The Actuarial Profession
making financial sense of the future

GIRO Convention

23-26 September 2008
Hilton Sorrento Palace

Robotic reserving

Greg Taylor
Taylor Fry Consulting Actuaries
University of Melbourne
University of New South Wales

Gráinne McGuire
Taylor Fry Consulting Actuaries

Overview

- Robotic loss reserving
  - What is it?
  - Why is it of interest?
- Requirements of a robot
- Main components of the robot
- Robot supervision
- Future development

Robotic loss reserving – what is it?

- P&C loss reserving
  - Estimation of liabilities for incurred but incomplete claims
    - Central estimate (i.e. mean value)
    - Stochastic properties (statistical distribution of the amount of liability)
- **Robotic** (or **adaptive**) loss reserving
  - Software that will produce this output over a sequence of valuation dates
    - Without human intervention
    - With no significant loss of accuracy due to lack of intervention

Robotic loss reserving – why is it of interest?

- Valuation revolving door
  - Many insurers now wish to conduct frequent liability valuations
    - e.g. quarterly
  - They may have 50 or more lines and sub-lines of business recognised for valuation purposes
    - These require separate identification of valuation liabilities because of structurally different models of the claim process
  - These may have many segments
    - State, distribution channel, etc
    - These require separate identification of valuation liabilities for management and/or strategic reasons, e.g. profit measurement
  - These requirements mean that the performance of a quarterly valuation can take about 3 months
    - One valuation ends, another begins

Robotic loss reserving – why is it of interest?

- Valuation revolving door
  - Obvious advantages in automating such quarterly valuations
  - Once an insurer contemplates a move to monthly valuations, conventional actuarial valuation ceases to be feasible at all
  - A robot is the only option

Reserving by means of roll-forwards?

- Rolling a valuation forward
  - Consider the case of full half-yearly valuations
  - Roll these forward to provide intermediate monthly valuations
    - Assume that each half-yearly valuation remains valid over the following 5 months

> Value of liabilities at any one of these months =
> Value at previous half-yearly valuation
> **less** claims paid since then
> **plus** allowance for claims incurred since then

- Problem here is that the monthly series of loss reserves tends to run smoothly for 5-month periods with 6-monthly shocks
  - Roll-forwards not reliable

The Actuarial Profession
making financial sense of the future

Requirements of a robot

Requirements of a robot

- Must recognise changes in its environment
  - e.g. door is now closed instead of open as it was a minute ago
- Must be able to respond appropriately to these changes
  - e.g. don't attempt to pass through doorway without first taking action to open the door

8 The Actuarial Profession making financial sense of the future

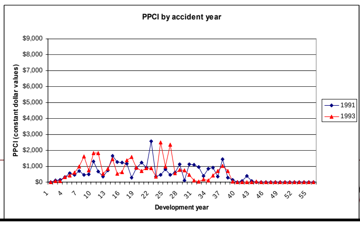
Requirements of a **loss reserving** robot

- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period

9 The Actuarial Profession making financial sense of the future

Requirements of a **loss reserving** robot

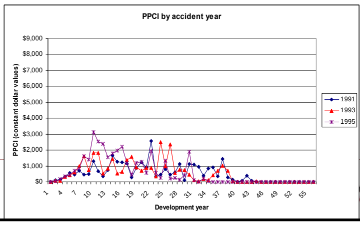
- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period
    - example of Payment per Claim Incurred (PPCI) by accident year



Actuarial Profession
ing financial sense of the future

Requirements of a **loss reserving** robot

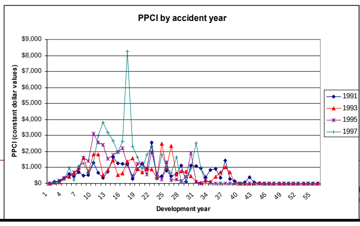
- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period
    - example of Payment per Claim Incurred (PPCI) by accident year



Actuarial Profession
ing financial sense of the future

Requirements of a **loss reserving** robot

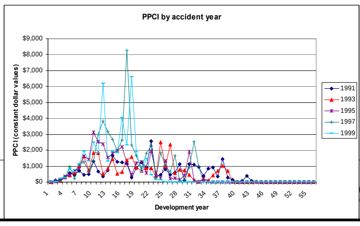
- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period
    - example of Payment per Claim Incurred (PPCI) by accident year



Actuarial Profession
ing financial sense of the future

Requirements of a **loss reserving** robot

- Must recognise changes in its environment, e.g.
  - The amplitude of the payment pattern tending to increase with increasing accident period
    - example of Payment per Claim Incurred (PPCI) by accident year



The Actuarial Profession
making financial sense of the future

Requirements of a **loss reserving** robot

- Must recognise changes in its environment, e.g.
  - The tail of the payment pattern tending to extend with increasing accident period
  - Case estimates tending to develop more rapidly in more recent accident periods
- Must be able to respond appropriately to these changes
  - Model of claim process must evolve over time to reflect these changes

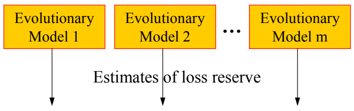
14

The Actuarial Profession
making financial sense of the future

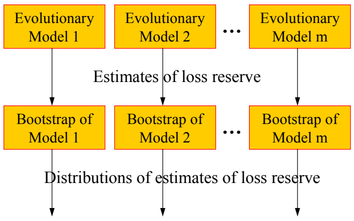
The Actuarial Profession
making financial sense of the future

Robot design

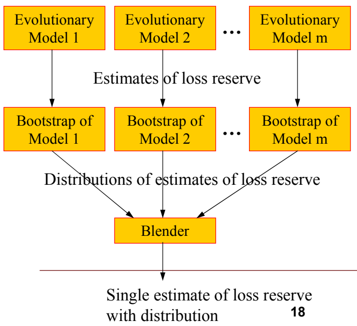
Main components of the robot



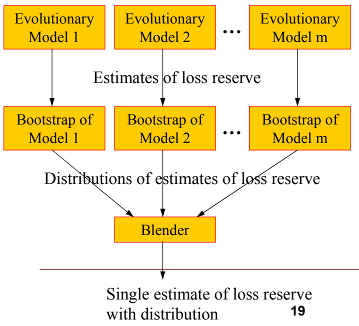
Main components of the robot



Main components of the robot

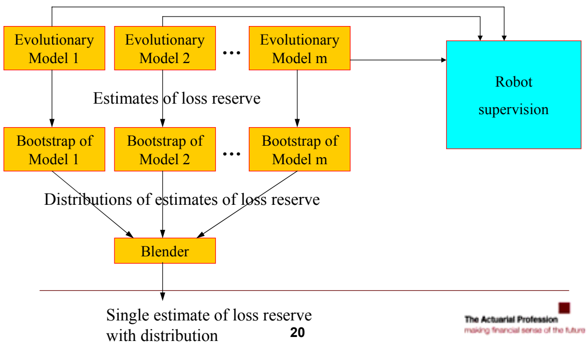


Main components of the robot



Taylor & McGuire (2007)

Main components of the robot



Evolutionary models

- These are **Dynamic Generalised Linear Models (DGLMs)**
- Model form is:

$$y_t = h^{-1}(X_t \beta_t) + \varepsilon_t \quad \text{[GLM for period t]}$$

Data vector – *Link function* – *Design matrix* – *Parameter vector* – *Centred stochastic error*

Evolutionary models

- These are **Dynamic Generalised Linear Models (DGLMs)**
- Model form is:

$$y_t = h^{-1}(X_t\,\beta_t) + \varepsilon_t \quad \text{[GLM for period t]}$$

Data vector (y_t); *Link function* (h^{-1}); *Design matrix* (X_t); *Parameter vector* (β_t); *Centred stochastic error* (ε_t)

$$\beta_{t+1} = G_t\,\beta_t + \eta_t \quad \text{[parameter evolution]}$$

Matrix (G_t); *Centred stochastic perturbation* (η_t)

Evolutionary models

- These are **Dynamic Generalised Linear Models (DGLMs)**
- Model form is:

$$y_t = h^{-1}(X_t\,\beta_t) + \varepsilon_t \quad \text{[GLM for period t]}$$

Data vector (y_t) – *e.g. row of a run-off triangle*; *Link function* (h^{-1}); *Design matrix* (X_t); *Parameter vector* (β_t); *Centred stochastic error* (ε_t)

$$\beta_{t+1} = G_t\,\beta_t + \eta_t \quad \text{[parameter evolution]}$$

Matrix (identity in our case) (G_t); *Centred stochastic perturbation* (η_t)

Forecasts

- Forecast of y_{t+1} by means of **adaptive filter**
  - Hence **"adaptive reserving"**
- Notation: let

$$Y_{t|s} = E(Y_t|\text{data from } 0,1,\ldots,s)$$

- Estimate

$$Y_{t|t} = Y_{t|t-1} + K_t\,\{y_t - Y_{t|t-1}\}$$

Gain matrix (K_t); *Realised value of* Y_t (y_t)

or (depending on link function and error terms)

$$Y_{t|t} = Y_{t|t-1} + K_t\,\{[\text{DIAG } Y_{t|t-1}]^{-1}\,y_t - 1\}$$

Adaptive filter

- This is a second order approximation to Bayesian updating of the parameter vector β_t (Taylor, 2008)
- It holds for following cases

h	ε	η
identity	normal	normal
log	Poisson	gamma
log	gamma	gamma

25

The Actuarial Profession
making financial sense of the future

Adaptive filter

- This is a second order approximation to Bayesian updating of the parameter vector β_t (Taylor, 2008)
- It holds for following cases

	h	ε	η
Kalman filter →	identity	normal	normal
	log	Poisson	gamma
	log	gamma	gamma

26

The Actuarial Profession
making financial sense of the future

Adaptive filter (cont'd)

- Proceed in 3 stages updating 1-step-ahead forecast from $Y_{t|t-1}$ to $Y_{t+1|t}$
  - Update $Y_{t|t-1} \rightarrow Y_{t|t}$ as just illustrated
    - Also update $Var[Y_{t|t-1}] \rightarrow Var[Y_{t|t}]$
    - Extract updated parameter estimate $\beta_{t|t}$

27

The Actuarial Profession
making financial sense of the future

Adaptive filter (cont'd)

- Proceed in 3 stages updating 1-step-ahead forecast from $Y_{t|t-1}$ to $Y_{t+1|t}$
  - Update $Y_{t|t-1} \rightarrow Y_{t|t}$ as just illustrated
    - Also update $Var[Y_{t|t-1}] \rightarrow Var[Y_{t|t}]$
    - Extract updated parameter estimate $\beta_{t|t}$
  - Update $\beta_{t|t} \rightarrow \beta_{t+1|t}$ by means of formula for assumed parameter evolution (in our case $\beta_{t+1|t} = \beta_{t|t}$)
    - Also update $Var[\beta_{t|t}] \rightarrow Var[\beta_{t+1|t}]$

28

The Actuarial Profession
making financial sense of the future

Adaptive filter (cont'd)

- Proceed in 3 stages updating 1-step-ahead forecast from $Y_{t|t-1}$ to $Y_{t+1|t}$
  - Update $Y_{t|t-1} \rightarrow Y_{t|t}$ as just illustrated
    - Also update $Var[Y_{t|t-1}] \rightarrow Var[Y_{t|t}]$
    - Extract updated parameter estimate $\beta_{t|t}$
  - Update $\beta_{t|t} \rightarrow \beta_{t+1|t}$ by means of formula for assumed parameter evolution (in our case $\beta_{t+1|t} = \beta_{t|t}$)
    - Also update $Var[\beta_{t|t}] \rightarrow Var[\beta_{t+1|t}]$
  - Update $Y_{t|t} \rightarrow Y_{t+1|t}$ using $\beta_{t+1|t}$
    - Also update $Var[Y_{t|t}] \rightarrow Var[Y_{t+1|t}]$

29

The Actuarial Profession
making financial sense of the future

Adaptive filter (cont'd)

- Proceed in 3 stages updating 1-step-ahead forecast from $Y_{t|t-1}$ to $Y_{t+1|t}$
  - Update $Y_{t|t-1} \rightarrow Y_{t|t}$ as just illustrated
    - Also update $Var[Y_{t|t-1}] \rightarrow Var[Y_{t|t}]$
    - Extract updated parameter estimate $\beta_{t|t}$
  - Update $\beta_{t|t} \rightarrow \beta_{t+1|t}$ by means of formula for assumed parameter evolution (in our case $\beta_{t+1|t} = \beta_{t|t}$)
    - Also update $Var[\beta_{t|t}] \rightarrow Var[\beta_{t+1|t}]$
  - Update $Y_{t|t} \rightarrow Y_{t+1|t}$ using $\beta_{t+1|t}$
    - Also update $Var[Y_{t|t}] \rightarrow Var[Y_{t+1|t}]$
  - Iterate

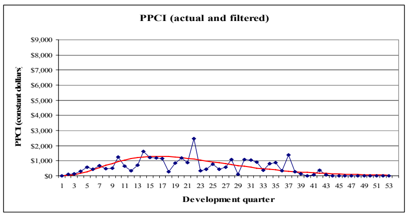
30

The Actuarial Profession
making financial sense of the future

Adaptive filter - examples

Data from earlier examples

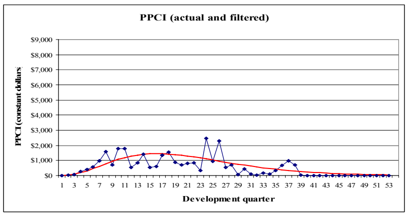
Accident year 1991



The Actuarial Profession
making financial sense of the future

Adaptive filter - examples

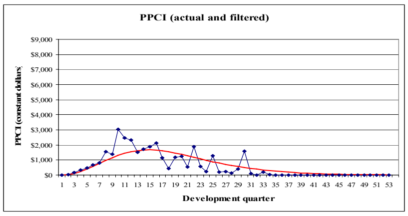
Accident year 1993



The Actuarial Profession
making financial sense of the future

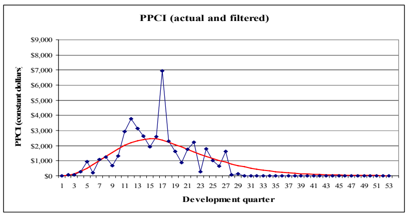
Adaptive filter - examples

Accident year 1995



The Actuarial Profession
making financial sense of the future

Adaptive filter - examples

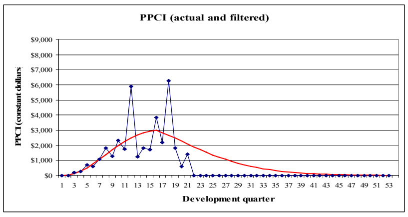


Accident year 1997

34

The Actuarial Profession
making financial sense of the future

Adaptive filter - examples

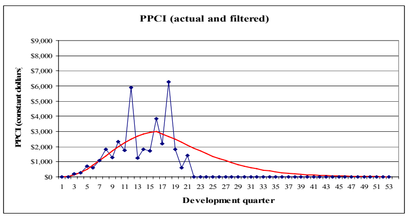


Accident year 1999

35

The Actuarial Profession
making financial sense of the future

Adaptive filter - examples



Accident year 1999

N.B. filter has been applied to accident periods

36

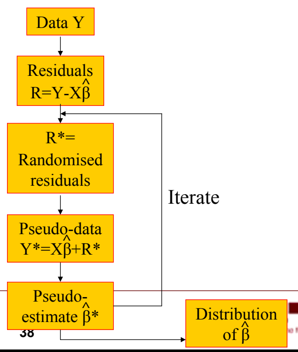
The Actuarial Profession
making financial sense of the future

The Actuarial Profession
making financial sense of the future

Bootstrapping

Bootstrapping a filter

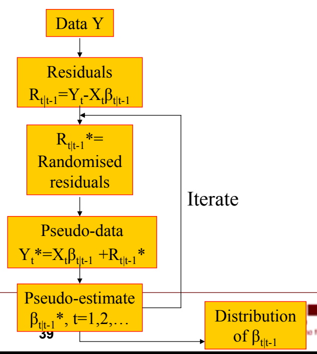
- Recall standard form of bootstrap for regression model



38

Bootstrapping a filter

- Recall standard form of bootstrap for regression model (adapted to filter)

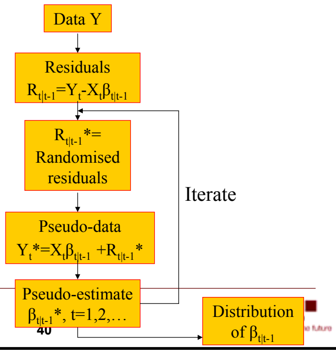


39

Bootstrapping a filter

- Recall standard form of bootstrap for regression model (adapted to filter)
- Problem here is that bootstrap assumes components of residual vector R are independent
- For a filter

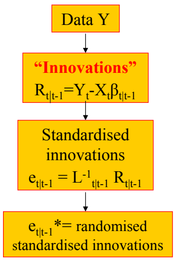
  $R_{t|t-1} = Y_t - X_t\,\beta_{t|t-1}$
- The components of $R_{t|t-1}$ are **NOT** independent because $\beta_{t|t-1}$ has been calculated from $R_{t-1|t-2}$, $R_{t-2|t-3}$, …



Bootstrapping a filter – correct version

- Procedure provided by Stoffer & Wall (1991)
- Let

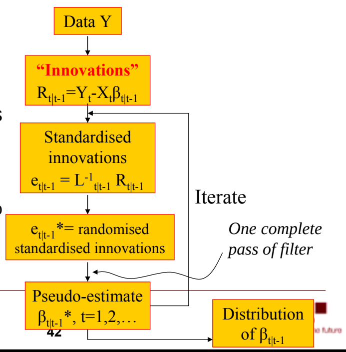
  $L_{t|t-1} = \text{Var}\,[R_{t|t-1}]$
- Standardised innovations are i.i.d.



Bootstrapping a filter – correct version

- Procedure provided by Stoffer & Wall (1991)
- Let

  $L_{t|t-1} = \text{Var}\,[R_{t|t-1}]$
- Standardised innovations are i.i.d.
- Note there is no need for pseudo-data
  - Filter is updated from t-1 to t using innovations only



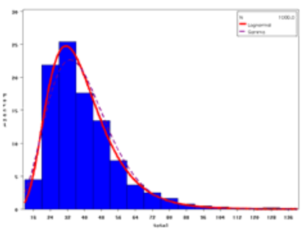
Example results of bootstrapping

- Using 3 forms of model
  - PPCI
    - Payments per claim incurred
    - Payment based
  - PPCF
    - Payments per claim finalised
    - Sensitive to the rate of settlement of claims
  - PCE
    - Projected case estimates
    - Sensitive to case estimates

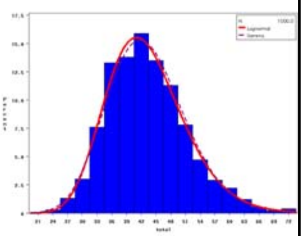
Accident year	PPCI Mean	PPCI CV	PPCF Mean	PPCF CV	PCE Mean	PCE CV
1	8	240%	132	55%	22	105%
2	20	213%	242	47%	56	108%
3	58	169%	165	58%	23	98%
4	110	132%	268	47%	70	90%
5	242	107%	861	30%	317	62%
6	291	74%	1,216	27%	671	64%
7	678	57%	1,257	27%	799	44%
8	817	52%	1,672	27%	1,319	40%
9	2,259	48%	3,366	25%	2,040	32%
10	3,544	48%	3,510	22%	2,368	31%
11	6,366	48%	6,041	21%	5,480	31%
12	7,182	44%	6,742	20%	6,700	31%
13	8,544	43%	8,664	21%	7,234	33%
14	9,001	43%	9,015	21%	3,749	98%
Total ex 14	30,119		34,136		27,099	
Total	39,120	42%	43,151	18%	30,848	22%

Example results of bootstrapping

PPCI



PPCF



Model blending

Model blending – inputs

- Results after filtering and bootstrapping m models consist of:
  - m sets of estimates of liability by accident year
  - m associated sets of standard errors of prediction
  - Case estimates by accident year

The Actuarial Profession
making financial sense of the future

46

Model blending

- Let
  $L_i^{(j)}$ = estimated liability for accident year i from model j
- Take final estimates as

$$L_i = \sum^{m}_{j=1} w_i^{(j)} L_i^{(j)}$$

with

$$w_i^{(j)} \geq 0$$

$$\sum^{m}_{j=1} w_i^{(j)} = 1$$

The Actuarial Profession
making financial sense of the future

47

Model blending - criteria

- We would like
  - MSEP[L] to be small where $L = \sum_i L_i$
  - $\sum_i \Delta^2 w_i^{(j)}$ to be small for each j
    - Smooth weights for each model
  - $\sum_i \Delta^2 [\log L_i/C_i]$ to be small where C_i denotes case estimates for accident year i
    - Smooth relation of final estimates to case estimates over accident years

The Actuarial Profession
making financial sense of the future

48

Model blending – objective function

- Problem addressed by Taylor (1985, 2000)
- Minimise the objective function

  $$Q = MSEP[L] + k_1 \sum_j \sum_i \Delta^2 w_i^{(j)} + k_2 \sum_i \Delta^2 [\log L_i/C_i]$$

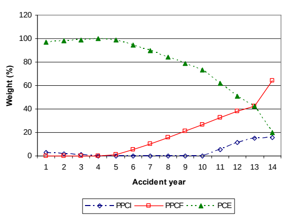
  with respect to the $w_i^{(j)}$, where k_1, k_2 are pre-determined constants that assign weight to the smoothness criteria

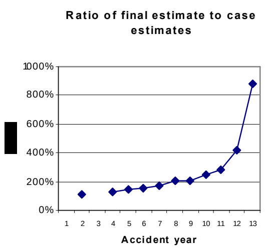
Model blending – example of results

Accident	PPCI		PPCF		PCE		**Blended**	
year	Mean	CV	Mean	CV	Mean	CV	**Mean**	**CV**
1	8	240%	132	55%	22	105%	**22**	**104%**
2	20	213%	242	47%	56	108%	**56**	**107%**
3	58	169%	165	58%	23	98%	**24**	**96%**
4	110	132%	268	47%	70	90%	**70**	**90%**
5	242	107%	861	30%	317	62%	**324**	**60%**
6	291	74%	1,216	27%	671	64%	**702**	**58%**
7	678	57%	1,257	27%	799	44%	**847**	**38%**
8	817	52%	1,672	27%	1,319	40%	**1,375**	**32%**
9	2,259	48%	3,366	25%	2,040	32%	**2,317**	**24%**
10	3,544	48%	3,510	22%	2,368	31%	**2,672**	**21%**
11	6,366	48%	6,041	21%	5,480	31%	**5,712**	**20%**
12	7,182	44%	6,742	20%	6,700	31%	**6,771**	**18%**
13	8,544	43%	8,664	21%	7,234	33%	**8,035**	**17%**
14	9,001	43%	9,015	21%	3,749	98%	**7,963**	**20%**
Total ex 14	30,119		34,136		27,099		**28,927**	
Total	*39,120*	42%	*43,151*	18%	*30,848*	22%	***36,890***	**13%**

N.B. smaller than for any individual model

Model blending – example of results





The Actuarial Profession
making financial sense of the future

Robot supervision

Need for supervision

- Robots affect business bottom line
- Need for strict supervision
- This takes the form of exception reporting
  - Using a range of diagnostics to test whether experience is deviating too far from model predictions

Example of supervision diagnostics

Accident year	Claim payments in latest period			
	Actual ($M)	Forecast ($M)	Actual: forecast	Significance
1991	0.7	1.6	44%	(>10%)
:	:	:	:	:
:	:	:	:	:
2007	25.4	21.0	121%	*** (<1%)
Total	**78.7**	**74.9**	**105%**	* **(5-10%)**

The Actuarial Profession
making financial sense of the future

Further development

Ample scope for further development

- Filter has been applied to accident periods (rows)
  - Could investigate application to diagonals
  - This could filter superimposed inflation parameters
  - Project currently under way
  - Appears more difficult

Further development (cont'd)

- Test performance of GLM filter against obvious alternatives
  - MCMC
    - Project currently under way
  - Particle filters
  - Neural nets
    - See e.g. Mulquiney (2006)

Further development (cont'd)

- Filter applied here to aggregate claim models
- Try application to micro-models (individual claims)
  - Excluding case estimate information (Taylor & McGuire, 2004)
  - Including case estimate information (Taylor, McGuire & Sullivan, 2007)

Questions?

References (1)

- **Mulquiney P (2006).** Artificial neural networks in insurance loss reserving. **Proceedings of the Joint Conference on Information Sciences 2006.** Atlantis Press. On-line proceedings at http://www.atlantis-press.com/publications/aisr/jcis-06/
- **Stoffer, D.S. and Wall, K.D. (1991).** Bootstrapping state space models: Gaussian maximum likelihood estimation and the Kalman Filter. **Journal of the American Statistical Association**, 86, 1024-1033
- **Taylor G (1985).** Combination of estimates of outstanding claims in non-life insurance. **Insurance: Mathematics and Economics**, 4, 321-438.

References (2)

- **Taylor G (2000). Loss reserving: an actuarial perspective**. Kluwer Academic Publishers. Boston, Dordrecht, London.
- **Taylor G (2008).** Second Order Bayesian Revision of a Generalised Linear Model. **Scandinavian Actuarial Journal** (to appear)
- **Taylor G & McGuire G (2004).** Loss reserving with GLMs: a case study. **Casualty Actuarial Society 2004 Discussion Paper Program,** 327-392.
- **Taylor G & McGuire G (2007).** Adaptive reserving using Bayesian revision for the exponential dispersion family. Appears at http://www.economics.unimelb.edu.au/SITE/actwww/wps2007/No165.pdf (submitted for publication)

References (3)

- **Taylor G, McGuire G & Sullivan J (2007).** Individual claim loss reserving conditioned by case estimates. Research paper commissioned by the Institute of Actuaries. Appears at http://www.actuaries.org.uk/files/pdf/library/taylor_reserving.pdf (submitted for publication)