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GENERAL INSURANCE PRICING SEMINAR

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Principles of Non-Proportional Reinsurance Technical Pricing
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Principles of Technical RI Pricing

- Technical price has three main components:
  - Expected claim payments
  - Expenses
  - Risk load
- Risk load determines the expected return on capital: it should be commensurate with risk (*ie* uncertainty)
- Main element of risk is uncertainty in claim payments
- So an essential part of technical pricing is finding probability distributions for RI claim payments (frequency and severity)

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Approaches to Technical RI Pricing

	Present values	Cash flows
Individual RI contract	Simple, quick, reasonableness check	
Portfolio of contracts		Theoretically best but complex

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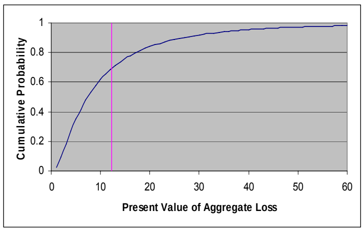
Individual Contract PV approach

- Suitable for:
  - Simple RI contracts
  - Quick calculations
  - Reasonableness check on more sophisticated methods
- Two main steps:
  - Find probability distribution of aggregate loss (present value) under the proposed RI contract
  - Determine pure premium and technical risk load from this probability distribution

Individual Contract PV approach



Risk load:
- Std deviation
- Percentile
- Tail VaR
- PHT

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Probability distribution of agg loss to RI

- Probability distribution of individual loss to reinsurer (X)
  - Distribution of gross individual loss (experience, exposure, industry data)
  - Apply limits and retentions (eg 90% of £2m xs £1m)
- Prob distribution of number of individual losses (N)
  - Variance > Mean (do not use Poisson!)
- Calculation of aggregate loss to RI
  - $T = X_1 + X_2 + \ldots + X_N$
  - Claim amounts independent of claim counts?
  - Find probability distribution F(T)
- Parameter and model uncertainty (sensitivity analysis)
  - Calculate mixture distribution: $F(T) = \Sigma p_i\, F_i(T)$

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Individual claim amount distribution

- Usually not enough experience to use distribution of actual past losses
- Instead use theoretical loss distribution (Log-Normal, Pareto etc)
- Calibrate to all relevant information:
  - actual past losses (FGU, losses to layer)
  - project individual losses to ultimate
  - adjust for claims inflation: care with losses to layer
  - industry data (eg increased limits factors)
  - percentiles (perhaps based on judgement)
- Allow for changes in exposure profile (different mix of loss distributions)
- Use a wide range of mathematical curves
  - To accommodate all relevant information, (eg Log-Normal fitted to past losses might give implausible value for 99th centile)
  - To allow for possible model error (particularly when extrapolating to high layers)

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Individual claim amount inflation

- Suppose:
  - True gross loss distribution is heavy-tailed eg Pareto
  - True rate of claims inflation is 10% pa
  - Losses excess of £100k observed for 5 years
- Mean loss excess of £100k understates true rate of claims inflation
- For outline solution see: www.casact.org/cotor/wright.pdf

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Claim count distribution

- Do not use Poisson distribution!
- Poisson has variance equal to mean
- Reasons why variance should exceed mean:
  - heterogeneity
  - contagion
  - parameter uncertainty
  - exposure uncertainty
- Negative binomial exists for any variance > mean
- Negative binomial usually adequate, but more complex distributions sometimes more appropriate
- Further details at:
  www.actuaries.org/ASTIN/Colloquia/Orlando/Papers/Wright.pdf
  www.actuaries.org/ASTIN/Colloquia/Orlando/Presentations/Wright.pdf

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Aggregate loss distribution

- Combine individual loss amount and count distributions
  - Method of moments
  - Panjer recursion
  - Fourier transform method (eg Heckman/Meyers)
  - Monte Carlo (random sampling) methods
  - Systematic sampling methods (eg Sobol sequence, Latin hypercube)
- Monte Carlo is most flexible but important to check for convergence
  - Repeat several times (eg 100,000 simulations each time) and look at stability of results
  - Check against method of moments

Aggregate loss – possible correlations

- Individual claim amounts correlated?
  - Generally yes, because of parameter uncertainty and model uncertainty
  - Solution: Calculate aggregate loss several times using different severity parameters and/or models, then mix the results.
- Amounts correlated with numbers?
  - Solution: Monte Carlo with different severity distributions depending on number of claims

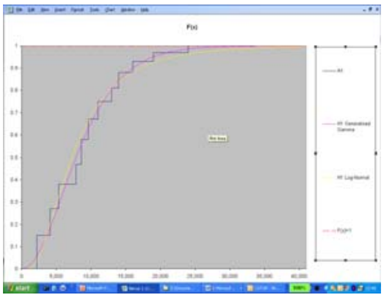
Example of individual contract PV approach

Proposed catastrophe excess of loss treaty:

- £5m xs £5m of each and every loss
- Aggregate deductible £10m
- Aggregate limit £10m

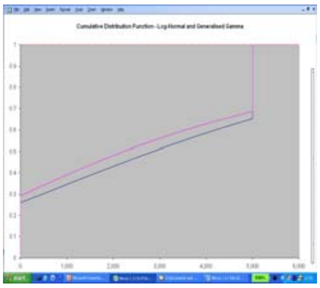
Individual gross loss distribution

Loss	Probability
2,200	0.15
4,100	0.12
5,300	0.11
7,800	0.09
8,600	0.11
9,600	0.09
11,000	0.08
13,000	0.06
14,000	0.07
16,000	0.05
19,000	0.04
24,000	0.03



Distribution of individual loss to layer

	Log Normal	Gen Gamma
Mean	2,449	2,670
Prob(0)	29.2%	26.0%
Prob(£5m)	31.5%	34.8%



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Claim count distribution

	Poisson	Neg Bin
Mean	1.20	1.20
Variance	1.20	1.68
0	0.301	0.364
1	0.361	0.312
2	0.217	0.178
3	0.087	0.085
4	0.026	0.036
5	0.006	0.015
6	0.001	0.006
7	0.000	0.002
8	0.000	0.001
>2	**0.121**	**0.145**

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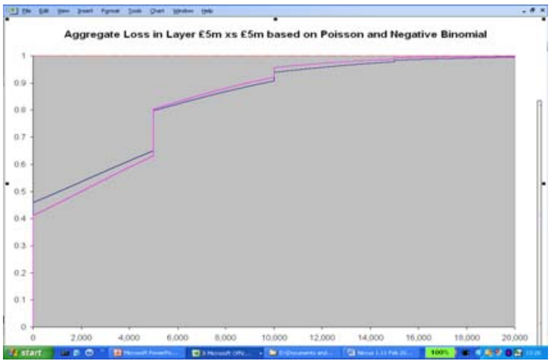

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Results based on Generalised Gamma individual loss distribution

	No agg limits		Agg limits £10m xs £10m	
	Mean	Std Dev	Mean	Std Dev
Poisson	3,204	3,746	145.5	857.5
Neg Bin	3,204	4,178	245.1	1,199.1

3,204 = 2,670 * 1.2 (= mean loss in layer * frequency)
This is the same for Negative Binomial as for Poisson

Pure premium for proposed treaty is 1.68 times higher for Negative Binomial than for Poisson (245.1 / 145.5)

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Results based on Log-Normal individual loss distribution

	No agg limits		Agg limits £10m xs £10m	
	Mean	Std Dev	Mean	Std Dev
Poisson	2,938	3,566	113.4	746.2
Neg Bin	2,938	3,949	196.2	1,059.7

2,938 = 2,449 * 1.2 (= mean loss in layer * frequency)
This is the same for Negative Binomial as for Poisson

Pure premium for proposed treaty is 1.73 times higher for Negative Binomial than for Poisson (196.2 / 113.4)

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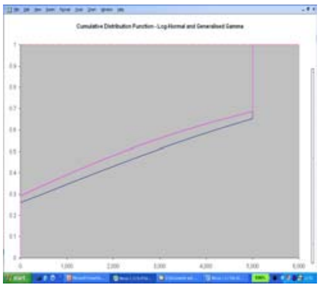
Distribution of individual loss to layer

	Log Normal	Gen Gamma
Mean	2,449	2,670
Prob(0)	29.2%	26.0%
Prob(£5m)	31.5%	34.8%



Sensitivity to individual loss distribution: ratio of Gen Gamma to Log-Normal

	No agg limits		**Agg limits £10m xs £10m**	
	Mean	Std Dev	Mean	Std Dev
Poisson	1.09	1.05	1.28	1.15
Neg Bin	1.09	1.05	1.25	1.13

1.09 = 2,670 / 2,449 (= ratio of mean losses in layer £5m xs £5m)

Ratio is magnified by aggregate deductible. Eg consider probability, given 3 losses, that they all go through layer £5m xs £5m:
- Gen Gamma: 0.348^3 = 0.042
- Log-Normal: 0.315^3 = 0.031

Conclusions from the example

- Poisson distribution can materially understate:
  - Risk load
  - Pure premium where there is aggregate deductible
- Individual loss model can have big impact on:
  - Pure premium for high layers of e&el
  - Pure premium where there is aggregate deductible
- These remain true in cash-flow pricing models

Cash-flow approach - Principles

- Technical premium can be based on the investment yield (IRR) of writing the contract
- Yield depends on capital allocated to the contract (or group of contracts)
- Capital plus premium must be such as to meet required solvency objectives (perhaps based on probability of ruin, or VaR)
- Given capital, premium and loss distribution: IRR is a random variables (because losses are random)
- Calculate probability distribution of IRR (for given capital and premium),
- Adjust capital and premium (subject to solvency constraints) until probability distribution of IRR becomes acceptable.

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Cash-flow approach – Simple Example

Assume:

- Capital (C) is allocated at start of year (and invested at 0%)
- Premium (P) is received at end of year
- Claims (X) are paid at end of year

Then **IRR = (P-X)/C** (except if X > P+C: IRR = -100%)

Suppose solvency requirement is 0.5% probability of ruin, that is

- P+C = Q (where Q is 99.5%ile of X)

Then **IRR = (P-X)/(Q-P)** (or -100% if X > Q)

Having found a probability distribution for aggregate claims X, we can calculate a probability distribution for the IRR.

Premium P is adjusted until the probability distribution of the IRR becomes acceptable (in terms of risk vs expected return).

When P is determined, so is C (by solvency requirement P+C=Q)

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Cash-flow approach – Simple Example

- IRR = (P-X)/(Q-P) (or -100% if X > Q)

Taking expectations (given P):

- E(IRR) ≈ (P-E(X))/(Q-P)

Note that expected yield increases rapidly with P because:

- Expected UW profit (numerator) increases with P
- Capital required to meet solvency objective (denominator) decreases with P

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Cash-flow approach – more realistic cases

In practice

- Premium cash-flows occur at different points in time
- Claim payment cash-flows modelled according to an assumed run-off pattern
- Allocated capital can be released as claims run off (assuming ruin does not occur)
- Premium plus capital are invested

Cash-flow model can be constructed to find IRR that will result from given claims experience

Hence probability distribution of IRR from random claims experience.

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Individual vs Portfolio approach

- Pricing RI contracts individually is simple but imperfect
- Reinsurer obtains diversification benefits by writing many contracts that are not perfectly correlated
- Aim to assess the effect of each additional contract on the reinsurer's portfolio
- Cash-flow calculation on entire pre-existing portfolio gives notional aggregate premium and risk capital for the entire portfolio.
- Cash-flow calculation on extended portfolio gives notional premium and risk capital for the extended portfolio.
- Capital and premium required for new contract can be obtained as the increases in capital and premium for the portfolio.

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Incremental portfolio approach in practice

- Incremental portfolio approach is often impractical for every individual additional contract
  - Too complex and takes too long
  - If many contracts are quoted for simultaneously, price for each depends on the order in which they are considered
- Practical solution is to use incremental portfolio approach for groups of contract on assumption that a certain volume will be written
- Use results to specify pricing guidelines for individual contracts in the group
- For example: price should be at least mean plus 1.2 times standard deviation for each individual contract

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