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Sources of Uncertainty and their Impact

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Agenda

- Different Sources of Uncertainty
- Measurement of Uncertainty
- Examples – Curve Fitting (Reinsurance Pricing)
- How to Manage Uncertainty

Different Sources of Uncertainty
Topics

- The Problem
- Parameter Uncertainty
- Model Uncertainty
- Stochastic Uncertainty

Different Sources of Uncertainty
The Problem

- The Actuarial Process involves:
  - Predicting the Future
  - Using:
    - Historical Data, information and Prior beliefs
    - Specifying Future behaviour (or Model)
- The Increase in:
  - Computing Power
  - Complex Modelling algorithms and processes
- Has:
  - Given us the ability to solve complex problems
  - NOT REMOVED the uncertainty around such modelling

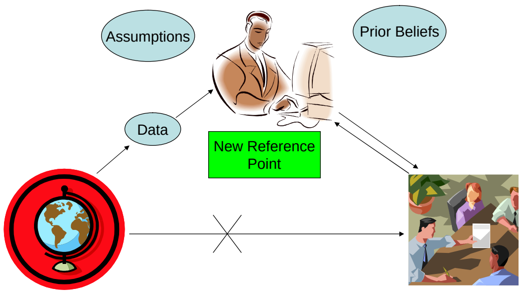
Different Sources of Uncertainty
The Problem

- Sources of Uncertainty are Often:
  - Not Recognised
  - If Recognised then frequently Under-modelled
  - Mis-communicated:
    - Not the same as "Reliances & Limitations" in Actuarial Reports
- Impact of Uncertainty can be Very Significant
- How to Communicate:
  - Sources of Uncertainty in a practical manner
  - One doesn't have Perfect Foresight
  - Avoid perception of the limitation of any analysis – Ranges ?

Different Sources of Uncertainty
The Problem

- CAS RWP (2005) on Risk Transfer:
  - Ultimate Loss estimates
  - Rate Level History
  - Prospective rate change
  - Historical Claim Trend estimates
  - Prospective Claim Trend estimates
  - Experience period might be too short to include large losses
  - The 'Best Fit' distribution is not the actual
  - Cash-Flow timing assumptions
  - Prospective Exposure mix
  - Multi-year Deals – Parameter Uncertainty increases with time

Different Sources of Uncertainty
The Problem



Different Sources of Uncertainty
Parameter Uncertainty

- What it is:
  - Parameters are Incorrect given that the Model is Correct
  - Parameters Change through Time
- How it arises:
  - Limitations in the amount of Data to estimate parameters
  - Greater Impact in the Tail than the Expected Value

Different Sources of Uncertainty
Model Uncertainty

- What it is:
  - Not having the "True Model" or having an "Incorrect Model"
- How it arises:
  - The Model from the 'Best Fit' may not be the "True" Model
    - The 'Best Fit' is not the only criteria → Predictive Power ?
    - Helps if there is some scientific / behavioural rationale as well
  - The Model imposes structural properties that may not hold

Different Sources of Uncertainty
Stochastic Uncertainty

- What it is:
  - The Modelled Outcome is not as Expected
    - Given Model and Parameters are correct
- How it arises:
  - Not having enough Outcomes – If Simulating Experience
  - Greater Impact in the Tail than the Expected Value

Measurement of Uncertainty
Topics

- Parameter Uncertainty
- Model Uncertainty
- Stochastic Uncertainty

Measurement of Uncertainty
Parameter Uncertainty

- Confidence Intervals for the Parameters
- Bootstrapping:
  - Resample with replacement many times
    - Easy to understand and implement / Sample Size ?
- Bayesian Techniques:
  - Can combine a Priori Belief with Actual Data
    - Overcome limitations of only Experience Data
    - Determination of Priori Distribution
- Can be recognised through the use of simulation:
  - Parameter uncertainty can be included within the simulation
  - Make use of information obtained in the claim fitting process
    - Parameter CVs

Measurement of Uncertainty
Model Uncertainty

- Investigation of a Range of Models:
  - Consider scientific rationale:
    - Independent Loss Events → Poisson Process
    - Prior Knowledge of typical models → Selected Severity
  - Test sensitivity of modelled outputs
  - Helps in the selection of the most appropriate model

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Measurement of Uncertainty
Stochastic Uncertainty

- Estimated through Simulation:
- Minimisation:
  - Large number of simulations / Convergence of results
  - Sampling Methods
  - Software choice (C++, VBA, @Risk)
  - Closed-Form Solution
    - Minimise uncertainty vs Accuracy of formulae

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Examples – Reinsurance Pricing
Topics

- Fitting Curves to Data
- Random Samples from Simple Pareto (Example 3)
- Example 1 – Curve Fitting: Low Uncertainty
- Example 2 – Curve Fitting: High Uncertainty
- Example 3 – Pricing: Parameter Uncertainty

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Examples – Reinsurance Pricing
Fitting Curves to Data

- Allows use of the client's own data instead of industry
- Allows Simulations - Distributions of Excess Layer
- Gives information where data is missing
- Provides smoothing where data is present
- Provides distributions for parameters if MLE is used

Examples – Reinsurance Pricing
Fitting Curves to Data – Adjusting Past Data

- Fitting adjusted historical data assumes:
  - Future Loss comes from stochastic process similar to past
  - Assumption common to all actuarial work
- Works best when:
  - Data can be adjusted correctly with confidence
    - Trend, individual loss development
    - Exposure change for claim counts
  - Have several years (e.g. 5 – 8) of stable claims per LOB w.r.t.
    - Limits or Line Sizes
    - Mixes of classes / regions
- One can always fit whatever data is available but:
  - When should you believe the fit – enough to use it
  - When are other methods preferable ?

Examples – Reinsurance Pricing
Fitting Curves to Data – Two Major Obstacles

- Individual Claims - Trended and Developed to Ultimate:
  - Trend is fairly routine (but need trends for 'Large' Claims)
  - Consistent methods for claim development; not well established
- Capping by Policy Limits:
  - Spikes in Data makes fitting difficult
  - Really need uncensored losses (" damage curve")
    - Then need policy limits profile in simulation
  - Can fit to ranges as a possible solution:
    - Will produce Damage Curve
    - Will increase Parameter CV

Examples – Reinsurance Pricing
Fitting Curves to Data – Parameter Uncertainty

- Is always present
- Comes from:
  - Limited Data
  - Lack of knowledge on what model to use
  - Extrapolation of Data
  - Judgement as to what data to use
- Will Push probabilities from the Mean to the Tail:
  - Mean is not affected much → Traditional pricing isn't either
  - High Excess of Loss can be substantially affected
  - Reserving example from US Homeowner Data shows:
    - 99% Loss - $11.5 bn (No PU) → $ 14.6 bn (PU) **+ 25%**
    - Expected - $9.96 bn (No PU) → $ 10.01 bn (PU) **+ 1%**

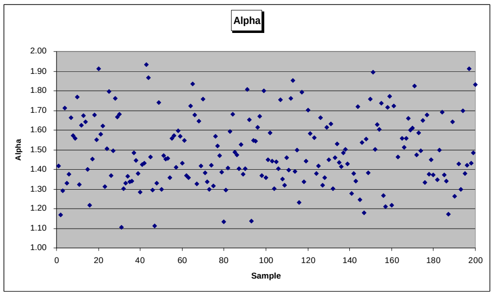
Examples – Reinsurance Pricing
Fitting Curves to Data – RI Distributions

- What distributions do we generally see in insurance ?
- Fitting to Conditional Distributions – Why Min & Max?
  - Minimum:
    - Data Availability, Convenience and Comparability
    - Quality of Fit
  - Maximum:
    - No such thing as infinity
    - Should be largest conceivable event – (3-4x largest observed)
    - Should do Sensitivity testing
- Tail Dependence:
  - Extreme Value Theory – Power Law
  - Usually expressed in terms of α

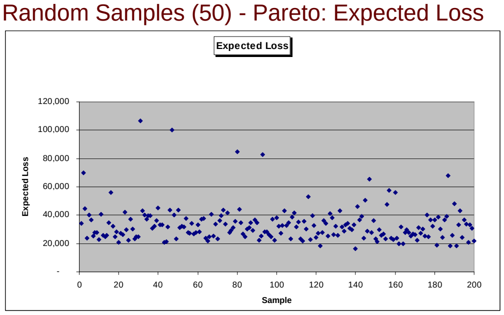
Examples – Reinsurance Pricing
Fitting Curves to Data – 'Best' Distribution

- Model Specification:
  - Look at many distributions
- How do we compare the distributions:
  - Model Specification Criteria - Akaike, Schwarz, A-D, K-S etc
    - Why parameter penalty is necessary
- Where in the Curve are we fitting
- Quality of Fit
- Empirical vs Fitted Distribution:
  - Mean, Standard Deviation and Percentile Matching
- Actual Parameters:
  - Expected Values
  - Parameter CVs

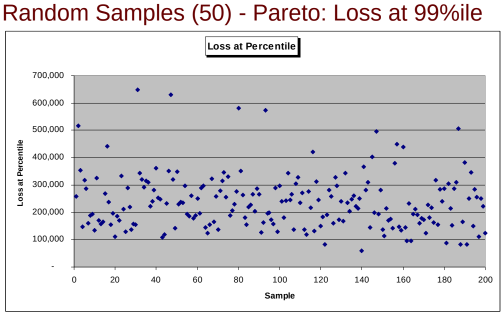
Examples – Reinsurance Pricing
Random Samples (50) – Pareto: MLE of α



Examples – Reinsurance Pricing
Random Samples (50) - Pareto: Expected Loss



Examples – Reinsurance Pricing
Random Samples (50) - Pareto: Loss at 99%ile



Examples – Reinsurance Pricing
Random Samples (50) - Pareto: Distribution

PARETO UNCERTAINTY MODEL

No.of Samples	**200**
Sample Size	**50**

PARETO PARAMETERS

α (Shape)	**1.50**
λ (Scale)	**10000.0**

Percentile	Expected Loss	Loss at 99% ile	Alpha
MODEL	**30,000**	**215,443**	**1.50**
Expected	**33,323**	**238,838**	**1.52**
1% ile	18,381	81,646	2.19
5% ile	20,980	111,351	1.91
10% ile	22,612	130,468	1.79
25% ile	25,389	163,028	1.65
50% ile	31,080	227,244	1.47
75% ile	37,060	288,629	1.37
90% ile	43,459	346,576	1.30
95% ile	53,067	419,872	1.23
99% ile	84,702	580,602	1.13

Examples – Reinsurance Pricing
Random Samples (Sample Size - 250 vs 50)

PARETO UNCERTAINTY MODEL

No.of Samples	**200**
Sample Size	**250**

PARETO PARAMETERS

α (Shape)	**1.50**
λ (Scale)	**10,000**

Percentile	Mean Loss	Loss at 99% ile	Alpha
MODEL	**30,000**	**215,443**	**1.50**
Expected	**30,792**	**222,499**	**1.50**
1% ile	23,087	136,054	1.76
5% ile	25,243	161,326	1.66
10% ile	25,737	167,073	1.64
25% ile	27,865	191,537	1.56
50% ile	29,618	211,218	1.51
75% ile	33,435	252,245	1.43
90% ile	36,196	280,186	1.38
95% ile	38,541	302,743	1.35
99% ile	42,648	339,662	1.31

PARETO UNCERTAINTY MODEL

No.of Samples	**200**
Sample Size	**50**

PARETO PARAMETERS

α (Shape)	**1.50**
λ (Scale)	**10,000**

Percentile	Expected Loss	Loss at 99% ile	Alpha
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99% ile	84,702	580,602	1.13

Examples – Reinsurance Pricing
Example 1 – Description (Low Uncertainty)

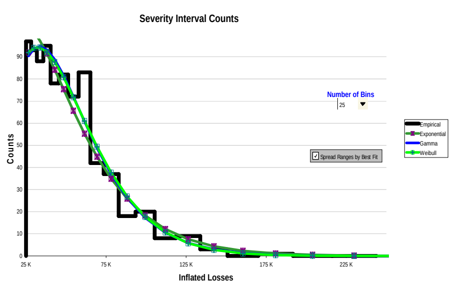
- Commercial Auto Physical damage:
  - Claim sizes $25k to $186k
  - 826 Claims
- Ideal for fitting:
  - No individual claim development
  - 7 years stable but growing business
  - No capping by limits
- No very representative of usual reinsurance situation:

Examples – Reinsurance Pricing
Example 1 – Data

Year	Trended Loss
1999	185,991
2000	141,998
1998	139,540
2000	134,234
2000	131,027
1994	130,615
2000	128,774
1998	127,221
2000	125,153
2000	125,153
2000	125,022
2000	120,585
1995	119,598
1996	116,867
2000	116,696
2000	116,649
1998	113,825
2000	113,775
1999	113,113
2000	111,500
1999	110,288
2000	105,098
2000	103,568
2000	103,207
1994	102,490
1997	102,287
1994	100,241
etc.....	

Minimum	25,026
Maximum	185,991
Empirical Mean	48,766
Empirical Std Dev	21,721
CV	44.5%
Min for fitting	25,000
Max for fitting	500,000

Examples – Reinsurance Pricing
Example 1 – Fitted Curves



Examples – Reinsurance Pricing
Example 1 – Parameters

Parameter	Exponential	Gamma	Weibull
Alpha			
Theta	23,765.89	17,253.52	32,136.69
Beta		1.936	1.250
Tau			
Split			
Mu			
Sigma			
Parameter 1 St. Dev.	826.86	1,895.75	2,750.73
Parameter 2 St. Dev.		0.356	0.089
Parameter 3 St. Dev.			
Parameter 4 St. Dev.			
Parameter 1-2 Corr		(96.82%)	94.56%

	Exponential	Gamma	Weibull
F(m)	65.1%	44.5%	51.8%
F(M)	100.0%	100.0%	100.0%
E(X | m < X < M)	48,766	48,766	48,766
Parameter 1 CV	3.5%	11.0%	8.6%
Parameter 2 CV		18.4%	7.1%
Akaike	9,153.70	9,154.81	9,154.97
Difference from Best	0.00	1.11	1.27

Examples – Reinsurance Pricing
Example 1 – Exponential vs Empirical

Loss	Modeled Percentile	Empirical Percentile
$25,000	0.00%	0.00%
$27,489	10.00%	9.32%
$30,301	20.00%	18.16%
$31,829	25.00%	23.24%
$37,161	40.00%	38.01%
$41,433	50.00%	46.97%
$46,749	60.00%	56.05%
$58,022	75.00%	73.61%
$63,296	80.00%	80.75%
$79,754	90.00%	90.56%
$95,700	95.00%	96.00%
$117,220	98.00%	98.43%
$135,245	99.00%	99.64%
$158,273	99.60%	99.88%
$176,536	99.80%	99.88%
$192,830	99.90%	100.00%
$308,155	99.99%	100.00%

Examples – Reinsurance Pricing
Examples 1 – Weibull vs Empirical

Loss	Modeled Percentile	Empirical Percentile
$25,000	0.00%	0.00%
$27,866	10.00%	10.9%
$30,980	20.00%	20.6%
$32,627	25.00%	25.3%
$38,189	40.00%	40.1%
$42,596	50.00%	49.0%
$47,890	60.00%	58.5%
$58,649	75.00%	75.1%
$63,510	80.00%	80.9%
$78,260	90.00%	89.8%
$92,326	95.00%	94.9%
$109,702	98.00%	97.5%
$122,816	99.00%	98.7%
$140,410	99.60%	99.8%
$152,480	99.80%	99.9%
$163,770	99.90%	99.9%
$223,815	99.99%	100.0%

Examples – Reinsurance Pricing
Example 2 – Description (High Uncertainty)

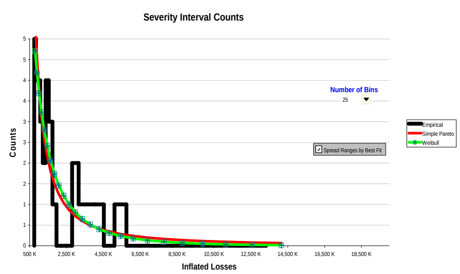
- Medium – Large Commercial Property:
  - Claim sizes $750k to $5.2m
  - 32 Claims over 10 years
  - More representative of typical reinsurance situation
- Illustrates:
  - Deciding between fits
  - Looking more closely at high parameter uncertainty
    - Weibull has high parameter CVs
    - Why does this happen and what is the effect in simulations ?

Examples – Reinsurance Pricing
Example 2 – Data

1995	5,211,708
2003	4,092,503
1998	3,581,179
1998	3,478,904
2003	2,923,505
2003	2,917,710
1999	1,828,737
1997	1,727,535
1999	1,566,039
2002	1,563,198
1995	1,524,288
2004	1,518,950
2002	1,478,829
2001	1,433,939
1998	1,342,704
2001	1,333,595
2000	1,116,007
1996	1,114,965
1996	1,087,296
1997	1,061,951
2004	1,045,000
1997	965,760
1996	957,261
1997	938,800
2001	916,866
1995	905,450
2003	894,902
1997	803,396
1996	799,281
2003	761,933
1996	755,425
2000	754,719

Minimum	754,719
Maximum	5,211,708
Empirical Mean	1,637,573
Empirical Std Dev	1,087,609
CV	66.4%
Min for fitting	750,000
Max for fitting	15,000,000

Examples – Reinsurance Pricing
Example 2 – Fitted Curves



Examples – Reinsurance Pricing
Example 2 – Parameters

Parameter	Simple Pareto	Weibull
Alpha	1.5378	
Theta		25,757.57
Beta		0.371
Tau		
Split		
Mu		
Sigma		
Parameter 1 St. Dev.	0.3071	198,584.71
Parameter 2 St. Dev.		0.430
Parameter 3 St. Dev.		
Parameter 4 St. Dev.		
Parameter 1-2 Corr		99.80%

Parameter CV	20.0%	771.0%
		115.9%
Min	750,000	750,000
Max	15,000,000	15,000,000
F(min)	0.0%	97.0%
F(max)	99.0%	100.0%
Simulated mean	1,750,000	4,170,000
Simulated std dev	1,690,000	4,000,000
CV	96.6%	95.9%

Very high parameter CV

Leads to unreasonable results

Examples – Reinsurance Pricing
Example 2 – Simple Pareto vs Empirical

Loss	Modeled Percentile	Empirical Percentile
$800,911	10.00%	12.5%
$863,889	20.00%	15.6%
$900,094	25.00%	18.8%
$1,037,356	40.00%	34.4%
$1,165,310	50.00%	50.0%
$1,343,862	60.00%	56.3%
$1,821,892	75.00%	78.1%
$2,100,063	80.00%	81.3%
$3,229,350	90.00%	87.5%
$4,815,926	95.00%	96.9%
$7,703,060	98.00%	100.0%
$9,994,746	99.00%	100.0%
$12,414,227	99.60%	100.0%
$13,644,360	99.80%	100.0%
$14,359,357	99.90%	100.0%
$14,934,111	99.99%	100.0%

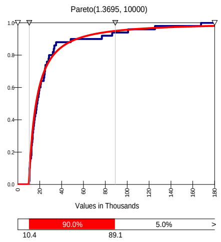
Examples – Reinsurance Pricing
Example 3 – Description (Parameter Uncertainty)

- Simple Pareto:
  - 'True' Population
    - α = 1.50
    - λ = 10,000
  - Sampled Data
    - α = 1.3695
    - α at the 75%ile of Expected Loss (25%ile of α)
- Illustrates:
  - Sampled Data → Fitted Distribution
  - Impact of Parameter Uncertainty on Pricing

Examples – Reinsurance Pricing
Example 3 – Data – Sampled 50 Losses

Loss	Loss
10,187	17,856
10,219	17,901
10,409	18,940
10,413	19,130
10,546	19,577
10,580	20,698
10,846	21,053
11,070	23,669
12,083	24,144
12,109	24,518
12,269	24,828
12,279	24,915
12,822	26,583
13,212	28,086
13,418	28,141
13,452	31,979
13,693	32,824
14,188	33,319
14,366	34,952
14,820	48,306
15,156	76,847
15,896	86,288
16,582	100,828
16,683	125,451
17,096	167,589

Examples – Reinsurance Pricing
Example 3 – Fitted Curves (Best Fit - GOF)



	Chi-Sq	A-D	K-S
Test Value	1.2	0.2764	0.06832
P Value	0.9909		
C.Val @ 0.75	4.2549		
C.Val @ 0.5	6.3458		
C.Val @ 0.25	9.0371		
C.Val @ 0.15	10.7479		
C.Val @ 0.1	12.017		
C.Val @ 0.05	14.0671		
C.Val @ 0.025	16.0128		
C.Val @ 0.01	18.4753		
C.Val @ 0.005	20.2777		
C.Val @ 0.001	24.3219		

Examples – Reinsurance Pricing
Example 3 – Fitted Curves

Parameter	Simple Pareto
Alpha	1.37
Theta	
Beta	
Tau	
Split	
Mu	
Sigma	
Parameter 1 St. Dev.	0.19
Parameter 2 St. Dev.	
Parameter 3 St. Dev.	
Parameter 4 St. Dev.	
Parameter 1-2 Corr	
Parameter 1-3 Corr	
Parameter 2-3 Corr	
Parameter 1-4 Corr	
Parameter 2-4 Corr	
Parameter 3-4 Corr	
Parameter 1 CV	14.1%
Parameter 2 CV	
Parameter 3 CV	
Parameter 4 CV	
Comments	

Parameter Standard Deviation

Parameter Coefficient of Variation = 0.19 / 1.37

Examples – Reinsurance Pricing
Example 3 – Loss vs Parameter Uncertainty

PARAMETER UNCERTAINTY

Parameter Uncertainty	Expected Loss	99% Loss	99.5% Loss
No	37,060	288,629	483,252
Yes	42,541	324,527	563,163

Examples – Pricing
Frequency (Claim Count) Fitting

- Similarities / Differences to Fitting Severity:
  - Still use MLE
  - Distributions are discrete
  - Must be consistent with Severity Fitting
    - Fitting Counts excess of severity minimum
- Issues:
  - Developing Claim Counts
  - Trending for Exposure
  - Choosing Distribution – Poisson, Negative Binomial, or Other
  - Parameter Uncertainty – Effective Years
    - Effective years = Parameter Mean / Variance
    - Intuitive measure – Actual years for untrended distributions

Examples – Reinsurance Pricing
Conclusions

- Misinterpretation of Tools can cause problems:
  - A "Best" Fit can still have unreasonable parameter uncertainty
  - A "Best" Fit may not be the best for the situation
- Judgement Essential – Not purely a mechanical process
- Understanding Conditional Fitting:
  - Quality of Fit
- Role of Policy Limits
- Bayesian Inputs:
  - Effectively weights fitting with:
    - Exposure rating
    - Company-specific knowledge

How to Manage Uncertainty
General

- Develop a thorough understanding of:
  - Problem to be Solved
  - Possible Models and Approaches
  - Risks and Uncertainties of the Selected approaches
- Understand:
  - What Risks are Captured by the Models
  - What Risks are Not Captured by the Models
  - The Exposures Units to be Modelled
    - Level of Granularity
    - Uncertainties in such Data
  - Mathematical Axioms underpinning the Model
- Recognise ALL Judgemental steps