might occur. What should his free assets be in order to achieve his desired strategy with, say only a 1 in 100 chance of ending up insolvent? Or, given his free assets, how can he plan his growth strategy so as not to raise the probability of insolvency above a fixed level?

The Solvency Working Party of the General Insurance Study Group has developed a simulation model of a general insurance company to enable these questions to be tackled. We believe that this will take a stage further the sort of work the authors have been doing and show how actuaries in general insurance can be of assistance not just in establishing provisions and in understanding the uncertainties there, but in the strategic management of a company.

**Professor A. D. Wilkie:** The first part of the paper describes in effect a straightforward linear regression method in which ultimate loss ratio is regressed on paid loss ratio. What the opener said, was as it were, $y=mx+c$, or what the authors show as 'ULR equals something times PLR plus something else'. In both cases these are just estimates of the actual ultimate loss ratio. There is another item to take into account. The actual loss ratio is equal to the estimate plus some residual deviation or error term, $e$.

The different sorts of models that the opener mentioned: $y=c+e$; $y=mx+e$; or $y=mx+c$ will all produce, once the data have finally been investigated, different error terms, and a feature of a good model is that it has a low error term. The authors have not told us anything very much about the error terms. I presume they have zero mean. They may not have, but it is usual for them to have a zero mean and the paths do show the scatter of them in some cases. The standard deviations of the error terms and something about the distribution of them would also like to be known. If they are normally distributed, then the usual methods produce good estimates of $m$ and $c$.

If they are not normally distributed and if there are some very large errors in one direction or another, then the estimates of $m$ and $c$ may not be very good. The estimates of $m$ and $c$ are only estimates based on a limited number of observations, and we do not know the true values of them, if indeed there are any true values, but the whole model assumes that there are true values underlying it. Therefore it is possible, using ordinary statistical methods, to widen the path in the way that Mr Craighead said by using an expanding funnel of doubt at both ends by taking account of the uncertainty of the estimates of $m$ and $c$, or in the example in Figure 5, say, the fact that the ultimate loss ratio is not really equal to $1{\cdot}325\times \text{PLR}+40{\cdot}24$, but is something times PLR of which a reasonably good estimate is 1·325 plus something else which is about 40·24, where we do not quite know what the true values are.

The method that the authors have used is simple linear regression. I am unhappy that ultimate losses are equal to the paid losses plus the future losses, and therefore the ultimate losses include part of the paid losses already and therefore there is correlation between those two. That does not matter too much, it may be better to describe it as future loss ratio $=(m-1)\times \text{PLR}+c+e$.

I do not think that the authors have used all the information that is apparent or available. At the end of several years you not only have this year's paid loss ratio, or paid losses to date, you have the position at the end of year one, the position at the end of year two, at the end of year three and so on. They could have used a multiple regression model basing the ultimate loss ratio on all the previous loss ratios. They may well indeed have done this and found that it does not give any better results and just have not told us, but it is another way of looking at it and it would be worth investigating.

It is also possible to see from Figure 2 that drawing a line joining the successive years usually there is not very far to travel between them. Beginning from the top, 65 to 66 is not very far, then to 67 and so on. This means that there is some auto-correlation, i.e. some correlation between results in successive years, and although it complicates the model yet more, it would also have been possible to take account of the paid loss ratios in the immediately preceding years on each occasion. I am now replacing not one figure by two, but two figures by about twenty.

Considering Mr Craighead's formulae in Appendix I, the curve fitting method is essentially a kind of multiple regression. I presume that the curves are fitted by using three or four values of the observed loss ratios, and using those not in a linear way but in the more complicated way to produce an estimated loss ratio, so that this ratio is some function of the paid loss ratios for years 1, 2, 3, etc., up to T. There is no comment about what the statistical properties of this estimator are. It is certainly a non-linear structure and so it is not so easy to see what the properties of the estimator are. Nor is there an obvious way of fitting in a residual error term. Again, I would be happier to see the model stated in a straightforward statistical way with error terms included and their features described. In the fitting process I do not see what it is that is being minimized. The golden section method is mentioned, which is incidental. The numerical method of fitting is secondary to deciding on what it is that is being minimized. That could do with some explanation.

I have emphasized the idea of estimates involving an error term because it seems to me quite appropriate, having worked the arithmetic of it, to say: because of additional information that we have about the real world, we think we will use a different error term from zero. We think that these reserves are too low, because we know that something is going is going to hit us next year, so for actual reserving we will use an arbitrarily higher error term, or contrariwise, if we know that something has happened recently that is not going to hit us again, we can use an arbitrarily lower one. That is where the statistics are completed by producing practical answers.

**Mr D. M. Hart:** As the method originates from a desire to improve the current system of minimum audit reserving, I would like to comment on the application of the minimum percentages, which is incompletely explained in § 5.1. The point that does not come out is that there are two separate sets of figures. One is the reinsurance to close, used for determination of the profit or loss to the Names on the year being closed. This is the equivalent of the estimation of Companies Act technical provisions for an insurance company. The second is the solvency test required under the Insurance Companies Act. This is applied to the syndicate and year of account into which the outstanding liability is transferred after closing, normally the oldest open year for the same syndicate. There is a significant difference between the application of the minimum audit percentages in these two situations. In the former, where the auditors are now required to give a 'true and fair' audit opinion, there is a requirement that any reserve shortfall below the minimum audit percentages must be reported to the Lloyd's authorities. However, in respect of the solvency test, no such shortfall is permitted, the Lloyd's minimum audit percentages being an absolute minimum. I can think of no other part of the United Kingdom insurance industry which is subject to a minimum reserving code. The existing structure, based on the minimum percentages, is more unsatisfactory than the chain-ladder proposals put forward by the regulatory authority in the early 1970's for insurance companies. These were successfully, and in my view, correctly repulsed. Lloyd's is certainly over-due for a change in the area.

Why do I contend that the current method is unsatisfactory? There are four basic reasons: because of the serious lack of homogeneity of the business; because of the inadequate cognizance of the often very substantial effect of reinsurance protections on the loss development. In fact, credit is given for the reinsurance premium ceded rather than the reduction in net liability resulting from the reinsurance, which seems totally illogical and can give rise to strange anomalies. The third reason is because of the inadequacy of premium income as a measure of exposure. This results in a likelihood that the more unprofitable syndicates also tend to be under-reserved—at least the minimum audit percentages do nothing to prevent this situation. The problems here are much more significant than in the domestic market because of the much greater variation from time to time in premium adequacy; for example, the current rating increases of 500% or more in parts of the non-marine market. The final reason is because of random variations.

Considering the impact of the paper in relation to these four problems, in my view very limited benefits accrue in the areas of varying reinsurance protections and differing portfolio mixes within an audit category, although the latter problem would be reduced to some extent if the individual syndicate's results are used as the base data rather than the all Lloyd's results, provided that there have been no recent changes in the syndicate's own portfolio mix.

One major development which the proposed method should bring is the removal of almost total reliance on premiums to measure exposure. Instead, in effect, the method is using claims paid-to-date as a measure of exposure. Although this is not ideal, I believe it is a substantial improvement.

The second major contribution which the paper makes is in the area of random variations. It is not possible to remove their effect, but the idea of estimating some form of confidence interval is a substantial step forward.

The third factor which I consider to be particularly important is the additional facility to enable an underwriter to compare his syndicate's development pattern with a relevant standard table. As the authors point out, this is a standard actuarial tool, but the existing system does not lend itself to such an approach. Instead an actuary involved in this area has to rely on such standard data bases as that produced by the Reinsurance Association of America, which may be relevant to part of the portfolio at best.

As with many new systems there are some practical problems arising from the use of the proposed method. In my experiments with the method intermittently over the last eighteen months, I have found two particular problems which are likely to be of more general interest. The first is one which has already been touched on and that is the determination of the ultimate reserve for old years which is required for use as base data, or even deciding which years are capable of reasonably accurate estimation of the ultimate. The effects of asbestos-related claims over the last five years and the prospective effects of pollution claims in future have dramatically worsened the results for long-tail non-marine business emanating from the United States. To illustrate this, I have seen a deterioration in the 1962 incurred loss ratio for one syndicate from 56% at the end of 1980 to over 300% by the end of 1984. No method can cope with such a sudden transformation, and I suggest the method should be applied excluding such items which should be dealt with separately.

The second factor is the width of the path. In my experience, what appears at first to be a narrow path can prove, in practical terms, to be extremely wide. As an example, take the case of the non-marine short-tail account, where, after five years of development, based on the all-Lloyd's data, the method gives an estimated ultimate loss ratio of 118% ± 16%. This sounds useful until it is realized that the paid losses amount to 115%, so that the reserves become 3% ± 16%—not very helpful!

Much as I like the idea of introducing a measure of fluctuation, I believe the 'path' approach often gives an unhelpfully wide range, and I welcome the ideas propounded on a more formalized confidence interval in § 18.

The authors, whilst concentrating on paid loss ratios, refer in several places to the possibility of applying the method on incurred loss data, incurred losses being paid losses plus reserves for known outstanding losses. I strongly favour this alternative, especially on long-tail business, as the results on incurred data tend to be helpful at a much earlier state of development. I have found that even on long-tail account the incurred loss ratio at the end of the first year gives a fairly good indication of the ultimate—or what I believe to be the ultimate—loss ratio.

I would take issue with the authors on two minor points. The first is that the Lloyd's system groups together risks which are signed in a particular calendar year, rather than written in a particular year as stated in § 4.3. The second is the use of the average historic figure if there is no correlation with the ultimate loss ratio: I do not believe that this is the best approach, because other data may be available regarding the level of premium rates which enable the investigator to make a more satisfactory estimate than a straight average.

**Mr P. S. Carroll:** It is attractive that the authors have concentrated on methods that will enable the differences to be apparent between different classes of business and different years in which business is originating, and this is a great step forward. As a lecturer in statistics, I am in the habit of stressing the assumptions implied in regression. Regression implies a linear additive model. This is quite a good start, and is a good first attempt to fit a model, but the risks may multiply. They may be multiplicative. They may escalate exponentially and so the model which may fit very well for some data may fail to fit at the time when it is most needed. Regression also implies, when the confidence intervals are calculated, that the errors are normal. Professor Wilkie has pointed that out. It also implies that the variances are homogeneous. Mr Hart has given an example where the variances are not homogeneous, again when things go wrong the model and the confidence interval may not apply.

The authors have concentrated on yearly data. The London market data, as they say, is accountancy-type data originating in quarters and is available in quarters. All the many examples, illustrations, graphs and tables in the paper seem to be yearly data. Perhaps one or two examples showing the quarterly developments would give us a more close insight into how this method really works.

The statement in § 22.3.2 that "we have found that a 90% confidence interval does the right job for our analyses of individual portfolios" might be quite good from an actuarial point of view, but is not very good from a statistical point of view. These confidence intervals are not very precise and I would echo what other people have said. We must not try to write into the rules or legislation these confidence intervals, whether 90%, 95% or any other arbitrary level of confidence.

The authors have very rightly addressed the task of making the best possible use of available data, but as Mr Craighead has pointed out, there are computers and there are possibilities of getting much better data than might have been possible in the past.

I have been told that there is a small error in § 4.2 that helps to illustrate this: "This risk will often (always at Lloyd's unless it is United Kingdom motor) be placed on a coinsurance basis, . . .". I have been told that domestic house insurance and personal accident insurance are often handled by one insurer alone, so this is not strictly true.

Why not find out how many claims there are? Why not find out the claim numbers, the claim incidence figures? It is surely possible to count up claims.

In reinsurance it is advantageous to skip details and reduce the amount of work and processing of *bordereaux*; but nevertheless, when a class of business is possibly in a dangerous situation, it is not wrong for the reinsurer to say we would really like more information in future. We would really like to have notification of claims. Why not press for this?

**Mr J. P. Ryan:** The method produced by the authors for the Lloyd's market minimum reserves produces superior results to the current system. It would, however, be useful to examine in more detail why. Essentially the formula that the authors derive is a factor times a paid—or incurred—loss ratio plus another loss ratio. This is a combination of a projection method approach and a Bornhuetter–Ferguson method approach assuming that these are credibility weighted in some way. The algebra for doing this is fairly straightforward.

Using the credibility weighted Bornhuetter–Ferguson method the ultimate loss ratio

$$= z \times df \times \text{paid loss ratio} + (1 - z)\ \text{ELR} \times (1 - \frac{l}{df})$$

where $z$ = credibility factor
$df$ = development factor to ultimate
ELR = expected loss ratio used in the Bornhuetter–Ferguson method

thus $z \times df$ is the authors' slope of line and is basically the estimate using projection technique times its credibility

and the constant term is $(1 - z)\ \text{ELR} \times (1 - \frac{l}{df})$

which is the Bornhuetter–Ferguson method times the complement of the credibility.

This assumes that the Bornhuetter–Ferguson method case is applied to premiums as a base. The Bornhuetter–Ferguson method for those who are not active in this field, essentially estimates the I.B.N.R. by multiplying an estimate of the unreported or unpaid percentage by the initial estimate of the ultimate losses, which is often an expected loss ratio times the ultimate premiums. This method is described in a paper by Bornhuetter and Ferguson published in 1970 in the proceedings of the Casualty Actuarial Society and is, in my view, essential reading for anybody attempting to do any work in this area.

Adopting this approach to the authors' method produces three unknown factors for each underwriting year: a credibility factor; an estimate of the development factor and an estimate of the expected loss ratio for each underwriting year. The authors' method can be regarded as being superior to the method presently utilized by Lloyd's in setting minimum reserves as it gives credibility to the paid loss ratio or incurred loss ratio. If a high degree of credibility is to be given to the paid loss ratio, then the results will be significantly different from those produced by the present Lloyd's method. This is closely related with the authors' slope of the line. However, where the credibility factor or the slope of the line is horizontal, then the results produced by the authors' methods are very little different from those currently produced by the minimum reserve percentages.

The authors provide limited information as to the success of the method between the various classes. Careful reading of the paper indicates that perhaps for the marine and aviation market the method works quite well, but for the non-marine and other class, their line is virtually horizontal and the reserves will not be very different from those currently set up, i.e. no credibility weighting is given to the paid loss ratio. As Audit Code A lines are likely to be the most problematic, this is an important limitation. The authors suggest that this could be resolved, at least partially, by greater sub-division of the Audit Code A data, and I agree with that. Indeed, it is essential if much further progress is to be made in the Lloyd's market in reserving this particular category.

It is also worth considering the impact of the second term, that is, the additional loss ratio, in greater detail. It is the Bornhuetter–Ferguson estimate based purely on market averages, and of course the existing Lloyd's percentages are effectively based on the Bornhuetter–Ferguson method of estimation using broad market averages. Much depends on the choice of selected loss ratio in that method, but this is an important, indeed essential, factor in adapting to changing market conditions. This is the response to Mr Hart's criticism of the inadequacy of the premium as a method of exposure. A judgemental weighting of that premium by way of an expected loss ratio goes some way towards meeting some of his objections.

Breaking down the authors' result into its two components, that is, the projected paid ratio and the expected unreported losses from the Bornhuetter–Ferguson method, allows the practitioner in the market to amend them suitably for changes in market conditions. The authors' methodology effectively ignores changes in market conditions even on a very broad brush basis, except to the extent that their analysis covers a very wide span of time. The present Lloyd's system makes some attempt to adjust for this by changing the reserves as time goes by. In my view, therefore, the authors' method could be improved if the same approach were adopted for their second factor. For most major classes of business, particularly the non-marine classes, results have deteriorated significantly in recent years. To use the second factor, unadjusted for changes in premium levels, would produce significant under-estimates of reserves for those years. Conversely, rates, as Mr Hart has pointed out, have risen very rapidly in much non-marine business recently and the authors' methods might well produce over-estimates, or at least estimates which are much stronger by comparison with the earlier years, for the latest underwriting year and the next underwriting year ahead. Certainly the two sets of estimates will not be consistent. These comments do not invalidate the authors' estimation progress, but are simply an indication of how the methods might be improved in a practical way that would be understandable to practitioners in the market. This approach, changing the second term, can also be used to allow for changes in the reporting or payment patterns by making allowance for that factor in relation to the changes in the expected unreported, or the expected unpaid. Factors can be applied to the second term that allow some judgement to be made to the individual syndicates or indeed to the market as a whole. It is probably fair to say, in many accounts that I have seen, that there is strong evidence that tails are lengthening. To use the authors' numbers unadjusted to this factor could lead to some degree of understatement. However, by adjusting the second term to some extent for the greater proportion of unpaid or unreported claims that would be expected in those cases, would produce more accurate estimates.

Another time when this factor should be changed is in respect of some of the short-tail casualty lines. Much of the non-marine or other class includes classes like bankers' business, personal accident and so on. Particularly in the bankers' class of business the results have deteriorated very significantly in recent years and looking at the incurred and paid claims ratio at three and four years of duration for underwriting years say, 1982 or 1983, these are, perhaps, much worse results than would have otherwise been anticipated if those losses had not been there, or in the case of a syndicate that had not been writing that business. By adjusting these factors for that, adjustments in the market overall can be taken into account for changing the percentage unreported and unpaid. It should be noted that the increased level of bankers' losses for example in this sort of business, will erroneously lead to higher estimates on the basis of the authors' estimate because they are putting credibility on those higher weighted ratios.

One important point to consider in the Lloyd's market if my approach, and indeed to some extent the authors' approach unadjusted is adopted, is that for certain general classes, such as the London market excess business, the premiums and claims are not accurately matched for audit code purposes. This does not matter when applying a straighforward percentage, but it is important when mixing the two methods.

The concept of variability introduced by the authors is important. Indeed their introduction of this concept is very much to be welcomed and this follows through into their discussion of solvency. Their analysis covers only part of the variation in weight loss ratios—that is, that based on observed historical variations. It does not deal with the variation which arises due to the change in the mix of business or market conditions. Their approach could therefore be further supplemented by breaking down the formula into its component parts and analysing the uncertainties of both further. This variation is not readily susceptible to mathematical measures, though sensitivity analysis can easily be carried out to adapt for changes in conditions. More importantly, they can also be used as a basis of discussion with underwriters or general market practitioners and this is a very fruitful source of additional information in carrying out reserving analysis.

Part II seems to have a slightly circular logic in that the ultimate loss ratios are initially estimated by Craighead's method and then regressed against development factors. Essentially this produces the same combination of a projection method and the Bornhuetter–Ferguson as before. It is not apparent why the Craighead method, or indeed any other method, should not be combined with the Bornhuetter–Ferguson method judgementally rather than the straightforward non-judgemental approach of the authors. The only real advantage would be if there is very little information about the underlying account and where there are only data to analyse and it is impossible to talk to underwriters or auditors in any way. The approach would be extremely useful for making estimates about another company, or syndicate of broad market data or D.T.I. returns, and presumably this is what the author intends. The method is particularly well suited for this and I see it as particularly suitable for a regulatory body using this sort of approach and asking a particular syndicate or company to justify differences from that—but greater elaboration would be very helpful to me. The introduction of confidence intervals is helpful, but it is important to realize that there is a much greater error in the actual estimates than that arising from the confidence intervals. The uncertainty due to change in mix of business and the change in market conditions rate levels is not taken into account by the authors and this is important when considering both solvency and other considerations. Indeed the variation from this source can in certain cases be much greater than that indicated by the authors.

**Mr J. Brecknell** (a visitor): As an accountant I should like to congratulate the authors, in that at this stage in Lloyd's history they have done a great service to those of us who are trying to manage syndicates, by producing something simple that our auditors and our underwriters will be able to understand. Even better those, such as myself, who have to act as Managing Directors can also get somewhere close to understanding it. This is the virtue which I see the authors as having contributed to the Lloyd's market, together with the fact that at this stage Lloyd's is about to embark upon a journey of unknown destination, a little *pas de deux* with the Revenue. I believe that the authors' contribution is going to be of great value to many syndicates in their battles to come.

**Mr G. Ward** (a visitor): As a chartered accountant and auditor, I welcome this most constructive paper. Professor Benjamin in particular has expressed concern over the years regarding the need to combine the professional skills of actuary and accountant in order to improve the quality of financial information. He has also worked hard to improve communication between these professions in order to improve the quality of presentation of that information. Those concerns are again coming up trumps. Carefully used, the paper should greatly assist auditors in taking a better informed view of the acceptability of their clients' claims estimates.

I have had staff working on applying the ideas of the paper to certain clients. Unfortunately it is too early as yet to assess the results of this work and report them to you. This enthusiasm, however, is tempered with realism. I would not wish formulae to be incorporated into legislation in a way which would encourage their mindless application to individual circumstances without taking proper account of the ways in which those individual circumstances differed from the mean.

Two difficulties greet the cynical eye of the auditor who sometimes has to view professional indemnity business from angles other than the merits of establishing the adequacy of the underwriter's reserves. He must be able and prepared to defend himself to the informed layman, sitting on the bench in the High Court of Justice. These two difficulties are firstly the changes in the nature of the account, in particular looking at the wide fluctuations in currency parities that can occur from year to year, and the changes in the scope of the risk that is being taken on by underwriters. This latter is especially important in indemnity and liability business, where the fundamental nature of the risks underwritten, and which people are willing to underwrite, seems to be re-assessed from year to year. The second point is the reliability of first and other early year data, where the auditor must be alert to manipulation as well as to random fluctuations and other changes in market circumstances: for example, recent business practice changes in London market operations generally; the impact of the recent hardening of rates; and the changing methods and speeds of processing claims notifications and claims settlements. Earlier speakers have referred to the need to make suitable adjustments for these matters. If only we auditors and our clients could so easily form a view of suitability.

**Mr P. H. Hinton:** The method proposed by the authors is to me intuitively very appealing, being simple and apparently easy to use. The form of the calculations leads me to suspect that it is likely to prove reasonably robust in practice. Quite how the statutory authorities would react to its use, I cannot of course anticipate.

In § 11, the authors state that they test "whether the calculated slope is statistically significantly different from zero and use a horizontal path where it is not". This is a somewhat odd null hypothesis. It implies that, for such a category of business, it is appropriate to assume that the higher the claims that have been paid, the lower the claims that are outstanding; that is, that high paid claims represent a bringing forward of payments rather than an increase in total claims. Although there will be situations where this is the case, I would take as my null hypothesis that future claim payments are independent of claims already paid. Where no premiums are outstanding this would involve testing whether the slope of the graph differed significantly from unity. In the first two years when significant amounts of premium may be outstanding, a somewhat more complicated test would have to be employed. Perhaps the authors could explain why they adopt their particular null hypothesis.

An advantage of the method is that it naturally gives rise to confidence intervals which provide further useful information, even though not all the variation can be regarded as stochastic. I agree with Mr Taylor that, since to use the data for the latest year it is necessary to project the estimated ultimate loss ratios, the width of the confidence intervals will normally be underestimated. Do the authors have a method of adjusting for this effect in their calculation of the confidence interval?

I would like to see some examples of the method applied to longer-tailed accounts, and how adequate reserves set up by this method proved to be in practice, as an account developed.

**Mr T. G. Clarke** (closing the discussion): This paper repeats the same writing techniques used by Professor Benjamin in his earlier papers to the Institute. In particular, they provide a clear, simplistic approach which all of us, whether mathematically inclined or not, are able to comprehend. Our profession has much to learn from this approach, especially those working in non-life business. It is a pity that some members are offended by this method of explaining the concepts being used on the basis that it does not incorporate profound mathematical techniques and formulae, and thus in some way devalues the professional advice and assistance, although after listening to the discussion, I think we all now accept the authors' approach.

It is important that in non-life we use methods which easily communicate the concepts to our non-life colleagues or clients and therefore I would agree wholeheartedly with the use of graphs for indicating the trends and emphasizing the problems of variability. We all know that there is no unique answer in projecting outstanding liabilities, but frequently there seems to be an acceptance in general insurance that the estimate of outstanding liability is sacrosanct and must be accorded a degree of accuracy or fact which it does not warrant. While it is important for the actuarial adviser to satisfy himself or herself on the size of the potential viability, using more complicated techniques in explaining the results to his clients, the use of the cruder method, or for instance the upper and lower limits of variability based on a parallel line or a funnel of doubt through the highest and lowest values experienced from the mean line, is a much easier method for the layman to understand than mathematical confidence limits.

What concerns me are the methods related to the longer-tailed accounts where there is a lack of data to produce a reasonable fit to the ultimate liability. Especially for the longer-tail accounts we could well find that the underwriter changes and therefore the constituent parts of the account also change, thus changing the statistical development patterns.

We frequently hear the phrase "but you cannot use actuarial techniques in non-life", sometimes even within this profession. What are actuarial techniques? I believe they are primarily the use of mathematical formulae. Actuaries are trained in logical thinking; they are expected to analyse a problem, create a model solution, make predictions and then follow these predictions by analysing in subsequent years why the model did not fit. If we as a profession are not capable of analysing non-life data, who is? Should those working with non-life data, be they underwriters, claims staff or management just guess, or should they have the problem analysed for them by professional people who are equipped in financial statistical analysis?

One of our greatest attributes as a profession is the fact that we know how wrong we can be and therefore because of this we should not avoid giving the non-life industry the assistance of our knowledge because we know we will get it wrong. Others will try to give this assistance without understanding the potential errors. While we will get it wrong, my experience, having worked in non-life for sixteen years, and for the last eight years working with London market data, suggests that we are more likely to indicate the probable area of the result than many others. It is time for this profession to take a higher profile in the non-life industry. The authors have shown how very difficult and frequently non-homogeneous data can be taken and made some logical sense of, and give very helpful information to the Lloyd's market as to how it is performing. I am certain it will be found that this method is more robust than the current Lloyd's factors, although at the same time the method will not, and should not, be expected to produce exactly the right answer. In this I totally support the opener and others.

**The President (Professor P. G. Moore):** Any paper from these two authors is likely to be interesting and trail blazing. This paper lives up to expectations and it is particularly exciting to see our newest and youngest gold medallist once more in action with more quality research on a fresh topic. The paper is to be welcomed the more, not only because it contributes to an area—namely, general insurance—that we have only opened up among our activities in recent years, but also because it demonstrates, dealing as it does with Lloyd's insurance market, that the Institute is capable of picking up a current issue of some importance, to research it and then to put forward perceptive and soundly based measuring rods that should assist in the better regulation of the market.

I was also particularly pleased to note the reference in § 3.3 to the power of the human eye to absorb information through visual and graphical displays, which has been commented upon this evening. The late Professor Egon Pearson, who was an Honorary Member of this Institute for many years, devoted his Presidential Address to the Royal Statistical Society to an analysis of the geometrical representation of statistical data as an aid to its effective analysis. The paper led actually to a subsequent spirited exchange between Pearson and Sir Austin Bradford Hill, also and still an Honorary Member of this Institute, in that Sir Austin asserted that he could spot data patterns through effective tabulation of differences ratios, etc. rather better than he could with geometrical displays. I think in the end it was suggested that it was a draw and individuals do differ in this respect, but both did agree on the principle that preliminary data analysis of one or other forms is highly desirable before esoteric forms of statistical analysis are even considered.

It gives me therefore much pleasure to propose a vote of thanks to Messrs Benjamin and Eagles for their paper tonight.

**Professor S. Benjamin** (replying): The work in the paper was intended to do two separate things. It was intended, first, to help the Lloyd's Members' Solvency and Security Committee, which I always used to call the Audit Committee, in a problem that had been posed to them. The terms of reference were a little vague, but essentially they meant "Can we improve on what we are doing at the moment?" We accepted that minimum reserving was going to continue in Lloyd's. I think it should, but there is a danger to which people have referred of putting things into legislation.

The second subject of the paper is the use of the method by individual syndicates and companies. What we say to them when we show the results of this analysis is: "What we are doing is asking you to look at your own data". A phrase that I use time and time again when talking to the Lloyd's people—and I gave something like twenty presentations of the method—is "If there is a pattern, you can use it. There may not be a pattern. That in itself is interesting". We take the results to a company or to a syndicate and say: "Let us discuss your own data and let us discuss what is happening. Are your reserves reasonable?" As Mr Ryan said, "Let us have a look at what is happening in the market. You probably know that better than we do". That ought to be a jumping off point for discussion.

Syndicates may feel that the method is authoritative. In fact, we have recently carried out another analysis as a result of a very large number of replies to our questionnaire that was sent out, and I would guess that the present minimum percentages are used as reserves when they cannot think of anything else. Maybe they will do that with the new method too.

The effect of the new method will probably be to increase some reserves at the lower end and allow some reserves at the stronger end actually to be lower. Overall, the interesting point is that the new method gives answers which in total for all of Lloyd's are very approximately the same as the present method, but it does swing the figures around.

I cannot talk about them in detail, but about the actual level of reserves that are set up I would like to say personally that I was very pleasantly surprised.

I was accused in one presentation that the method is mechanical and uses no judgement. My reply was along the lines suggested tonight: "We can use your judgement, but perhaps we could start by seeing how well your judgement has worked in the past. Let us take your paid plus outstandings and use those instead of the paid, because they incorporated your judgement. Let us see how well it did. If we plot those on the horizontal axis and plot the ultimates along the vertical axis, then if you have got it right you should have a straight line going right through the origin at 45 degrees. In so far as that is not the result, it is actually a test of how well you did in the past. Let us suppose for example that the path we get on this simplistic method is wider when we put your past judgement in than if we just use it mechanically on the paid-to-date, then presumably you really ought to justify whether we should be using your judgement now".

That is another example of the way in which this simplistic method can be used to force people to contemplate harder what they are really doing. I do not see this actually as a statistical method. I see this as what the President called preliminary data analysis. I think the usual jargon is "exploratory data analysis", which is a subject which is developing in its own right. Let us not put it in the legislation!

I think that we could have better statistical approaches. My own favourites are to move along the lines that Mr Taylor mentioned to use the variability of the development factors themselves, and the other is to use a paper that was printed in *J.I.A.* **110,** 157, Claims Reserving, State-space Models and The Kalman Filter by Piet de Jong and B. Zehnwirth, where the two Australian authors suggested using a linear combination of standard run off patterns and controlling the coefficients using adaptive control by the Kalman filter. I recommend the paper to you.

I thought you might like to hear about the present work that is going on in the Institute's Research Committee. Tonight's paper gets away from the chain ladder method. We are in the odd position that in the General Insurance Study Group's conferences year after year, culminating in the report by Mr Truckle's working party, we keep panning this chain ladder method, and his working party, I would remind you, actually said that the method was hopeless.

What we are trying to do is to follow up the paper and the booklet by Mr G. Lyons' working party and the G.I.S.G. conference and to aim at what I personally would like to see as a loose-leaf publication from the Institute on methods of reserving for outstanding claims.

A start has been made. We have some volunteers. We are going to build on that booklet and we should like some more volunteers. At the same time, as a separate activity, a few companies have been asked to fill in some data forms, firstly to help the working party on solvency under Mr Daykin, and secondly to examine a method of assessing the "overall financial condition"—I quote from the legislation—"of a company". So I do hope that those actuaries who are associated with the companies who have been asked to do this will use their good offices to help the project forward.

Dr Coutts has been asking for some while that we should set up a data base. I have been pessimistic about it because I have so often met the paranoia that exists in the general insurance world about revealing any piece of information at all. But I think it is an excellent idea. Perhaps the Department of Trade and Industry and the accountants, can help us in an informal fashion, to do that. Let us try to get some better data.

Can I put the point that perhaps as computers and telecommunications develop, we may reach the stage where the reinsurer can actually tap in to the files of the ceding company. Then the whole situation about doing business without information will change altogether.

There is a terrible problem for us of presentation. I would agree with Professor Wilkie for example that you could use more of the the data in a multiple linear regression, but once you go beyond two dimensions you have a real problem of presentation, and presentation, as Mr Clarke said, is terribly important because you need to involve the practitioner who has actually got to see what you are doing, otherwise he just cannot join in the discussion, and that is no good for anybody.

In one of the presentations I was showing how the "path" was developed. I said we draw a line through the points and then we take the point that is furthest away, draw a parallel line through that and then we reflect it on the other side so that we get a symetrical path. In one of the presentations there was a man attending who was a lawyer. I think he was a Name. He had been once before and he had come back. He had decided this was an important subject and he was determined to understand everything I said. Of course, that is the best sort of audience, but when I came to describing how we developed this path he just could not understand it. I could not get him to understand how we had done it. In the end, I went round and had a look at his copy of the report with the graphs. By some quirk of photocopying or computer out-put, that poor man had an extra cross on his graph!

## WRITTEN CONTRIBUTIONS

**Mr P. N. Matthews and Mr R. A. Hurst:** The paper represents a major leap forward by the Institute in demonstrating to Lloyd's and the London Reinsurance Market the practical usefulness of involving actuaries to assist in the interpretation of underwriting results. We applaud the authors for producing such a readable and comprehensive paper.

Having tested the model on our company's reinsurance data, we make the following comments.

The model in the paper has been used on very old data. Since the late 1970's there has been an increasing variability in results due to larger and a greater incidence of claims. We wonder how robust the model is to these changing circumstances. Having said this, the model gave reasonable results on Proportional business, be they long tail or short tail. When looking at Direct and Facultative business, no conclusive results could be found. Even at the four year stage, no trend could be established.

The model does not appear to give meaningful results if there is very little spread in the ultimate loss ratio. In the paper there is a spread of 70 points in the ultimate loss ratio. In our Property D&F business there was only a spread of 20 points which resulted in a bunching of points, and no real trend.

For the long tail classes of business, there may be zero paid claims for several years. An alternative method could project based on Paid Claims plus Case Outstanding Losses. For Product Liability business for instance, the notified claims after one year may consist solely of outstanding loss cases and represent less than 10% of the final ultimate claims. Projections based purely on paid losses generate tremendous gearing variances in the early years on a long tail account. We would be interested to see the results from the model, by using Notified Losses, instead of just Paid Claims, to see how consistent the two sets of results are.

Figure 4 suggests that in cases where the paid loss ratio at the end of the first year is in excess of 25%, the Lloyd's minimum percentage gives a much lower result than the new method. It follows that in all these cases where the line gradient exceeds unity there could be considerable under-reserving at Lloyd's. In our opinion that is most likely to occur during the early years of development. Has any measure of this been assessed, and if so what are the results?

Within the last three underwriting years there have been considerable losses from:

| | |
|---|---|
| 1983 | Alicia |
| | Winter Storms in U.S.A. |
| 1984 | Satellite Losses |
| 1985 | Aviation Losses |

The catastrophes must have a severe impact on Lloyd's results, and it would be useful to see how the model fits or has to be adapted for these years.

Finally, we congratulate the authors once again on their useful contribution towards the actuarial exploration of general reinsurance business.

**The authors:** There were many interesting points made in the discussion. Mr Gillott has pointed out that a line of best fit which is negative does not imply that the ultimate will be less than the paid-to-date. He is correct. As he says, it implies that the lower the paid loss ratio the higher the ultimate and *vice-versa*. If the points are in chronological order then the speed of settlement is decreasing. Whether or not the line of best fit is negative, the points on the line of best fit above a line $y=x$ imply that the ultimate loss ratio will be greater than the paid.

We do not understand his suggestion that the path should narrow as it runs away from the historical data. Normally the confidence path of regression widens into a funnel of doubt as it moves outside the data on which it is based, as several other speakers have mentioned. We have chosen a parallel path for simple presentation, but we do vary our approach according to the sophistication of the client.

We suspect Mr Taylor is right that the incestuous nature of the method constrains the variability of the results but, we think, not necessarily towards a linear path.

Professor Wilkie has drawn attention to many statistical points which ought to be explored. We agree. With regard to his point about Figure 2 that there seems to be some auto-correlation between the years we have received a letter from an American statistician, Mr Peter K. Reilly, that he split off the observations from 1970 onwards using regression with a dummy variable and obtained a significant answer. We have reproduced his figures but do not think that in this particular case there is an improvement. However, we do think the method is worth using.

Professor Wilkie asked what function we are minimizing. It is the square of errors weighted by time; it is the same function as was used by Mr Craighead.

We would comment on one of Mr Hart's remarks to the effect that an estimate of ultimate loss ratio of $118\% \pm 16\%$ sounds useful until it is re-expressed allowing for a paid loss ratio of, say, 115%, when it becomes a reserve of $3\% \pm 16\%$. There are two aspects. The first is that the reserve could be heavily negative. That is a problem related to the distribution of the error terms to which Professor Wilkie referred. The second implies that the considerable width of the path makes the result useless. However, suppose that this is a good representation of the statistical distribution of the outstandings i.e. suppose that that is the reality. In what sense is that result unacceptable or useless? We would argue that a philosophy which cannot incorporate that size of fluctuation as a fundamental feature of reality into its approach to reserving, to emergence or surplus and recognition of profit, is a deficient philosophy. Our § 19 indicates how it can be done.

With regard to Mr Ryan's discussion of the Bornhuetter–Ferguson method, we do not ourselves see our approach as having anything to do with that method. It seems to depend on a purely judgemental estimate of the ultimate, whereas we are attempting a more direct estimate from the data.

Mr Hinton suggested looking at whether the slope of the line was significantly different from +1. We think he is looking at an interesting but different model which is worth investigating.

There are a few points where the discussion indicates that our paper may have given a misleading emphasis.

For the Lloyd's minimum reserving work only the paid was available. Where the incurred is also available we look at both and mostly use the incurred. Similarly, quarterly data are often available, and we use them.

In practice we use our familiarity with the Craighead curve to apply some judgement to the choice of parameter values in difficult cases.

Finally, Dr Coutts has hit the core of the problem. A professional report would be better than any formula method.

We would like to thank all contributors. They have given us much to think about.