SOME ASPECTS OF THE STATUTORY VALUATION

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1. INTRODUCTION

1.1 THE burden of legislation upon authorized insurers in the U.K. continues to grow and this is particularly true for the statutory valuation. In recent years regulations have been introduced to cover such matters as the valuation rates of interest, the provision for changes in investment conditions and the setting up of solvency margins.

1.2 The purpose of this paper is to summarize some of the more important aspects of the Regulations and comment upon their interpretation with reference to current practice. A substantial part of the paper is concerned with the provision for changes in investment conditions. Some implications for financial control and product pricing are also mentioned.

1.3 The paper will concentrate on the valuation of liabilities and is written in the context of assets taken in accordance with the asset valuation regulations, that is to say broadly at market value. Most of the paper is addressed to conventional business but a few comments on linked business are included. The material is discussed under various headings but is so interdependent that it is difficult to concentrate on one aspect without impinging on several others.

2. RATES OF INTEREST

2.1 Regulation 59 places limits on the rates of interest that can be used for the Schedule 4 valuation, so that they are linked closely to the yields on the existing assets backing the long-term liabilities. Ignoring points of detail, the maximum rates allowed are the redemption yields on gilt-edged securities and the running yields on equity or property investments. To establish a contingency margin the resulting yields are reduced by 7·5% so that if 10% is earned then up to 9·25% may be assumed in the valuation.

2.2 One problem which has always faced actuaries is the rate of interest to assume in future years; part of the "expanding funnel of doubt" as Redington so vividly described it in 1952 (1). He talked of probability distributions which would be compact in the immediate future but quite widely dispersed in 5 years time. Markets seem to be so volatile these days I am wondering if we can even be as sanguine as that. In recent years there have been several occasions when interest rates have changed by 2 or 3% over a few months. Perhaps nowadays the future is not so much a funnel as a tunnel! Faced with this problem the authorities appear to have settled on an upper limit of 7·2%, before allowance for tax but

after allowance for the 7·5% contingency margin, for the yield to be assumed on any investment made more than 3 years after the valuation date.

2.3 *Hypothecation*

2.3.1 The Regulations allow assets to be apportioned notionally between different categories of contracts. This facility can be very helpful. For example, a portfolio of gilts backing level immediate annuities will experience disinvestment and hence the valuation rate of interest can be based on the full redemption yield. The 3 year concession enables very short term contracts to be valued similarly.

2.3.2 As a simple example of how assets may be hypothecated in practice, consider the following office:

Assets:
£500 million in gilts yielding 10%,
£550 million in equities yielding 4%,
£150 million in properties yielding 5%,

Liabilities:
£100 million in respect of immediate annuities if valued at 9·25%,
£100 million for other non-profit business if valued at 7·2% gross or an equivalent net rate,
£500 million for single premium with-profit deferred annuities in the pension business fund and £300 million for annual premium with-profit business in the life fund if valued at rates within the average yields (gross and net respectively) on the hypothecated mix of assets.

2.3.3 Suppose £100 million of gilts are apportioned to the immediate annuity business and a further £100 million to the remaining non-profit business. The office could use the remaining £300 million of gilts plus the £150 million of properties and £350 million of the equities as backing for the with-profit business. This would give an average running yield, after taking the 7·5% contingency margin, of 5·95% gross. By leaving the lowest yielding assets out of the calculation the office has maximized the valuation rate of interest. The Regulations appear to allow this technique to be used in any way which the office feels is justified. For example, high coupon gilts, which would normally carry the highest redemption yields, could be selected for immediate annuity business. Gilts could be hypothecated to a combination of immediate and deferred annuities to enable the combined business to be valued at the full redemption yield. Moreover, it may be technically advantageous, as well as appropriate, to hypothecate deposits and certain net current assets to liabilities maturing over the next year. This might well apply to terminal bonuses since the office may wish to consolidate equity gains so far and hold its rate for a period.

2.3.4 Since the liability for any category will depend on the rate of interest used, and the rate of interest may depend on the liabilities for each category, the

hypothecation process may need to be iterative. In any case the office needs at least to carry out a simple check that the rates of interest used are justified.

2.4 *Some implications for product design and pricing*

2.4.1 Consider an office which issues immediate annuity contracts priced in accordance with the yield obtainable on a matched portfolio of gilts. Ignoring any differences between the valuation and premium bases in respect of expenses and mortality the office must necessarily incur a valuation strain due to the 7·5% contingency margin on the interest rate. When gilts are yielding 10% this strain is approximately 5% of the consideration for a male aged 65. If the office wishes to achieve a return on capital in excess of the return on gilts it would need to take a small margin on the gilt yield in setting the rates. This would be in addition to allowing for the effect of the solvency margin.

2.4.2 At the other end of the scale, if an office issues a single premium with-profit deferred annuity in its Pension Business Fund and wishes, notionally, to maintain a high equity backing then the level of guarantee in the rates should be close to the expected dividend yield in equity-type investments.

3. CHANGES IN INVESTMENT CONDITIONS

3.1 Regulation 55 requires the Actuary to make "appropriate provision against the effects of possible future changes in the value of the assets on their adequacy to meet the liabilities". I suspect that in recent years those few words have caused plenty of extra work. Actuaries have always taken a keen interest in the nature and term of the assets representing the long term fund and many learned papers have been written on the subject of matching and immunization. Many offices, particularly mutual offices with a large volume of with-profit business, will probably not be close to a matched position. Resilience to changes in conditions is therefore most important. A full discussion of this subject is outside the scope of this paper though some recent developments and their practical effect are illustrated. Some aspects are currently being investigated by a Research Working Party on Valuation Regulations.

3.2 The subject has recently given rise to practice notes from the Government Actuary (2) and the Institute and Faculty of Actuaries (3). The simple guideline adopted is that a company should be able to absorb the effect of a change of 3% in the rate of interest and a fall of 25% in the market value of equities (and presumably properties) without prejudicing its ability to hold reserves which satisfy the valuation regulations. If assets have been hypothecated to all liabilities any assets entirely free after such an apportionment can be ignored for the purpose of determining the mismatching reserve. However, what exactly is 'free' in this context is, I believe, open to doubt and is discussed in 8.3.

3.3 The above guidelines became apparent during 1985 and were referred to in a presentation to the First Actuarial Convention in Birmingham. In a paper formed from those proceedings C. L. Cannon (4) explained that they were only

an example of what might be appropriate. He felt it might be reasonable to allow for a smaller increase in yields when yields were considered to be particularly high and vice versa. I think there is a danger in such an approach. Interest rates at any time reflect a balance of buyers and sellers and it could be argued that rates have as much chance of rising 3% as of falling 3% no matter what the current level is. It hardly requires pointing out there is no upper limit for interest rates. The Government Actuary's Department have stated that actuaries are expected to test for extreme changes in conditions, though as the tests become more stringent it would be reasonable to cut into investment assumption margins contained in the minimum standards or even additionally to rely on the explicit solvency margin. However, such guidelines are still vague and there is no standard procedure in detail. The actuary must still rely on judgement.

3.4 An office has 2 extreme choices in presenting its compliance with Regulation 55.

(i) It can use valuation bases which only just comply with the Regulations, excluding Regulation 55 (though there are some difficulties in determining what that means) and set a full explicit mismatching reserve.
(ii) It can use valuation bases which are strong enough to cover the mismatching provision.

In case (i) the mismatching reserve must itself allow for the fall in asset values. If M represents the difference between the change in liabilities and assets hypothecated to meet those liabilities, then the mismatching reserve is $M/\cdot75$. This assumes that equity-type assets are amongst those hypothecated. This point is taken up again in 8.3.

A survey of Returns from companies as at the end of 1985 showed a considerable preference for the second approach though more companies seem to be making an explicit provision in 1986 indicating a mix of approaches (i) and (ii).

3.5 It is interesting to compare the above guidelines with market movements over the past 20 years. According to the de Zoete and Bevan Equity Index it was only during 1973/74 that prices fell by more than 25% within one year, although they did so in style, falling by about 55% during 1974. A change of 3% or more in gilt yields has been more common. Between June 1973 and June 1974 yields rose by more than 4%. On several occasions since then they have failed to rise quite so much quite so fast, but have sometimes fallen by more than 3% over a short period.

3.6 It appears that only a fall in equity values need be considered, though one would expect a rise to be beneficial since the reduction in dividend yield would be modest. Whether a rise or fall in interest rates is easier to accommodate will depend to a large extent on the direction of mismatching. However, companies will normally find a rise in interest rates to be more onerous for several reasons.

3.6.1 The sensitivity of the net premium valuation to changes in interest rates

has long been in question. It was often considered to be rather insensitive, but the problem is not straightforward as was demonstrated by an Institute Working Party in 1975 (5). Sensitivity depends on the level of interest rates and the outstanding term of the liabilities. When the interest rate is high and the outstanding term is long the net premium method can be over-sensitive. However, I would expect that for most offices the average term of the liabilities is such that the net premium method becomes progressively more stringent as interest rates rise.

3.6.2 Even if assets and liabilities are matched exactly by cash flow a reserve may nevertheless be required due to the 7·5% contingency margin in the interest rate. A simple example is a block of immediate annuity business matched perfectly to some dated gilts. If interest rates rise from 10% to 13% the maximum valuation rate of interest will rise from 9·25% to 12·025% and clearly the fall in liabilities will be less than the fall in the market value of the gilts.

3.7 Other possible changes in investment conditions are considered briefly later in the paper, but most of the attention is given to a rise in interest rates. Two types of mismatching will be considered. The first, which compares the change in assets with the change in liabilities, has already been touched upon and will be referred to subsequently as 'global mismatching.' This is the test which is most likely to apply to an established conventional life office. The other method is to compare asset and liability cash flows, period by period, which I will refer to as 'cash flow mismatching'.

4. MODIFIED NET PREMIUM FORMULAE

4.1 For global mismatching an approach suggested by the Government Actuary is to assume that interest rates and asset values change as described in 3.2 above and then to determine the minimum value of liabilities allowed by the Regulations. The mismatching reserve is simply the extent to which the fall in assets exceeds the fall in liabilities.

4.2 For regular premium business the procedure seems very straightforward in principle. However, it is certainly not so in practice since it is not clear just how the minimum reserves under the Regulations should be determined. The Regulations are expressed by reference to a net premium valuation so we may assume that any method used must remain faithful to that principle, unless of course it produces stronger reserves. Regulation 57 requires that the premium to be valued should be calculated at the same rate (or rates) of interest used for calculating the liability. If a constant rate of interest is used we have the normal net premium method. However, if the rate of interest varies with duration, due to the use of an initial rate of interest and a different reinvestment rate, then a reasonable interpretation of the Regulations is that the net premium should also vary. Over the years a considerable amount of work has been done to explore this issue and the purpose of this section is to outline some of these developments.

4.3 *The V_2 Formula*

4.3.1 The normal net premium reserve for a non-profit endowment assurance is given by

$$_tV_{x:\overline{n}|} = A_{x+n:\overline{n-t}|} - P_{x:\overline{n}|}\, \ddot{a}_{x+t:\overline{n-t}|}$$

In simplified notation

$$V = A - P \cdot \ddot{a}$$
$$A = V + P \cdot \ddot{a}$$
$$\quad = V + P \cdot \ddot{a} + V \cdot A - V \cdot A$$
$$\quad = V \cdot A + P \cdot \ddot{a} + V - V\,(1 - d \cdot \ddot{a})$$
$$\quad = V \cdot A + P \cdot \ddot{a} + d \cdot \ddot{a} \cdot V$$
$$\quad = V \cdot A + \left(\frac{i}{1+i} \cdot V + P\right) \ddot{a}$$

4.3.2 This expresses that the future income to provide the sum assured will be financed by the remaining premiums plus investment income from V, both accumulated at rate i, with V returned at expiry of the policy. This rearrangement of the formula was given in the Institute paper referred to in 3.6.1 and enables one to consider the effect of allowing for the current return on assets. If this is denoted by g we can use it to replace i in the numerator of the above expression. The 1975 paper called this modified reserve V_2 so that

$$A = V_2 \cdot A + \left(\frac{g}{1+i} \cdot V_2 + P\right) \ddot{a}$$

$$\quad = V_2\left(A + \frac{g\ddot{a}}{1+i}\right) + P\ddot{a}$$

Substituting $1 - d\ddot{a}$ for A and rearranging, we obtain

$$V_2 = \frac{A - P\ddot{a}}{1 + \left(\dfrac{g-i}{1+i}\right)\ddot{a}}$$

4.3.3 Clearly, if $g = i$ this reduces to the normal net premium formula. The authors of the 1975 paper argued that strictly speaking i should vary with outstanding term, declining from g to some cautious value for long terms.

4.3.4 There is considerable appeal in this approach. The snag is that the net premium is calculated at rate i whereas the effective rate of interest at any valuation date during the term of the contract is somewhere between i and g.

4.4 *The W_2 Formula*

4.4.1 A method which avoids the objection described above has been developed by A. E. M. Fine and others and is generally known as W_2. I am indebted to Mr Fine for notes on this subject on which the rest of the section is based.

For a non-profit endowment assurance,

$$W_2 = \frac{A_{x+t:\overline{n-t}|}^{i} - {}_tP_{x:\overline{n}|}^{(i,g)} \ddot{a}_{x+t:\overline{n-t}|}^{i}}{1 + \frac{g-i}{1+i} \cdot \ddot{a}_{x+t:\overline{n-t}|}^{i}}$$

Where
$${}_tP_{x:\overline{n}|}^{(i,g)} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \quad \text{at rate } i'$$

4.4.2 An acceptable approach for determining i' might be to use simple interpolation so that

$$i' = \frac{tg + (n-t) i}{n}$$

For whole life policies we could use

$$i' = \frac{tg + (e_x - t) i}{e_x}$$

where e_x is the expectation of life for the age at entry.

4.5 The W_2 formula assumes that g is obtainable from current assets over the whole of the remaining term. Therefore, some adjustment should be made if there are fixed interest securities maturing earlier than the liabilities to which they are hypothecated. An important strength of the W_2 formula is that it allows a number of such modifications. For example, we could reason as follows.

Suppose all the fixed interest securities mature early so that rate g is only obtainable for a period of m years (measured from the outset of the contract), where $m < n$, and rate i is assumed for reinvestment over the remaining $(n-m)$ years.

The value of future premiums is now

$${}_tP_{x:\overline{n}|}^{(i,g)} \cdot \ddot{a}_{x+t:\overline{m-t}|}^{i} + P_{x:\overline{n}|}^{i} \ \ddot{a}_{x+m:\overline{n-m}|}^{i} \cdot D_{x+m}/D_{x+t}$$

Furthermore, excess interest of $g-i$ is only obtained for a further $m-t$ years. Hence the modified formula is

$$W_2 = \frac{A_{x+t:\overline{n-t}|}^{i} - {}_tP_{x:\overline{n}|}^{(i,g)} \cdot \ddot{a}_{x+t:\overline{m-t}|}^{i} - P_{x:\overline{n}|}^{i} \ \ddot{a}_{x+m:\overline{n-m}|}^{i}}{1 + \frac{g-i}{1+i} \cdot \ddot{a}_{x+t:\overline{m-t}|}^{i}} \cdot D_{x+m}/D_{x+t}$$

The other resilient aspect of the W_2 formula is that the specially-derived net premium ${}_tP_{x:\overline{n}|}^{(i,g)}$ can easily be restricted to a suitable proportion of the office premium.

4.6 In summary, therefore, the W_2 method can lay a very strong claim to adhere to the Regulations both in the letter and the spirit. At the time of writing, however, no approval from the supervisory authorities exists for the use of this modification. Its practical use is nevertheless illustrated in the next section.

5. SOME MISMATCHING EXAMPLES

5.1 *Global Mismatching*

5.1.1 Suppose an office transacts the following business.

(i) Non-profit and with-profit endowment assurances in the Life Fund.
(ii) Non-profit level immediate and deferred annuities in the gross funds.
(iii) Compound bonus deferred annuities, by single premiums, in the Pension Business Fund.

We will look at the possible effect of the guidelines for Regulation 55 on each class of business, assuming that the office does not set out to adopt either of the extreme approaches described in 3.4. For this purpose I shall concentrate on the effects of the guidelines set out in 3.2 and ignore the implications of 3.3.

5.1.2 Assume that at the valuation date gilts are yielding 10% gross to redemption and equities are providing a 4% rate of dividend. The office holds no property investments. For hypothecation the office allocates 100% gilts to the non-profit liabilities. The with-profit liabilities are backed 75% equities, 25% gilts. Unless otherwise stated I have assumed that the office retains this hypothecation when testing the change in investment conditions (though in practice it seems that the office could re-hypothecate in almost any way it wished).

5.1.3 Tax is allowed for at the rate of 35% on gilt income (assume that the coupon is 10% p.a., all stocks priced at par) and 30% on equity dividends. Since the 35% rate is not pegged there is some risk here, but I have ignored it. Assume that no tax liability arises in either the General Annuity Fund or the Pension Business Fund.

5.1.4 The change in investment conditions would mean gilts yielding 13%. The office could probably justify a small reduction in the allowance for tax, due to the tax-free gain on redemption, but we will ignore this. For equities, assume the dividend is unchanged. The yield can be taken as $4/\cdot75 = 5\cdot33\%$.

5.1.5 *Valuation rates of interest.*

5.1.5.1 Using the notation in Section 4 we have the general formula:

$$g = (1 - t_1) \times \cdot925 \times PE \times EY + (1 - t_2) \times \cdot925 \times PG \times GY$$

where t_1 = equity tax rate
t_2 = gilt tax rate ($t_1 = t_2 = 0$ for the gross funds)
PE = proportion of equity backing
PG = proportion of gilt backing
EY = equity running yield
GY = gilt redemption yield.

If t_3 is the weighted average tax rate

$$\frac{t_1 \times PE \times EY + t_2 \times PG \times GY}{PE \times EY + PG \times GY}$$

Then i is the lower of g and $(1 - t_3) \times 7 \cdot 2$.

5.1.5.2 Results for the office and investment conditions in question are shown in the table below.

| | With Profit | | | | Non Profit | | | |
| | g | | i | | g | | i | |
	Gross %	Net %	Gross %	Net %	Gross %	Net %	Gross %	Net %
Before the change	—	—	5·1	3·4	9·25	—	7·2	4·7
After the change	6·7	4·54	6·7	4·54	12·0	7·8	7·2	4·7

Before the change g is only shown for non-profit gross business since we will assume that it is only used in the published basis for immediate annuities, or a combination of immediate and deferred.

5.1.5.3 Assume that in the published valuation the office uses the following rates of interest.

With-profit endowments:	3·0%
Non-profit endowments:	4·7%
Immediate annuities:	9·25%
Non-profit deferred annuities:	7·2%
With-profit deferred annuities:	5%

In other words, the office takes a small margin on the with-profit business. We can notice that, in this example, the W_2 formula will be of real help for non-profit endowment business, but does not seem to be of assistance for with-profit business. The weighted average tax rate for with-profit endowments is approximately 32% so, for W_2 to be of assistance, g must exceed $\cdot 68 \times 7 \cdot 2\% = 4 \cdot 9\%$.

5.1.6 *Non-Profit Endowments*

5.1.6.1 Some examples of the ratio of the W_2 reserve at the higher yields to the unmodified net premium on the published basis are shown in Table 1. i' has been calculated using the interpolation approach in 4.4.2.

5.1.6.2 It is interesting to note that the ratio dips at medium durations. The lowest ratio is about 81% for an outstanding term of 30 years. For a 10 year gilt with a 10% coupon and ignoring tax the market value will fall by approximately 16%. For long-dated stocks the fall can exceed 20%. Therefore it seems that a mismatching reserve is likely to be required and a 5% margin might be sufficient. Without resorting to W_2 it would be much higher.

5.1.7 *With-Profit Endowments*

5.1.7.1 To show the effect of different hypothecations and methods consider 3 approaches the office may be able to take.

(i) The office hypothecates 25% to gilts. Assume that the average tax rate is 33%. Therefore the valuation rate of interest is

$\cdot 67 \times \cdot 925 \times (\cdot 75 \times 5 \cdot 33 + \cdot 25 \times 13) = 4 \cdot 5\%$ approximately.

Table 1. *Non-Profit Endowment Assurances for £1,000
Sum Assured*

Net Premium Reserves on A67–70 ult.
Males aged 30 at entry

		(1) Unmodified Reserve $i=4\cdot7\%$	(2) W_2 Reserve $g=7\cdot8\%, i=4\cdot7\%$	(3) (2)/(1)
Term	Duration			
10	5	442	415	·939
20	5	172	151	·877
	10	387	344	·889
	15	658	608	·924
30	5	90	76	·849
	10	202	170	·842
	15	340	289	·848
	20	512	445	·869
	25	725	663	·914
40	5	57	47	·826
	10	127	103	·814
	20	313	255	·815
	30	580	498	·859
	35	759	690	·909

(ii) The office hypothecates 32% to gilts. This produces a maximum gross yield for valuation purposes of the upper limit of 7·2%, a net rate of 4·8% say.

(iii) The office hypothecates nearly half to gilts. With the notation of the previous section this gives a value of 5·5% for g, with reinvestment at 4·8%. This would be of some help if the formula W_2 is used.

5.1.7.2 The effect of these 3 approaches on the change in published liabilities is shown in Table 2.

5.1.7.3 The ratios in columns (5), (6) and (7) in Table 2 should be considered bearing in mind that the market value of assets is likely to fall by at least 20%. It

Table 2. *With Profit Endowment Assurances—Basic Sum Assured £1000
Compound bonus 5% p.a. throughout*

Sample Net Premium Reserves A67–70 ult.
Males aged 30 at entry

Term	Duration	(1) $i=3\%$	(2) $i=4\cdot5\%$	(3) $i=4\cdot8\%$	(4) $g=5\cdot5\%$ $i=4\cdot8\%$	(5) (2)/(1)	(6) (3)/(1)	(7) (4)/(1)
10	5	701	666	660	647	·950	·942	·923
20	5	376	319	308	293	·848	·819	·779
	10	896	789	780	749	·881	·871	·836
	15	1623	1529	1510	1474	·942	·930	·908
30	5	250	189	178	167	·756	·712	·668
	10	601	477	455	426	·794	·757	·709
	15	1100	919	886	836	·835	·805	·760
	20	1809	1599	1559	1490	·884	·862	·824
	25	2830	2654	2620	2550	·938	·926	·901

should not be difficult for an office to justify the column (3) valuation basis. However, even the ratios in column (6) suggest that one cannot be sure that a 3% net premium basis is sufficiently strong to meet the guidelines for Regulation 55. It will depend on the scope for further zillmerization, the mix of business, the degree of gilt-edged matching and the reduction in the provision for future capital gains tax as a result of the 25% fall in equity values. A large value of free assets can be helpful in this respect because it appears that an office can take account of the reduction in tax provision on its total assets, not just those required to match the mathematical liabilities.

5.1.8 *Immediate Annuities*

5.1.8.1 The effect of the 7·5% contingency margin on mismatching reserves has already been mentioned in 3.6.2. An accurate assessment of this effect would require a hypothecation of gilts by term for each age of the annuity portfolio so that the effect of a 3% rise in interest rates on those gilts could be calculated. For simplicity I have set out below a table of annuity values at various rates of interest. The ratio between the change in liabilities from a 10% basis to a 13% basis and the change from a 9·25% basis to a 12% basis does, I believe, approximate to the minimum mismatching reserve which the office should hold for this business.

5.1.8.2 Values of a_x on a (90) ult. Males

Age at Valuation Date	(1) 9·25%	(2) 12%	(3) (2)/(1)	(4) 10%	(5) 13%	(6) (5)/(4)	(7) $100 \times ((3) - (6))$
50	9·003	7·468	·8295	8·652	6·982	·8070	2·25
60	7·765	6·607	·8509	7·506	6·228	·8297	2·12
70	6·157	5·400	·8771	5·991	5·143	·8585	1·86
80	4·312	3·905	9056	4·225	3·761	·8902	1·54

5.1.8.3 Column (7) expresses the minimum mismatching reserve as a percentage of the liabilities on the published basis. The percentages may look small but in practice such close matching will be very difficult, if not impossible, to achieve. I consider that, taken in isolation, a mismatching reserve of 3% to 5% may be required. However, there is more on this point in 5.2 below.

5.1.9 *Non-Profit Deferred Annuities*

5.1.9.1 I will assume these are all by single premium. When testing for mismatching the office could make use of the 3 year period for which reinvestment at current yields may be assumed. Furthermore, if the assets are sufficiently long-dated the current yield can be taken for reinvestment at 7·2% after 3 years. In the special case of gilts at par with the coupon rate equal to the redemption yield the valuation discount factor for a term of n years, ignoring mortality, is given by

$$1 \div (1+g)^3 \cdot (1+g \cdot s_{\overline{n-3}|}^{i})$$

where $g = 12\%$ and $i = 7·2\%$.

5.1.9.2 Table 3 shows a few examples of the effect of revaluing on this basis.

Table 3. *Single Premium Deferred Annuity.*
£1000 Cash Option

Term	(1) Reserve At 7·2%	(2) Reserve At g = 12%, i = 7·2%	(3) (2)/(1)
5	706	570	·807
10	499	348	·697
20	249	149	·598
30	124	70	·565

5.1.9.3 Clearly, if the above approach is acceptable, the margins in the published basis are more than adequate to meet the contingency of a 3% rise in interest rates.

5.1.10 In 5.1.8 and 5.1.9 I have considered the business in isolation. In practice an office could probably take immediate and deferred annuities together to justify valuing at 12% throughout and in any case it should be able to offset the small theoretical requirement for mismatching of immediate annuities against the spare margins for deferred annuities. However, this assumes that no requirement arises under a cash flow test—see 5.2 below.

5.1.11 *Compound Bonus Deferred Annuities*

5.1.11.1 This business is likely to be the substantial part of the office's liabilities. Assume it is funded-to-cash and single premium costed. The published basis is 5% interest and after the fall in assets the rate is 6·7%, as shown in the table in 5.1.5.2. Therefore we are interested in the ratio

$$\left(\frac{1\cdot05}{1\cdot067}\right)^n$$

For a 5 year term this is 0·92 and for a 20 year term it is 0·725. If we assume that the underlying assets fall in value by an average of 22·5% (25% on equities and an average of 15% on gilts) then the effective average outstanding term of the business would need to be no less than 16 years. If not a mismatching reserve may be required.

5.1.11.2 This is only a crude example, but it does illustrate the importance of product design. The actuary strives for harmony between premium rates, investment policy, the valuation basis and distribution of surplus. Ignoring the provision for expenses in the premium and valuation bases the example in 5.1.11 suggests that the underlying interest rate guarantee in the premium rates is 5% and this is consistent with the investment policy (namely 75% invested in equities) assuming current dividend and gilt yields are sustainable. Nevertheless, if the office used an underlying guarantee of 3·5% interest in the premium rates, with valuation also at 3·5%, then clearly there is much more scope for provision under Regulation 55. We would be interested in the ratio

$$\left(\frac{1\cdot035}{1\cdot067}\right)^n$$

For the liabilities to fall by at least 22·5% n needs to be only 9 years. The benefits of using lower guarantees and a higher degree of 'with-profit' are emphasized yet again.

5.1.12 On the few assumptions made it is impossible to say what mismatching reserve the hypothetical office in this section may need to set up. However, one can see how some offices have been able to argue, in their 1985 Returns, that sufficient margins are contained in these bases. Some even claim to have tested for a rise in interest rates up to 17% and a fall to 5%. Nevertheless, I suspect that for a few offices it must have been a tight calculation.

5.2 Cash Flow Mismatching

5.2.1 C. L. Cannon (4) and A. E. M. Fine have described a method for determining such a reserve. For convenience I will summarize the method here. A tranche of business will give rise to an expected liability flow. Assets are hypothecated such that their present value at the market rate of interest equals the value of the liability flow at the valuation rate of interest. To allow for the 7·5% contingency margin the asset flow is reduced by the ratio of the liabilities at the market rate i to their value at rate $·925i$. The net cash flow for each period is the adjusted asset flow less the liability flow. A negative cumulative cash flow is increased at rate $i + ·03$. Positive amounts are accumulated at $i - ·03$, limited to 7·2% after 3 years. If the accumulated cash flow at the time when all liability and asset flows cease is negative then a mismatching reserve is set up at the outset. The exercise is repeated until a mismatching reserve is found such that there is ultimate zero surplus. If this exceeds the difference between the change in assets and liabilities on a 3% rise in interest rates, then it is the required reserve.

5.2.2 The method is illustrated by the following example.

Liability Flow:

A temporary annuity of £1000 p.a., payable at the end of each year, for 5 years. Ignore mortality and expenses.

Asset Flow:

A one year gilt, present value £2000
plus a five year gilt for £120
plus a six year gilt for £1745
all with a 10% annual coupon.

5.2.3 The value of the assets at 10% p.a. is 3865 which equals the value of the liabilities at 9·25%. Table 4 shows the results of the cash flow matching test.

5.2.4 The test indicates that a cash flow mismatching reserve of 100 is required assuming, for simplicity, that it is on deposit. The interesting point is to compare this with the changes in assets and liabilities. I have chosen the above asset flow because it virtually immunizes the business at 10% interest. If we adjust the asset flow (or the liability flow) to eliminate the effect of the 7·5% contingency margin we can test the effect of Redington's immunization equations (1).

Table 4

Year	(1)	First Trial				Successful Trial
		(2)	(3)	(4)	(5)	(6)
1	1000	2386·5	2340·9	1340·9	1340·9	1447·9
2	1000	186·5	182·9	−817·1	617·8	732·2
3	1000	186·5	182·9	−817·1	−156·1	−33·6
4	1000	186·5	182·9	−817·1	−993·4	−855·0
5	1000	306·5	300·6	−699·4	−1821·9	−1665·5
6	0	1919·5	1882·9	1882·9	−175·9	0·9

Description of columns:

(1) Liability flow,
(2) Asset Flow,
(3) Adjusted asset flow,
(4) Net cash flow (3)–(1),
(5) Cumulative net cash flow if no mismatching reserve is set up,
(6) Cumulative net cash flow if mismatching reserve is 100.

5.2.5 We find that the first derivative of the value of assets is almost equal to the first derivative of the value of liabilities and the second derivative for assets exceeds the second derivative for liabilities. Thus the conditions for immunization are met. A check for a rise of 3% in the interest rate reveals a very small difference between the new values for assets and liabilities. Hence the global mismatching provision is close to the figure of 1·5% of the fund demonstrated in 5.1.8.2. Yet the cash flow mismatching provision of 100 represents about 2·5% of the fund. The only way to reach the approximate 1·5% minimum reserve is to match by cash flow, described by Redington as 'absolute matching'.

5.3 Further Points

5.3.1 If interest rates fell one would expect that, in most cases, asset values would rise by more than the liabilities for reasons mentioned in 3.6. However, in exceptional circumstances an extra problem may arise—that of investment guarantees such as guaranteed annuity rates.

5.3.2 For with-profit we should test for the combination of a substantial fall in interest rates and equity prices, but this does not seem likely. Although there have been many times when the gilt and equity markets have moved in opposite directions, these have been small divergences. I am not aware of any occasion in modern times when long dated gilts have risen by about 20% and equities fallen by 25% simultaneously.

5.3.3 So far consideration has only been given to conventional business. However, for a company transacting only linked business and keeping a fully matched position with creation and cancellation of units, there are a number of potential problems. These could probably justify a paper in themselves. One area of interest is sterling reserves. If positive reserves are held there is the problem of

their investment. Even if gilts of suitable term are available a problem could arise with the 7·2% maximum valuation interest rate. If sterling reserves are negative there could be some vulnerability to a fall in unit values.

5.3.4 If investment conditions changed dramatically just before the valuation date, then the office would need to provide for a further rise in interest rates. Whether this is 3% or a bit less is a matter of judgement but in any case the current limitation of reinvestment at 7·2% may then prove quite onerous.

5.3.5 So far I have only looked at the effect of changes in the mathematical reserves and assets, with a passing reference to capital gains tax. The solvency margin may also have a part to play. Rather than attempt to deal with these matters at this point I have deferred them to Section 8 since they are bound up with the final presentation of results.

6. POLICYHOLDERS' REASONABLE EXPECTATIONS

6.1 Ever since this notion was conceived it has been prominent in many valuation discussions. Protection of the principle was a major reason for stipulating a net premium valuation as the statutory method. Yet there is not, nor probably ever will be, wide agreement on exactly what it means. I see the concept encompassing more than just future bonuses. Here are some areas for consideration.

6.1.1 Future bonuses.

6.1.2 Bases for calculating claim values or alterations to policies where these are not guaranteed, but current practice is used as a major selling feature.

6.1.3 Periodic management charges where these are explicit but not guaranteed.

6.2 I venture the opinion that, of these 3 areas, it is with future bonus rates that the policyholder should have the *least* expectation. A policyholder has every right to expect the office to manage its affairs sensibly and fairly. This means running the business economically, not indulging in rapid expansion on terms which reduce the bonus earning power of the funds and dealing as equitably as possible between different generations and classes of policyholders. However, the office cannot control, except to a small degree, the investment return on the funds. If it experiences a sustained period of low inflation and low interest rates with steady, but unspectacular economic growth, bonus rates could fall substantially. Yet the policyholder may be better off in real terms than if inflation and interest rates had stayed high and bonus rates increased. Perhaps the industry's, and the public's concern with maintaining existing bonus rates has unduly influenced our thoughts when considering the valuation of liabilities. Some actuaries have suggested that at least two-thirds of the current ordinary rate should be reserved, whether implicitly or explicitly. There are worries, very understandably, that if an office were alone in reducing its rate there would be a catastrophic loss of confidence which would precipitate a self-fulfilling crisis. Nevertheless, I feel that so long as the office has a high proportion of equity-type

assets, a reserve of approximately half the current rate, implicit or explicit, is more than adequate. The rest must come from dividend growth. Let us hope that the introduction of projections based on hypothetical investment returns, soon to be extended under LAUTRO, eventually removes the influence of bonus expectations.

6.3 An example of the type of expectation described in 6.1.2 would be a basis of early retirement under an executive pension plan. One possibility is a stated practice of providing benefits that would have arisen if that date had been selected at the outset. To a degree the office can arrange its investments to reduce the risks of having to depart from that practice. If it does not one could argue that it is not providing the care and maintenance that the policyholder expects.

6.4 My third area is that of explicit but non-guaranteed periodic fund charges. If the charge is expressed as a percentage of the fund then the office has the power to raise that percentage. If a policy fee is charged, linked to some index such as the Retail Prices Index, there is probably more incentive for cost control. However, the office will probably have the power to change the index or to raise the policy fee to a new level if it considers the circumstances appropriate. Hence the office may feel justified in assuming that a sterling reserve will never be required and may regard this as the benefit of good policy design. But surely the policyholder has a reasonable expectation that any such changes in the periodic charge will be in accordance with economic circumstances and not either a licence for the office to make extra profits or to rescue itself from inadequate expense control. If an office discovers that negative cash-flows are likely in future unless charges are raised well above the market level then perhaps it should set up a sterling reserve, recognizing that its powers, in practice, are not unlimited. Clearly the office would not wish to be faced with a sudden and significant requirement of this nature which it may find very difficult to meet. One approach might be to project cash-flows every valuation on pessimistic assumptions for inflation and unit cost control, but assume the fund charge to be ·25% higher or the policy fee 25% higher than the current level.

7. EXPENSES

7.1 Provision for future expenses, where explicit, attracts much diversity of approach both in principle and detail. The Insurance Company Regulations require particular attention to be given to the following aspects:

  (i) Consistency with the current level of expenses experienced by the company. (Although not spelt out, offices should allow for inflation of unit costs).
  (ii) Taxation.
  (iii) The possibility that the company may cease to transact new business.

7.2 Point (i), with an allowance for tax relief, can be checked roughly by the authorities by comparing the expense provisions shown in Form 55 and

paragraph 7 of Schedule 4 with the break down of the actual expenses for the year shown in Form 41 of the Returns. Allowance for inflation should be consistent with the long-term valuation rate of interest used. For conventional business, in current conditions, I would regard 5% as the minimum rate that should be allowed for and a rate of 7·5% as more than adequate.

7.3 If a company is unable to relieve all its expenses against tax because of a shortfall in investment income it can value this 'excess E' at a suitably high rate of interest. However, most companies appear to use it as a broad offset against future capital gains tax liability.

7.4 *Closure to new business*

7.4.1 The possibility that the company may cease to transact new business is one which appears to be ignored by many companies. Until recently offices generally may have felt such a possibility to be remote. Besides, it may have seemed insulting to managements keen on expansion! However, with increasing statutory controls and competitive pressures who can say which company can feel completely free from such a restriction. Even a sudden need to reduce new business could cause a short, sharp expense overrun.

7.4.2 Suppose an office decides to make a special reserve against this contingency. Clearly the appropriate sum will depend on the office's circum-stances and it is probably impossible to lay down specific rules. One might assume that commission payments for acquisition would cease immediately but that other acquisition costs would take at least a year to eliminate. Since those costs will include a share of premises it may not be possible to eliminate them entirely until the office relocates to smaller premises and either sells or lets the existing ones. In addition there would probably be redundancy payments to meet. As a rough guide, therefore, perhaps a reserve of one year's 'non-commission' acquisition costs would be a reasonable provision, less any margin already in the valuation expense loadings over and above the reported level of maintenance expenses.

7.5 For linked business the capacity to meet future expenses, along with all other aspects of such contracts, should be tested by carrying out cash-flow projections and a description of the whole process is given in an excellent paper presented to the Institute in 1978 by Brown et al (6). At present there are no specific regulations covering linked business and in testing the adequacy of future charges to meet future expenses current practice is to use techniques and parameters much in line with those recommended in the above mentioned paper. Perhaps the most interesting aspect is the approach to voluntary discontinuance. Three forms which this can take are:

(i) Surrenders or lapses,
(ii) Conversion to a paid up contract,
(iii) Part surrender.

If the cash-flow projection is carried out policy by policy it can be argued that no

allowance for lapses (for the purpose of testing for adequacy in meeting future expenses) needs to be made. However, there could be a problem with regular premium policies being converted to paid-up contracts. The need to provide for this option in the valuation basis was pointed out in the discussion to the 1978 paper mentioned above. A scrutiny (not exhaustive) of DTI Returns for 1985 for linked business suggests that offices feel they need not do so. This is also true of allowance for part surrenders. Considering the attractions of regular withdrawals on Single Premium Bonds I find this surprising if the fund charge is related to fund size. On the other hand, when one considers the effect of regular part surrenders on the cash flow projection one can appreciate that it is a very convenient omission!

8. PRESENTATION OF RESULTS

8.1 The advent of solvency margins has necessitated drawing a clear distinction between liabilities and margins, regardless of the treatment of assets in the Schedule 4 valuation. In this section we will look at a few of these aspects.

8.2 *Capital Gains Tax*
8.2.1 Provision for future capital gains tax can be presented in one of several ways.

(i) Schedule 4 can show it as an explicit margin or it could be implicit in the margins in the valuation basis.
(ii) Part of the investment reserve can be earmarked. This would be done by providing a footnote to Form 14 of the DTI Returns stating how much of the investment reserve in line 51 of that form is required for this provision.
(iii) It could be treated as deferred taxation and included in line 44 of Form 14.

8.2.2 A disadvantage of the first 2 methods described above is that since the provision is treated as a mathematical reserve it would attract a solvency margin. Method (iii) overcomes this, but since Form 14 is subject to audit the agreement of the auditors is required. Normally there should be no great difficulty in obtaining such agreement and therefore it seems a sensible approach.

8.2.3 Two other implicit methods of provision are to make an allowance in the valuation of the admissible assets or to offset future gains tax against unrelieved expenses if appropriate.

8.3 *Mismatching (or Regulation 55) Provision Revisited*
8.3.1 The alternative approaches of an explicit provision out of the investment reserve and reliance on margins in the valuation basis have already been discussed. Whatever approach, or combination of approaches the office uses, a solvency margin will still be required. This is because the provision is a mathematical reserve. At the same time it will be affected by the change in the capital gains tax liability resulting from the change in the value of assets.

8.3.2 One point which does not seem very clear, at least to the present writer, is just how the mismatching provision calculated as illustrated in Section 5 should be adjusted to allow for these factors. A number of approaches are no doubt being considered by many actuaries at present. With the help of a little bit of algebra I offer my own interpretation here. It is based on the concept that the office should be able to comply with the Regulations, covering all liabilities and margins except Regulation 55, in the new conditions. I will assume that the office only has equities left for hypothecation before consideration of any explicit reserve for mismatching. The procedure determines the mismatching reserve from scratch assuming that global mismatching applies.

8.3.3 Let R = mathematical liabilities before testing under Regulation 55.

MR = the explicit extra required for Regulation 55, to be determined.

CGT = provision for future capital gains tax.

SM = solvency margin calculated as if R were the total mathematical reserves. The final solvency margin in the Returns will be greater by the amount $\cdot 04 \times MR$.

A_r = assets hypothecated to R $(A_r = R)$.

A_e = additional assets in respect of the mismatch provision, CGT and solvency margin. Thus $A_e = 1 \cdot 04 \ MR + SM + CGT$.

Any remaining assets are considered free and do not enter the calculation.

8.3.4 With the change in investment conditions:

R falls to R',

Regulation 55 is ignored,

CGT falls to CGT' due to a fall in equity prices,

SM falls to SM' (calculated using R'),

A_r falls to A_r',

A_e falls to $A_e' = \cdot 75 \ A_e$

To comply with the necessary regulations in the new conditions we must have

$$A_r + \cdot 75 \ A_e = R' + CGT' + SM'$$

Therefore

$$A_r + \cdot 75 \ (1 \cdot 04 \ MR + SM + CGT) = R + CGT' + SM'$$

Rearranging, we have

$$MR = \frac{\dfrac{1}{\cdot 75}(R' + CGT' + SM' - A_r') - SM - CGT}{1 \cdot 04}$$

8.3.5 A numerical example

Let
$$A_r = R = 800$$
$$CGT = 25$$
$$SM = 40$$
$$R' = 700$$
$$A_r' = 620$$
$$CGT' = 15$$
$$SM' = 36$$

Therefore
$$MR = 105\cdot45$$
$$A_e = 174\cdot67$$
and
$$A_e' = 131$$

The total solvency margin is $SM + \cdot04\ MR = 44\cdot22$

Results before and after a change in investment conditions are summarized below.

	Before	After
Required assets	800	620
	174·67	131
	974·67	751
Total liabilities	800	700
and margins	105·45	—
	25	15
	44·22	36
	974·67	751

In effect this interpretation requires an allowance for the fall in assets backing the solvency margin (offset by the fall in the solvency margin itself) and may be unnecessarily onerous. On the other hand, if the office held assets of less than 974·67 it would be unable to cover the solvency margin explicitly in the changed conditions. Is this acceptable? The answer may lie in the authorities' attitude to Implicit Items.

8.4 Implicit Items

8.4.1 Implicit items come under Regulations 10 to 13 and are granted by the DTI under Section 68 orders. The 2 items usually applied for are Future Profits and Zillmerization with the former being far more common. Perhaps this is not surprising since Zillmerization seems more naturally dealt with directly in the valuation basis. Pre-valuation estimates should indicate the degree of Zillmer appropriate for the office. With Future Profits the office is taking credit for part of the future bonus reserve it is setting up in the valuation basis. Both items, in

effect, are a weakening of the valuation. The Zillmer item, plus any direct allowance in the valuation basis, is limited to what could be extracted under the Regulations relating to the Schedule 4 valuation. The Future Profits item, however, may not be available directly due to the statutory limitation on valuation rates of interest. What the Regulations taketh away with one hand they giveth back with the other!

8.4.2 Details for the calculation of Future Profits are set out in the Regulations and Guidance Notes. Broadly, this item is the product of half the average annual profit disclosed over the preceding 5 years and the average term to run. The profit figures must allow for any relative weakening of the valuation basis and exclude the contribution from explicit items covering the solvency margin and any other exceptional items. The average period to run should allow for all expected decrements and in any case is limited to 10 years.

8.4.3 Implicit items approved under a Section 68 Order can be used to cover part of the office's solvency margin. However, the explicit assets must at least cover one-sixth of the solvency margin, a figure which in turn is subject to an absolute minimum known as the Minimum Guarantee Fund. The use of implicit items therefore increases the office's free asset ratio (see 8.5 below). In this sense they are acting as assets. This is emphasized on Form 9 where they are added to the explicit assets available for the solvency margin, not deducted from the liabilities.

8.4.4 Only a few offices have so far applied for Implicit Items and there still appears to be some stigma attached to them. The GAD have made it clear that no such stigma should be attached. However, I suspect most offices will be more concerned at the reaction of the financial press and the financial intermediaries and we must expect the cautious approach to continue. So far there has been little need anyway since the strength of equity markets has enabled most offices to meet their solvency margins in comfort. However, if a substantial rise in interest rates and fall in market values occurred it might be an appropriate way to meet the solvency margin and at the same time maintain the ordinary bonus rate. This assumes that the terminal bonus would have been cut and that the office's investigations showed that under the new investment conditions the ordinary bonus rate was sustainable. Whether or not this would be regarded as a sign of weakness remains to be seen.

8.5 *Free Assets*

8.5.1 This is the figure which appears at the bottom of Form 9 of the DTI Returns and is equal to

Admissible Assets (including Implicit Items if applicable)

Less all liabilities (including provision for mismatching and future CGT)

Less solvency margin.

8.5.2 From this the ratio of free assets to admissible assets can readily be calculated. Although Form 9 of the returns is a useful summary of an office's

position it has gained undue prominence just recently. We have even had free asset ratio league tables in the financial press. As the authorities and the profession have been at pains to point out, there is a great deal more to assessing the financial strength of a life office than any single statistic can provide and we must try harder to get this message across.

9. EPILOGUE

The main purposes of the statutory valuation are to demonstrate solvency (whatever that means) and, further, to demonstrate the office's ability to meet policyholders' reasonable expectations (whatever that should mean). The present system is a mixed bag of actuarial science, prudent assumptions and *ad hoc* reserves and margins. It may be tempting to regard the Regulations as no more than a series of hurdles to be negotiated. For example, one could look for the optimum hypothecation of assets to minimize statutory reserves. Nevertheless, for all their faults, the Regulations do provide a framework which should be interpreted responsibly. At least they do not yet place unreasonable burdens on offices' technical resources. Furthermore, the deterministic approach is readily understood and has a long tradition. In recent years actuaries have been experimenting increasingly with stochastic models, a leading example of which is a paper presented to the Faculty of Actuaries in 1984 (7) to investigate the solvency of life offices. Will we see a stochastic statutory basis in our working lifetimes? I feel it is a possibility, but despite the unsatisfactory nature of the present system I do not particularly wish it.

10. ACKNOWLEDGEMENTS

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