TECHNICAL RESERVES WORKING PARTY 1977

SUMMARY OF THE PAPERS PRODUCED:

Following the suggestions made at the York conference, members of the working party have this year produced a number of separate short reports on practical aspects concerning technical reserves. These papers should not be regarded as definitive but more as discussion documents.

It was felt that one way to build on the past work of this working party would be to obtain sets of actual office data and use these as illustrations, with commentaries of the application of various methods for reserve calculations (which have previously been discussed mainly in theoretical terms). Three such reports are presented dealing with motor, liability and short term reinsurance business.

The motor paper

This paper concentrates on methods based on payment data, since recent discussions have indicated that these are particularly suitable for motor business although there has been some criticism of their widespread use. Initially we are given a table of average payments per reported claim, adjusted to 1976 money values using RPI, for claim years since 1966 by development years. This shows remarkable consistency, although some areas for further investigation are pointed out.

It seems strange that the many changes in conditions affecting the account have apparently resulted only in changed claim frequency with the effects on the payment pattern cancelling out. This feature may not be true of future changes in conditions. It is noted that the RPI may not always be the most suitable index, particularly for later years of development where payments are almost all in respect of bodily injury.

Three methods of projecting the outstanding amounts are then discussed. The first is an intuitive method termed Method A, which involves completion of Table 1 using average values for each column and the application of assumed future rates of inflation, the determination of which is assumed to lie in the area for the judgment of the actuary. Judgment may also be allowed if there are grounds for believing that the average value indicated by past years' data may be misleading - for example, weightings (other than unity) may be applied or continuation of a trend. The other two systems illustrated are the Grossing-Up method and the Separation method which have been widely discussed elsewhere. These are more rigorous mathematically, but the conclusion to the paper is that the simpler Method A will probably prove sufficient in most circumstances.

The liability paper

The outstanding claims for an Employers Liability account are estimated using two methods. First, the patterns of development of numbers of claims notified and settled are considered. The methods used are simple but some special investigation was required since it was obvious that a basic change in the procedures for dealing with nil claims had taken place over the period being analysed. Then average settled costs by development year are considered, standardised to 1971 values, by extracting the weighted mean increases evident in the data itself. There is remarkable consistency for the settlement years analysed. This must be regarded as an unexpected result for the type of business, particularly as the rates of increases (due to inflation etc.) in the original figures show no pattern and vary widely between development years. It is most unfortunate that data was not available for claim years prior to 1971, as it is apparent that about 20% of the total claims cost remains unsettled after five years, although only a small number of claims are involved, and so the analysis presented is deficient in this important area.

The second method is based on payments, standardised as before, and this again suffers in this example from the lack of data for the later years of development. It is noted that further investigation would have been carried out given more time so that better assumptions could be made regarding the tail of payments. It may be that after five years' development, the manual estimates on the relatively few cases then outstanding provide greater accuracy than any statistical system of estimating.

In an Appendix, alternative approaches, using the same data, are graphically illustrated. In particular a method of deflating payments using a lagged index of average earnings gives interesting results.

The short term reinsurance paper

It is pointed out early in this paper that despite the long delays in obtaining information, time is of the essence in calculating profit for classes of business so that changes in terms for new writings may be made quickly if necessary. Once the actuary can be sure that his result is in "the right parish", it is likely to be a great deal more useful to provide an approximate answer now rather than a more accurate answer in two years' time. The degrees to which the business should be subdivided into blocks showing essentially different features are discussed. It should be noted that for some types of reinsurance, the market has developed systems where a minimum of data has to be provided to the reinsurer, and that these systems are unlikely to be upset readily. In particular, numbers of claims are well understood and used by the underwriters, and so development triangles are produced for these. Although there is considerable delay in notification of claims, it appears that if there can be consistent patterns here, patterns building up to ultimate loss rates can be found. Various examples are presented. It is suggested that the apparent loss ratio for a given underwriting year x at elapsed time t may be expressed as $l_{x:t}/L_x=1-e^{-(t/k)^a}$ where L_x is the ultimate loss ratio. The parameters a and k may be sufficiently constant over a period of years to be useful for approximations. This is illustrated using Canadian motor statistics.

A paper on the effect of non-uniform exposure on the calculation of earned premiums

For certain classes of business (such as those covering yachts and motor boats, etc.) there can be obvious seasonal variation in claim frequencies or other non-uniform exposure to risk. A simple model was constructed to demonstrate the differences in UPR under the traditional 24ths method and the true requirement. Results are given for a variety of situations with peak periods of risk, high rates of premium growth and claim inflation. It is concluded that the error involved in using the 24ths method for year end earned premium figures in annual accounts is satisfactorily small, although the UPR values carried forward at each year end may be distorted. Also the earned premiums for particular quarters may be distorted, making interpretation of the profit position more difficult.

J. E. Lockett.

Technical Reserves Working Party 1977

Motor outstanding claims

by M.C. Bennett and J.M. Taylor

In this report we propose to consider only motor insurance, although some of the points will be applicable to other classes of non-life business. We intend to concentrate on methods of estimating outstanding claims which are based on PAYMENTS. In particular we assume that the number of claims in each year is available so that we can consider average payments per claim.

The object of the report is to give the reader a feel for different estimating methods as they are applied to motor insurance data.

A great deal of criticism has been levelled at some methods which have been proposed in recent years to calculate outstanding claims by the use of payments data. Some of the criticism arose because of a proposal to embody one method in government regulations, but this proposal was subsequently dropped. Further criticism has been made about the suitability of payments methods for general estimating purposes.

To what extent can we rely on past payments in motor insurance as a guide to the future? At the non-life conference at the University of York in 1976 a number of members reported encouragingly stable patterns of average claim payments in "real" terms from year to year. We believe it is important to illustrate this stability in some actual data. Having considered the level of stability in the payments we can then look at ways of estimating outstanding claims based on the past pattern of average payments and discuss their limitations.

Payments adjusted by using a published index

During inflationary periods, payment distributions are affected by the impact of inflation on these payments over time. To reduce or eliminate this distortion, therefore, we can translate average payments per claim into real terms by adjusting them to a common currency base, using the value of some published index relating to the time the payments were made. Which index to use is considered later but in the example in Table 1 below, which gives figures for one large motor insurer, the retail price index (RPI) has been used. The table shows the development of claim cohorts based in this case on year of notification of the claim. The year of development is the year when the payment was made, counting from the claim year.

TABLE 1

Average payments (£) per reported claim adjusted to end of 1976 money values, using RPI.

Year of claim	Year of development 1	2	3	4	5	6	7	8	9	10	11
1966	102.71	40.96	14.76	8.18	4.90	1.93	1.02	0.19	1.05	0.25	0.11
67	101.42	40.06	14.73	10.08	5.64	2.78	0.99	0.06	0.08	-	
68	93.49	37.83	12.03	9.36	5.23	2.20	0.72	0.61	0.41		
69	92.50	41.07	14.18	10.32	4.21	2.20	1.73	0.66			
70	92.58	40.61	14.21	6.41	6.06	3.14	0.77				
71	91.15	42.34	13.19	7.71	4.49	4.13					
72	94.87	41.07	11.51	9.94	4.65						
73	96.00	42.51	12.61	7.24							
74	95.69	39.34	9.99								
75	93.22	37.52									
76	95.09										
Average	95.34	40.33	13.02	8.65	5.03	2.73	1.05	0.38	0.51	0.12	0.11

The figures in Table 1 are based on over 100,000 reported claims per year, and relate to all classes and covers of motor business combined.

(N.B. If the data are based on reported claims, a separate allowance must be made in the reserves for IBNR claims, although payments on reopened reported claims are already included in the above table.)

In adjusting for inflation it has been assumed that payments in year of development 1 are all made in the middle of the third quarter of the year and payments in subsequent years of development are all made in the middle of the second quarter of the year. This is a reasonable enough assumption for the purpose of adjusting payments by a published index. In fact it would not have mattered much if we had merely assumed, e.g., that all the payments were made at the mid point of each year of development, provided the whole of any given column of payments had been treated in a consistent way.

On examining the figures in Table 1 we see a remarkably consistent pattern of payments. In any of the first few years of development the figures in a column are much the same from one claim year to another; in other words, for these development years the coefficient of variation is low. It is the consistent pattern among the years of claim, rather than the variation of individual figures, which is the striking feature of the table.

On closer examination it will be seen that the amounts along the bottom diagonal other than for year of development 1 (that is, the payments in 1976 on pre-current claims) are lower than average - in fact only 90% of the average of previous payment years. This should raise questions in our minds about the future represented by the blank triangular area in the bottom right of the table. In particular :

(a) Is the company behind with its payments? If so, next year's payments could well be higher than average in real terms if the company has by then become up-to-date with its payments.

(b) Alternatively, do the lower payments herald lower payments again in future years?

(c) Are the lower figures merely a temporary feature, with future payments likely to be much in line with the payments of earlier years?

(d) Has the RPI become less suitable for reflecting changes in the cost of motor claims than it appears to have been in earlier years?

The answers to these questions are not immediately obvious, and we shall give further consideration to them later in this report. The important point is that having observed the figures in the table and thereby noted the lower 1976 payments, we are in a position to ask the above, and other, questions and we can then carry out appropriate investigations to try to determine the answers.

Equally, there are questions to be asked if the latest payments are <u>higher</u> than average, and in this case we need to be particularly alert to the possibility of a deterioration in average claim costs.

<u>Choice of index</u>

The use of the RPI in Table 1 gives as constant-looking a set of values down each column as one might imagine to be possible given the basic data on payments. This need not necessarily mean that the RPI is the best published index to use here. For example, if the severity of claims has been declining steadily over the years, and if we had preferred the use of an earnings index (which has increased more rapidly than the RPI over most of the period) the payments would have shown a slight decreasing trend over the period instead of the "spurious" constancy brought out by using the RPI.

The practical differences between the use of different indices are unlikely to be great, and the best course is probably to select an index which, on general grounds, is likely to be close to reflecting changes in the cost of motor claims. We might argue that the RPI would be most suitable for at least the first column, and that an earnings index would be best for the years in which almost all the payments are in respect of bodily injury. In practice the use of just one index for all years of development is likely to be good enough, although we must be prepared to look, where necessary, for evidence that the index is, or has become, unsatisfactory for our purpose.

Further consideration of Table 1

Not all motor payments data, of course, are as well ordered as those summarised in Table 1; indeed there is no telling whether the Table 1 figures will look nearly so well ordered by the time another couple of years' payments have been made. If the payments pattern began to deteriorate substantially the actuary would have to consider how to modify his method of calculating reserves in the light of the changed circumstances.

During the eleven claim years covered in Table 1 there was a modest shift towards non-comprehensive cover from comprehensive cover, an oil crisis and changes in legislation, e.g. breath tests and lower speed limits. Over the period the claim frequency declined fairly steadily but from Table 1, if the RPI is accepted as being a satisfactory index for deflating the payments, the combined effect of the above measures on the pattern of payments per claim is not easy to discern.

If data for only the last, say, 5 years of claim had been available there would have been only one entry in column 5 of Table 1, and no entries in subsequent columns. Year 6 and later accounts for only about 3% of the real cost of a cohort's claims, according to the figures in Table 1, and the total amount for these later years can if necessary be obtained from the case estimates for claim year N-4, when N is the latest year.

Choice of method

Having had a look at the pattern of average payments in a particular case, what method(s) should we use to project the outstanding claims? Clearly we should consider only methods which make some kind of allowance for inflation.

Two methods which have been well documented elsewhere are :

1. Grossing up method based on payments which have been adjusted for inflation, alternatively known as the adjusted chain ladder method. (This method does not require the number of claims in each year to be given.)

2. Separation method, described by G.C. Taylor.

In this note we shall look at another method which we call Method A, and then compare this method with the above two methods.

METHOD A: AN AVERAGE PAYMENTS METHOD

If we can find suitable values for the bottom right hand triangle in Table 1, representing our estimate of future payments in real terms, it will then be a simple matter to adjust these payments back into money values in each future payment year, corresponding to the levels of future inflation we are proposing to assume after December 1976.

Having referred to Table 1, an intuitive approach to the problem of reserving is to take the average of each column as given on the bottom line and assume that this average applies to the remainder (i.e. the future payments part) of the column. Thus the future payments are not obtained by grossing up the payments to date, but by ignoring the payments to date and taking simply the average payments in each development year. We shall call this approach Method A. Unless we know that the character of the claims or the pattern of payments has changed over the period - and there is little evidence from Table 1 that this has been so in this case (although other evidence might be available) - we might easily be prepared to adopt Method A. Because of the relatively low level of payments in 1976 we might wish to adjust the method to allow for a higher than average level in 1977. However, there is a lot to be said for taking the averages in Table 1 as they stand - without even any smoothing applied to the later development years - unless we have evidence that these averages are inadequate (and we need to check carefully to see if there is any such evidence). Special considerations apply to the latest claim year and these are considered later. Great precision in the choice of values in the bottom right hand triangle can often be out of place since rates of future inflation are liable to be very different from those used in the calculations.

Indeed, especially if we are dealing with such regular figures as those in Table 1, the variation of payments from year to year can be small in relation to the range of answers obtained by using different assumptions regarding future rates of inflation of claim costs. If a margin is required in the reserve for outstanding claims it seems natural to provide this by taking cautious future rates of inflation, since this is the area in which so much of the uncertainty lies.

How much margin, if any, is required in the reserves in a particular case is a general policy decision, but it seems appropriate not to release all the "expected profit", on which tax will be payable, while there is a strong possibility that for recent underwriting years the experience may later prove significantly less favourable than is expected at present. How we determine a "suitably cautious" rate of inflation, and how much "margin" there is in any given rate will not be discussed here, being matters for the judgment of the actuary!

The latest claim year

About half the outstanding liability in real terms relates to 1976 claim year. If the payment per claim in year 1 differs much from the average, as it is liable to do for a variety of reasons (e.g. administrative delays or claims spread over the year in an untypical fashion), the payments in future years might well be affected in the opposite direction to the year 1 payments.

For the latest year's claims there is a lot to be said for making an estimate of the total ultimate cost of the claim cohort, ignoring the payments already made, and then subtracting the actual payments in year 1 to give the estimate of what is still outstanding for this cohort. If Method A is being used the approach suggested in this paragraph implies that, having used Method A for all years of claim, an adjustment be made for the latest year to allow for the difference between the actual payments in the year and the average of column 1.

Another approach to estimating the total ultimate cost of the latest year's (year N's) claims is to apply a factor to the estimated ultimate average cost (obtained after using, say, Method A) of claims in year N-1. The factor would represent the assumed weighted inflation between the year N-1 and the year N claim cohorts. This approach is similar to using Method A for the latest year except that the actual payments which enter into the estimate of the ultimate cost of the year N claim cohort are those for the first two payment years of year N-1 rather than the first year only of year N.

The actuary needs to consider which is the most suitable approach in respect of the latest year in the light of the particular figures and portfolio he is assessing, bearing in mind that the latest year is where about half of the outstanding liability in real terms, and probably most of the uncertainty as to the outstanding liability in real terms, lie.

Weighting the past experience

If there has been a changing experience in the average payments over the years for which data are available, we will wish to have less regard to the earlier years than to the more recent years. In Method A we used a weighting of 1 for each claim year to derive averages for the different development years. Alternatively we might in some cases prefer to take other weightings, including the possibility of zero for some years. One such set of weights is W^k $(0 < W < 1)$ for claim year $(N-k)$, for $k = 0, 1, 2 \ldots$..

We must be aware of the limitations of this and any other set of weights we may choose. If the experience has been changing substantially we are unlikely to increase our confidence in the future by applying any automatic weighting formula to the experience of past years. Indeed we may in some cases, e.g. where a continuing trend has been observed in the past, consider that future payments in real terms are likely to lie outside the range of those for earlier years.

Comparison of Method A with other methods

METHOD A: AN AVERAGE PAYMENTS METHOD

As a general rule, when using Method A, the spaces in the bottom right triangle of adjusted average payments are filled automatically by the averages of the columns, so the calculations required to project the future payments are very simple. Further, it is easy to see whether the future pattern so derived is reasonable in relation to the figures in the top left triangle. Further still, it is simple to modify if necessary any of the future average payments if, in the actuary's judgment, this action is called for by reference to the figures of past average payments or in the light of any other knowledge about the portfolio the actuary may have.

METHOD B: GROSSING UP METHOD (ADJUSTED CHAIN LADDER)

The grossing up method again adjusts the payments in each year by use of a published index to a common currency base and then uses ratios (for the earlier claim years combined) of cumulative adjusted payments for successive development years. It applies these ratios to cumulative adjusted payments to date to give projected cumulative adjusted payments at the next year end, and the process is repeated across all the development years. By subtracting successive values along the line of each cohort, we next reduce the cumulative projected payments to actual projected payments in each year, and as in Method A these values can then be multiplied by factors to allow for any chosen rates of future inflation.

This method does not require the number of claims to be given, since it is based on total rather than average payments. The information need not be in triangular form. In particular, if payment totals by cohort are available only from a certain calendar year of payment, the data will comprise a number of diagonals as in diagram (ii) below for Method C, and this layout of data will be quite satisfactory.

In this method the columns are not treated independently as in Method A but the future values in any column depend on the payments to date in previous columns. Consequently, if the payments to date have been on the high side the future payments will be correspondingly high. If the payments to date have been on the low side the future payments will be correspondingly low. The actuary needs to judge whether this feature of this method is what is required for the estimating in a particular case. It may not be since, as was indicated earlier, high payments to date may be followed by high, low or merely average payments in the future. If the actuary decides he must modify the results of this method, since those produced automatically were unsatisfactory, it would probably be better for him to use a Method A approach where he might see more easily than for this method what modifications to the automatically produced values he should make.

METHOD C: SEPARATION METHOD

As an alternative to using a published index to remove the effects of inflation from the payments, we can examine the payments themselves to see what the effect of inflation (and other factors) appears to have been on the actual portfolio.

This method was devised by G.C. Taylor and was described by him (1) and in last year's report on outstanding claims. The thinking behind the method was that a published index might well not appropriately reflect changes in claim costs and that this method would eliminate not only the true inflation experienced by the portfolio but also any other influences which have had some effect.

Our triangle of average payments per claim is represented by a model with terms $r_i\lambda_j$ where i is the development year and j represents a calendar year of payment, so along any diagonal j is constant. The values of r and λ are fitted by summing each column and each diagonal. Having fitted the model to the data we can divide each figure of actual average payments by $\frac{\lambda_j}{\lambda_1}$ to obtain average payments in real terms corresponding to Table 1.

As originally described, the separation method assumed a triangle with 5 cohort years and 5 development years. However, if data for earlier cohorts are available and it seems appropriate to use them, this can be done. Indeed, as for the other methods, the shape of the payments information need not be triangular, and, for example, the following sets of data could be used, where a line represents the development of a cohort :

```
(i)  x x x x x          (ii)             x
     x x x x x                         x x
     x x x x x                       x x x
     x x x x x                     x x x x
     x x x x                     x x x x
     x x x                     x x x x
     x x                       x x x
     x                         x x
                               x
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In Table 2 below we show the figures corresponding to Table 1 obtained from the same payments data by applying the separation method to an 11 by 11 triangle :-

TABLE 2

Payments (£) per claim in 1976 values (with subsequent effects of inflation and other influences removed)

Year of claim	Year of development 1	2	3	4	5	6	7	8	9	10	11
1966	86.14	35.21	13.23	7.53	4.41	1.73	0.88	0.16	0.95	0.23	0.09
67	86.83	35.90	13.55	9.10	5.08	2.38	0.90	0.02	0.07	-	
68	84.29	34.75	10.83	8.43	4.55	1.99	0.62	0.55	0.37		
69	85.21	36.96	12.79	8.91	3.79	1.94	1.55	0.60			
70	84.22	36.66	12.28	5.79	5.36	2.77	0.69				
71	83.41	36.62	11.89	6.81	3.97	3.81					
72	83.44	37.03	10.14	8.77	4.29						
73	87.85	37.49	11.15	6.65							
74	86.53	34.82	9.21								
75	86.07	34.61									
76	89.69										
Average	85.79	36.01	11.67	7.75	4.49	2.44	0.93	0.33	0.46	0.12	0.09

Note: The averages in this table are somewhat lower than those in Table 1, largely because in Table 2 the 1976 values represent values for the whole of 1976 whereas in Table 1 the payments have been adjusted for inflation to the end of 1976.

If we compare the figures in Table 2 with those in Table 1 it is not obvious in this case which set of figures is the more generally acceptable. In the first few development years the figures in Table 2 show a slight increasing trend compared with Table 1 where little or no such trend is noticeable. There is, however, one feature of the data in Table 1 of which the separation method clearly cannot take proper account: the payments in 1976 are higher than those in 1975 for development year 1, but lower for development years 2 to 4, whereas in the fitting process the separation method assumes the same exogenous influences are acting along the whole of each diagonal. Incidentally, this implies that a full year's inflation is acting from one diagonal to the next, whereas in practice there is considerably less than a full year's inflation acting between years of development 1 and 2, but about a full year's inflation between subsequent development years.

This method does not lend itself easily to manual adjustments after the calculations have been carried out.

Comparison of results

Table 3 shows in summary form the results of applying the three methods to the data considered previously.

TABLE 3

Total expected future payments (£) per claim — All motor classes

YEAR OF CLAIM	Inflation after 1976 at 15% p.a. METHOD A*	GROSSING UP METHOD/	SEPARATION METHOD	Inflation after 1976 at 30% p.a. METHOD A*	GROSSING UP METHOD/	SEPARATION METHOD
1967	0.1	0.2	0.1	0.1	0.2	0.1
68	0.3	0.4	0.3	0.3	0.4	0.3
69	0.9	1.0	0.8	1.0	1.1	0.9
70	1.4	1.5	1.3	1.7	1.8	1.6
71	2.7	2.7	2.6	3.4	3.4	3.2
72	6.0	5.8	6.0	7.5	7.3	7.3
73	12.2	12.2	12.0	15.3	15.3	14.9
74	23.2	22.4	22.6	29.5	28.5	28.6
75	40.5	38.6	39.0	52.8	50.5	50.9
76	89.2	88.6	86.2	113.5	113.2	109.6
Total of the above lines	176.5	173.4	170.9	225.1	221.7	217.4

* The future payments for 1976 claims for Method A have not been adjusted (as described earlier in this note) to allow for the actual payments to date differing from those expected on the basis of earlier claim years.

/ Results for this method adjusted to give averages per claim.

It will be seen that the three methods produce broadly similar results in this case.

A second set of data

By comparison with the well ordered set of data considered previously, we now consider in isolation a subset (motor cycle claims) of this same data. We could have chosen to exclude motor cycles from the data considered previously, but it was regarded as satisfactory for purposes of illustration to leave them in.

TABLE 4

Average payments (£) per reported motor cycle claim adjusted to end of 1976 money values, using RPI

Year of claim	Year of development 1	2	3	4	5	6	7	8
1969	44.2	18.8	7.1	9.5	1.0	0.2	-	-
1970	42.2	20.3	10.3	1.2	0.5	0.5	-	
1971	43.0	22.0	5.6	2.0	1.5	0.5		
1972	45.4	22.9	11.7	4.9	4.1			
1973	48.6	31.7	9.0	10.5				
1974	55.2	26.9	3.2					
1975	59.6	33.7						
1976	69.3							
Average	50.9	25.2	7.8	5.6	1.8	0.4	-	-

Data for motor cycles separately prior to 1969 are not available.
The numbers of claims reported in the year has varied during the above period from about 3500 in one year to over 9500 in the latest year.

It is known from other sources that the mix of the portfolio has been changing in the last couple of years or so, the movement being towards younger policyholders, more powerful machines and (especially) a much higher proportion of policyholders taking comprehensive cover. During this time the number of claims has been increasing considerably, with a consequent weighting of notified claims towards the end of the year, a feature which is particularly pronounced for 1976.

NONE of the three methods mentioned copes automatically with either a change of mix of portfolio or a changed distribution of claims during the year. How, then, should we proceed?

It might be of considerable help to have separate figures for comprehensive and non-comp claims, if these were available. The figures for each cover separately would show less of an increasing trend than the aggregate figures, but the increasing trend would by no means be removed as a result of considering the covers separately.

Given the distribution of notified claims during 1976 we might, using payment distributions for monthly or quarterly cohorts, be able to determine that, say, only 80% of the normal first year payments were expected in year 1, for 1976 claims. The "normal" first year average payments would then be estimated as $\frac{69.3}{.80} = 86.6$ If the best estimate we could make of the second year payments, for a year in which the notifications were evenly spread, was that they were likely to be about half the first year payments, then we might take the expected second year average payments for 1976 claims as

$$\frac{86.6}{2} + \text{the shortfall of payments in year 1,}$$

$$\text{i.e. } 43.3 + 17.3 = 60.6 \text{ (say 61)}$$

This approach, although fairly crude, is about the best likely to be available in the circumstances. We do not know how reliable our figure of 61 in column 2 will turn out to be, but we know that using, say, the separation method will not help here. (It will give values which are too low - see Table 5).

The entry of 61 calculated above accounts for a substantial part of the remaining liability. The rest of the table can be completed using Method A, but perhaps adjusting to allow for a deterioration in the later years of development. Since some of the deterioration already experienced for the initial years of development has resulted from a change to a higher proportion of comprehensive business, less deterioration is likely for the later years of development in which payments are more or less independent of cover.

TABLE 5 Motor cycles only

Total expected future payments (£) per claim
Inflation after 1976 at 15% p.a.

Year of claim	(i)*	(ii)**	GROSSING UP ∅ METHOD	SEPARATION METHOD
1972	0.52	0.77	0.74	0.39
1973	2.40	3.60	3.15	2.94
1974	8.68	13.03	9.59	10.88
1975	18.24	27.36	25.10	22.26
1976	85.42	95.90	68.47	60.74
Total of the above lines	115.26	140.66	107.05	97.21

* Year 2 payment for 1976 claims taken as £61 in end of 1976 values (or approx. £65 in 1977 values). Other figures obtained using Method A unadjusted.

** As * except that the future values used in columns 3 to 6 are 50% higher than the averages given in Table 4.

∅ Results for this method adjusted to give averages per claim.

Unlike Table 1, the figures in the later columns of Table 4 are extremely variable on account of the incidence of random large payments in a small portfolio. For such a portfolio as this, <u>any</u> method of statistical estimating of the gross amount of outstanding claims for cohorts prior to the latest three or so is liable to be wildly inaccurate for those earlier cohorts.

As an indication that the grossing up and separation methods do appear to give an inadequate reserve for 1976 claims, it may be noted that between January 1977 and the end of April just over half of the £65 referred to in * above had already been paid per 1976 claim. It is thought that the outcome may lie somewhere between (i) and (ii).

This example shows that situations can easily arise in which formulae must not automatically be applied.

CONCLUSION

An intuitive approach to estimating future claim payments - Method A modified if necessary to cope with particular circumstances - is likely to be as satisfactory as any other method in the situations which can be envisaged in motor insurance, and possibly in various other classes also.

Method A has those useful twin virtues of simplicity and flexibility. If it breaks down in a given situation, even though the required basic data are available, then so, it is suggested, will any other method of formula estimating. Expressed differently, Method A may work when the others do not, but it is most unlikely that the others would work if Method A did not.

REFERENCE

(1) G.C. Taylor, "Separation of Inflation and Other Effects from the Distribution of Non-life Insurance Claim Delays", The ASTIN Bulletin, 1977, Vol IX, Parts 1 and 2, p.219.

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Technical Reserves Working Party

Liability Sub-report by J.E. Lockett and C.J. Mellor

1. Summary

The data analysed are derived from a large Employers' Liability account. Analysis was first made for results to December 1975, but December 1976 results have subsequently been added (bracketed). Several methods of providing a basis for estimating outstanding liabilities have been attempted.

The first approach has estimated numbers of claims, using the development of both claims notified and claims settled, and has applied the estimates of unsettled claims to average costs, standardised to 1971 values.

The second approach has ignored the detailed information regarding numbers of claims, but has analysed the development of both payments, and settled amounts, again standardised to 1971 values.

In this paper, the effect of inflation on claims costs has been estimated from the data, but Appendix 2 demonstrates that in this particular example a lagged earnings index achieves a considerable degree of fit. It would be of interest to discover whether this finding applies also to other employers' liability data.

2. The data

The data are tabulated in Appendix 1.

The terminology employed will be as follows:-

n_d^y - claims notified in the d^{th} year

s_d^y - claims settled at some cost

t_d^y - claims settled at no cost

a_d^y - amounts settled

b_d^y - amounts paid

Let $\bar{n}_d^y$ - etc. represent claims notified up to the end of the d^{th} year.

y refers to year of loss, d to development year.

3. Numbers of claims

The faster development of notified claims compared with those settled at cost should provide a better estimate of the eventual number of claims. If there is a consistent pattern of quick development of claims settled at no cost it will be possible to estimate the number of claims remaining to be settled.

It will be seen from the subsequent analysis that this office's procedures for dealing with nil claims have changed; leading to very variable proportions of nil claims. Thus it was also felt necessary to analyse the development of settled claims.

It will also be noted that the development in 1976 does not conform well to the patterns estimated on the basis of 1975 data.

The development of notified claims is shown below as n^y_{d+1} / n^y_d (The cumulative ratios $\bar{n}^y_{d+1}/\bar{n}^y_d$ were investigated but the higher variance of n^y_2/n^y_1 caused higher variances for subsequent ratios).

Year of loss	Development year d+1/d 2/1	3/2	4/3	5/4
1971	0.36	0.097	0.31	0.21
1972	0.31	0.096	0.29	(0.33)
1973	0.27	0.093	(0.29)	
1974	0.31	(0.093)		
1975	(0.29)			

The ratios appear reasonably consistent. It is not clear whether the ratios between first and second years might be correlated to the proportion of nil claims (shown below). Projections based on the average ratios (including a small number of claims in year 6 onwards) will be shown at the end of the section.

The variation of the proportion of nil claims is shown by considering $(\bar{s}^y_d / \bar{t}^y_d)\%$ $(= \bar{r}^y_d)$

Year of loss	Year of development 1	2	3	4	5
1971	71	114	134	143	148
1972	53	79	96	104	(109)
1973	44	69	83	(91)	
1974	62	98	(118)		
1975	74	(99)			
1976	(56)				

The regular development of these ratios can be seen by considering $(\bar{r}^y_{d+1} / \bar{r}^y_d)\%$

(However, year of loss 1975 has an unexpected increase in 1976) This will enable a projection of the eventual ratio.

Year of loss	Year of development d+1/d 2/1	3/2	4/3	5/4
1971	159	117	107	104
1972	151	121	108	(105)
1973	156	121	(109)	
1974	157	(120)		
1975	(134)			

To see whether there are changes in the delay to settlement (assuming the notification pattern is stable, as appears to be the case) we consider $\left(\bar{s}_d^y + \bar{t}_d^y\right)/\hat{n}_d^y$ %

Year of loss	Year of development 1	2	3	4	5
1971	39.5	75.6	88.4	94.1	97.3
1972	37.7	75.2	88.6	94.7	(97.1)
1973	36.3	76.3	89.5	(94.9)	
1974	33.7	73.6	(88.0)		
1975	31.7	(71.1)			
1976	(28.8)				

There is evidently a slowing of total payments in the first year and this does not seem to be correlated to variations in the proportion settled at no cost.

In view of the variations of the proportion of nil claims the developments of nil claims and claims at cost are considered separately. First, for claims at cost, $\left(\bar{s}_{d+1}^y / \bar{s}_d^y\right)$% are:

Year of loss	Development year d+1/d 2/1	3/2	4/3	5/4
1971	332	129.5	110.4	105.1
1972	332	133.5	111.8	(105.1)
1973	355	133.2	(111.7)	
1974	367	(133.6)		
1975	(340)			

An apparent trend towards slower settlement in the first year has been checked for 1975 year of loss, and estimates for 1975 based on the 1975 trends will need to be altered.

Second, for claims at no cost, $\left(\bar{t}_{d+1}^y / \bar{t}_d^y\right)$% are

Year of loss	Development year d+1/d 2/1	3/2	4/3	5/4
1971	210	109.2	103.0	101.4
1972	220	110.1	103.4	(100.3)
1973	228	110.3	(102.3)	
1974	234	(111.1)		
1975	(254)			

It appears that development is slowing down through years 1, 2 and 3. The faster development of nil claims can be seen (42% in first year of 1971 compared with 20% for settled at some cost).

To project the results up to 1975, the following development ratios were used:

Year of development	Notified n_{d+1}^y / n_d^y	Settled Some Cost $\bar{s}_{d+1}^y / \bar{s}_d^y$	Settled No Cost $\bar{t}_{d+1}^y / \bar{t}_d^y$
2/1	0.30	3.76	2.40
3/2	0.091	1.332	1.103
4/3	0.28	1.111	1.034
5/4	0.20	1.051	1.014
∞/5	0.30	1.040	1.007

These result in the following estimates:

Year of loss	Some Cost (1)	Nil Cost (2)	Total Settled (3)=(1)+(2)	Ratio (2)/(1) (4)	Notified (5)	Difference (5)-(3) =(6)
1971	3 753	2 452	6 205	1.53	6 215	10
1972	3 892	3 494	7 386	1.11	7 387	1
1973	4 286	4 484	8 770	0.96	8 750	-20
1974	4 000	2 943	6 943	1.36	7 005	62
1975	3 832	2 376	6 208	1.61	6 280	72

The differences between the two sets of estimates are a little large and the developments should be adjusted so that differences are reduced. However, they give some idea of the possible margin of error on claims remaining to be settled at some cost. In the analysis that follows the estimates for settled at some cost have been used (column(1)).

4. Average settled cost of claims

The average settled costs (a_d^y / s_d^y) are as follows:-

Year of loss	Development year 1	2	3	4	5
1971	170	345	860	1 280	2 270
1972	190	395	835	1 495	(2 680)
1973	210	430	915	(2 075)	
1974	225	475	(1 100)		
1975	265	(535)			
1976	(310)				

If the average settled costs are denoted by (q_d^y) then the increases to costs (q_d^{y+1} / q_d^y) % are shown below, together with a mean. Since the variance of each average cost will depend on the number of claims involved and on the claim size variance, the mean has been weighted by the number of claims involved (as an approximate weighting).

Year of Settlement	Development year 1	2	3	4	5	Mean
1971/2	111.8					111.8
1972/3	110.5	114.5				113.3
1973/4	107.1	108.9	97.1			105.8
1974/5	117.8	110.5	109.6	116.8		112.2
1975/6	(117.0)	(112.6)	(120.2)	(138.8)	(118.1)	(118.5)

There is no evidence that the increases are dependent on delay. The average costs standardised to 1971 values are as follows:

Year of settlement	Development year 1	2	3	4	5
1971	170	309	679	955	1 510
1972	170	312	623	994	(1 504)
1973	166	321	609	(1 165)	
1974	168	316	(617)		
1975	176	(300)			
1976	(174)				
Average	170	315	637	975	1 510

It can be seen that there is a reasonable consistency with no noticeable trends, and to show this the development ratios are calculated.

Year of	Development year d+1/d			
loss	1/2	3/2	4/3	3/4
1971	182	220	141	158
1972	184	200	160	(151)
1973	193	190	(191)	
1974	188	(195)		
1975	(170)			

In view of the appreciable number of claims unsettled after year 5 it is clearly important to carry the development of average costs beyond the fifth year.

5. Estimated eventual cost

It is simple to calculate the unsettled costs at 1975 values from the numbers unsettled and average costs. The average amount after five years has been taken to be £2 500 (1971 value), but there is clearly a large variance to this guess.

Year of loss	Settled to 1975 £000's	Estimated Unsettled £000's	Total £000's
1971	2 123	541	2 664
1972	2 024	973	2 997
1973	1 780	1 650	3 430
1974	1 006	2 325	3 331
1975	167	3 052	3 219

It can be seen that on this basis 20% is estimated unsettled after 5 years, which is a high percentage to base on a guessed average cost of £2 500.

The development of paid claims provides another source of information which has its advantage in the slightly higher proportion paid at any time.

If there is a substantial pattern of partial payments, analysis by year of payment may make possible a more accurate correction for inflation. Let $\bar{s}^y_\infty$ be the estimated paid at some cost, and consider $\left(b^y_d / \bar{s}^y_\infty \right)$ as a standardised measure of costs.

Year of	Development year				
loss	1	2	3	4	5
1971	3.65	18,91	18.17	12.90	10.02
1972	3.91	19.30	19.94	15.75	
1973	4.06	22.24	22.66		
1974	4.25	12.35			
1975	4.96				

The average rates of inflation are: 1971/2 - 1.071; 1972/3 - 1.059; 1973/4 - 1.120; 1974/5 - 1.150 (These averages are weighted in the same way as in the previous example).

Then the standardised development is

Year of loss	Development year 1	2	3	4	5
1971	3.65	16.82	16.02	10.16	6.86
1972	3.65	17.02	15.70	10.78	
1973	3.58	17.51	15.51		
1974	3.35	17.35			
1975	3.40				

The corresponding development factors are:

Year of loss	Development year d+1/d 2/1	3/2	4/3	5/4
1971	4.61	0.95	0.63	0.68
1972	4.66	0.92	0.69	
1973	4.89	0.89		
1974	5.18			

There appears to be a tendency towards slower payments in years 1 to 2. There is clearly a high proportion unpaid at the end of five years and as a reasonable guess an exponentail run-off is assumed after year five with a factor of two thirds, the estimates are as follows:

Year of loss	Paid to 1975	Estimated unpaid	Eventual cost
1971	2 355	750	3 105
1972	2 292	1 200	3 492
1973	2 098	1 940	4 038
1974	1 184	2 710	3 894
1975	190	3 700	3 890

When these estimates are compared with those on an average cost/frequency basis they are consistently about 17% higher (except for 1975). It is thus apparent that the guesses of payments/settlements after five years are far apart - the average cost of settlements has been under-estimated, or the run-off of payments increases at a sharper rate than the exponential distribution assumed (or a mixture of both).

The divergence of results emphasises the need for a full development of run-off. In this case a period of five years is inadequate, and the same will be found to be true of other liability classes - e.g. products liability, professional indemnity. It is unfortunate that no time was available to obtain the full incurred development which might have provided an indication of the likely development of the tail.

Appendix 1 - Employers' Liability data

Year of loss	Year of delay	Number of claims: Notified (1)	Closed at nil (2)	Closed at cost (3)	Amounts: Settled £000 (4)	Paid £000 (5)	o/s £000 (6)
1971	1	4 408	1 016	724	123	137	
	2	1 591	1 117	1 678	579	676	
	3	155	197	808	669	682	
	4	48	71	324	415	484	
	5	10	34	175	397	376	430
1972	1	5 530	1 366	719	137	152	
	2	1 642	1 639	1 667	658	751	
	3	158	305	799	667	776	
	4	45	112	376	562	613	1 093
	5	(15)	(21)	(183)	(490)	(434)	(693)
1973	1	6 705	1 691	746	157	174	
	2	1 814	2 161	1 905	819	953	
	3	168	395	879	804	971	2 209
	4	(49)	(97)	(414)	(859)	(926)	(1 536)
1974	1	5 208	1 082	674	152	170	
	2	1 589	1 445	1 799	854	1 014	2 969
	3	(147)	(336)	(831)	(916)	(986)	(2 566)
1975	1	4 672	850	630	167	190	3 341
	2	(1 377)	(1 305)	(1 513)	(812)	(949)	(4 065)
1976	1	(4 208)	(779)	(434)	(134)	(167)	(4 928)

TECHNICAL RESERVES WORKING PARTY
LIABILITY SUB-GROUP

APPENDIX 2
by B.D. HUDSON

ESTIMATION OF OUTSTANDING CLAIMS

Introduction

This report is a brief account of one method which has been examined for the estimation of payments still to be made on claims (whether reported or not) arising in a particular accident year. The methods have been applied to data in respect of one office's EL account - the basic data is contained in the attached Table 1 - and some limited success has been obtained. It would be highly desirable to investigate this further, and to use this approach on data for other liability sub-classes, and for data from other offices, but this has not proved possible in the time available.

Again, a report such as this should comment on the basic data - features such as the varying number of claims and the relative number of zero cost settlements are certainly of interest - but again, there is no mention of these features outside this paragraph.

Approach

Clearly the payments made in a particular year will depend on a very wide range of factors - e.g. the type of claim reported, legislation, "social inflation" (i.e. higher awards for specific types of injury) and general wage or price inflation.

This latter factor has arguably been the most important of those mentioned in recent years, and certainly the reserves set by an office to meet outstanding claims will be critically dependent on their view of future inflation.

In considering claim development, it is therefore felt highly desirable - if it is possible - to remove the effects of general inflation. There may be scope for discussion on the most suitable index for this purpose, but this report has taken the index of average earnings as an appropriate measure of inflation.

Study of Deflated Payments

Assuming that all payments in the end column of Table 1 are made mid-year, then using the index of average earnings, they can all be put on a 1971 cost basis. These figures are given in Table 2 (a).

It is possible that these figures could be used in some kind of chain ladder approach - for example, expressing payments made in a particular development year in relation to the payments made in the first year. The figures on this basis are given in Table 2 (b).

Whilst these could be promising and could merit further investigation, it may be that the company involved has - for whatever reason - altered the rate at which it deals with claims over the period concerned; i.e. it may have speeded up or slowed down payments. One measure of this would be the percentage of claims closed at the end of any development year.

There are two approaches to this problem. One would be to express the number of claims settled (either cumulatively or non-cumulatively) as a percentage of the claims reported to date; the other is to estimate the total number of claims likely to be reported for a given accident year in the light of the numbers so far notified, and to express settlements in relation to this.

Four different methods of estimating the eventual number of reported claims were used, and they gave very similar results to each other. Averages of the four methods were taken and the resultant values are given in Table 3; the number of settlements was then expressed in relation to these.

Various graphs were then plotted relating costs (either total or averages) to claim settlement rates (using all claims, or just positive cost claims). Two such graphs are attached as an illustration of those drawn.

Figure 1 plots the total cost against the percentage of all claims settled. Figure 2 uses only positive cost claims, and plots the average settlement cost of these claims against the percentage of them which had been settled.

Whilst Figure 1 may seem encouragingly smooth, it is not, in fact, related to the number of claims arising - for example, if the number of claims reported doubled, then it would not be reasonable to expect the development of those claims to follow the line shown since the costs involved would be higher.

For this reason, averages were used in plotting Figure 2, but - as can be seen - the averages paid for a given settlement proportion have been consistently decreasing over the period examined, and so the graphs emerging are not particularly helpful.

Lagged Inflation

It was suggested that the deflation process used might be too severe and might be over-compensating for the way in which inflation affects claim payments.

Two further exercises were therefore carried out in which actual payments were deflated by the index of average earnings, in one case lagged by six months and in the other by 12 months.

An example of what this lagging can achieve is shown in Figures 3 (a) to 3 (c). These are based purely on positive cost claims, and show the average settlement cost of such claims against the proportion of those claims which have been settled.

In 3 (a) the payments have been fully deflated; in 3 (b) they have been deflated by the index of the average earnings lagged by 6 months; and in 3 (c) with the index lagged by 12 months.

The pattern is very clear. In 3 (a) there is virtually a separate line for each year of accident; in 3 (b) distinct lines are no longer obvious, but in 3 (c) the points are all very closely related to a single line.

Conclusion

Further study is very much needed on associated figures, but it may be that deflating EL claim payments by an index of average earnings lagged by up to 12 months may provide a stability in claim payment developments which would enable estimates of outstanding claims to be made with some degree of reliability. In this case, the figures would still have to be grossed-up depending on the company's view of future inflation.

TABLE 1

Basic Data
Employers Liability

Accident Year	Development Year	Claims Notified	Claims Closed at Nil	Claims Closed at +ve Cost	Total closed costs £ '000	Payments made in develop. year £ '000
1970	1	4408	1016	724	123	137
	2	1591	1117	1678	579	676
	3	155	197	708	609	682
	4	48	71	324	415	484
	5	10	34	175	397	376
1972	1	5530	1366	719	137	152
	2	1642	1639	1667	658	751
	3	158	305	799	667	776
	4	45	112	376	562	613
	5					434
1973	1	6705	1691	746	157	174
	2	1814	2161	1965	819	953
	3	168	395	879	804	971
	4					926
1974	1	5208	1082	674	152	170
	2	1589	1445	1799	854	1014
	3					986
1975	1	4672	850	630	167	190
	2					949
1976	1					167

TABLE 2 (a)

Claim Payments (£'000)
deflated by Index of Average Earnings

Development Year	Accident Year 1971	1972	1973	1974	1975	1976
1	137	133	135	110	95	70
2	593	582	615	504	395	
3	529	501	483	411		
4	312	305	386			
5	187	181				

TABLE 2 (b)

Figures in Table 2(a)
indexed to development Year 1

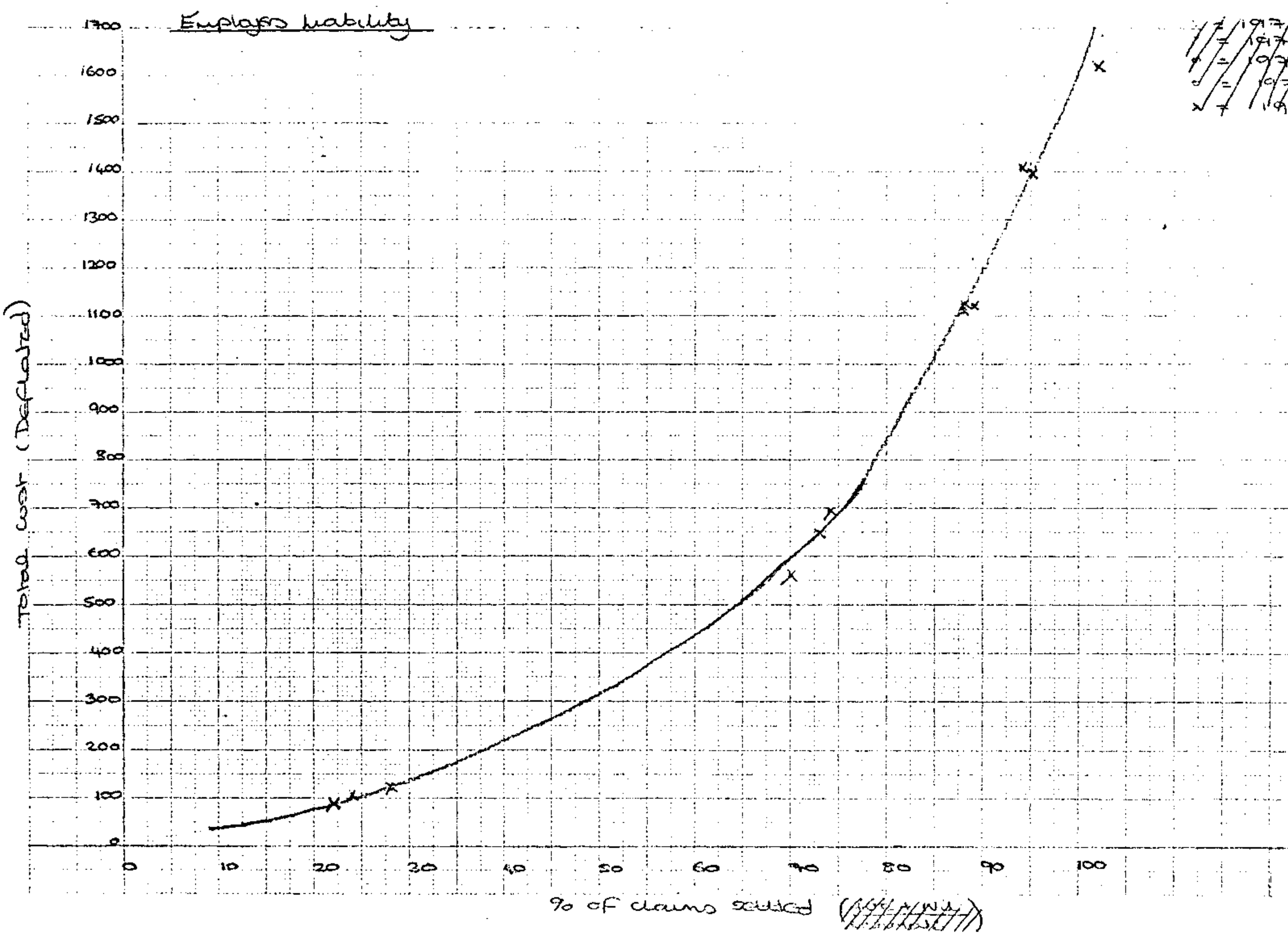
Development Year	Accident Year 1971	1972	1973	1974	1975
1	100	100	100	100	100
2	433	438	455	458	415
3	386	376	357	373	
4	227	229	286		
5	136	136			

TABLE 3

Estimated number of claims
which will eventually be reported

Accident Year	All Claims	Positive Cost claims
1971	6212	3730
1972	7387	3870
1973	8753	4257
1974	6998	4058
1975	6280	3885

FIGURE 1


Employers liability
Total cost (Deflated)
% of claims settled
1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0
0
10
20
30
40
50
60
70
80
90
100

Employers Liability
Positive cost claims only

FIGURE 2

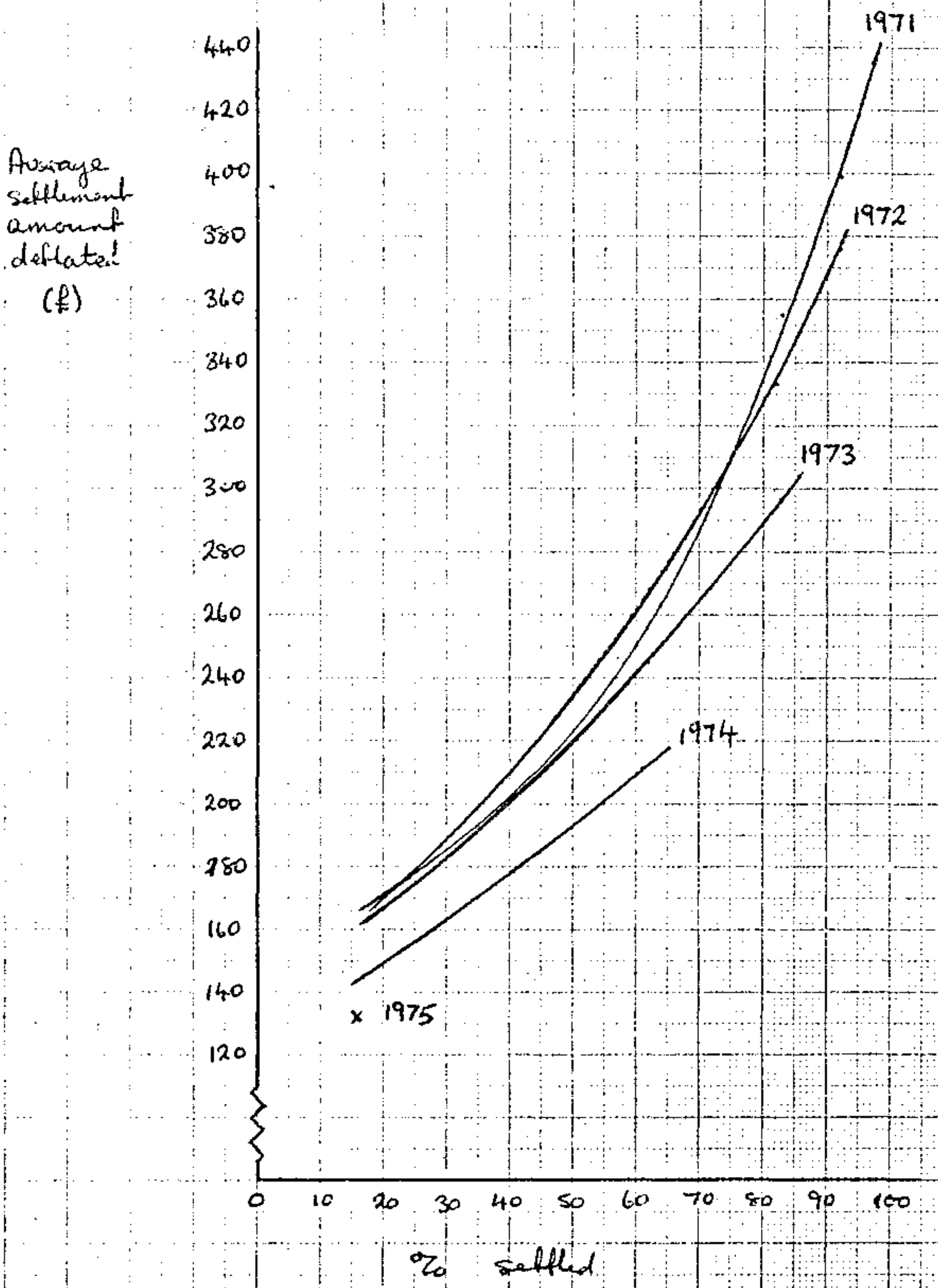
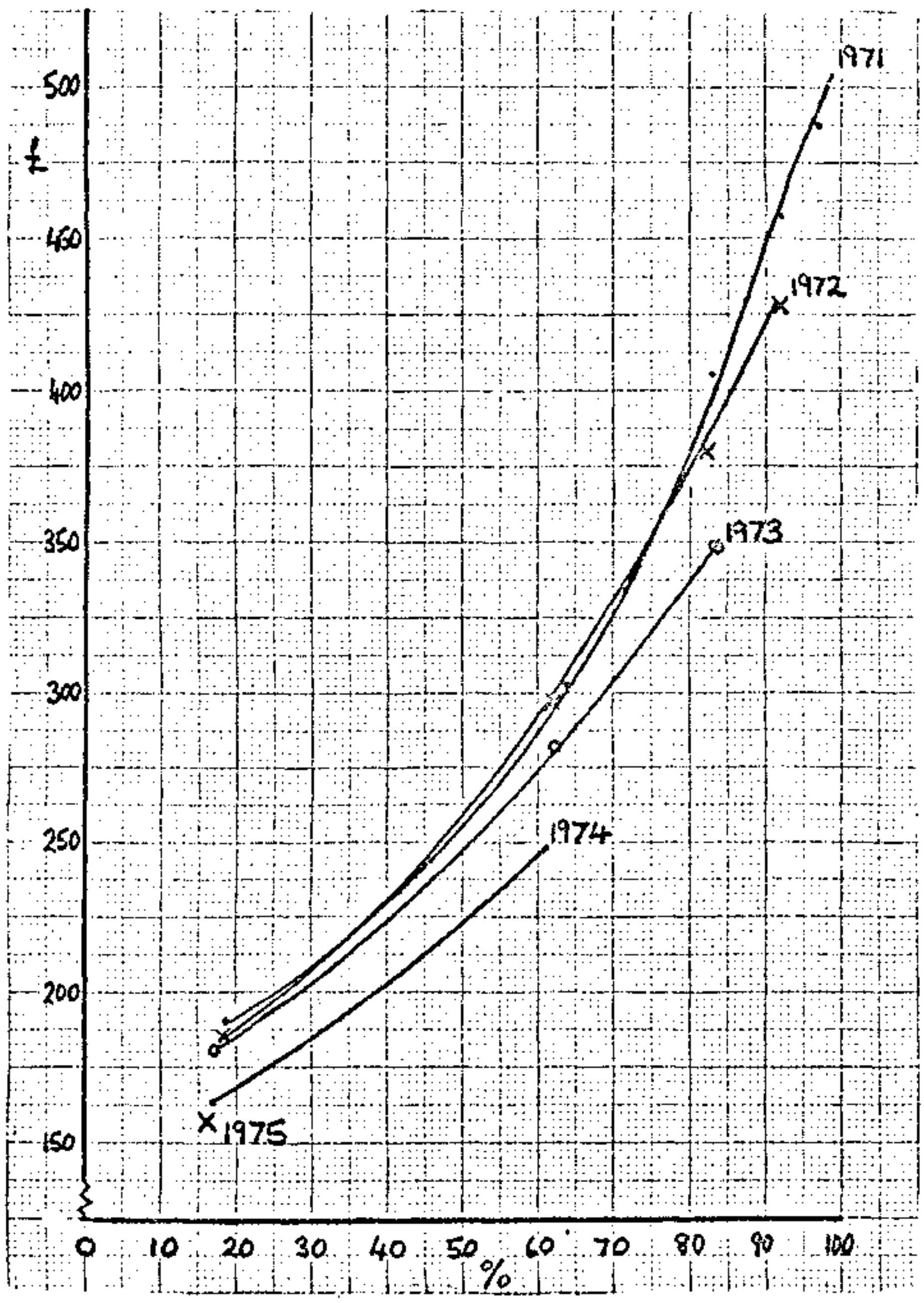


Figure 3 (a)

Payments fully deflated by the Index of Average Earnings

Average Amount of Positive Cost Claims Settled to date

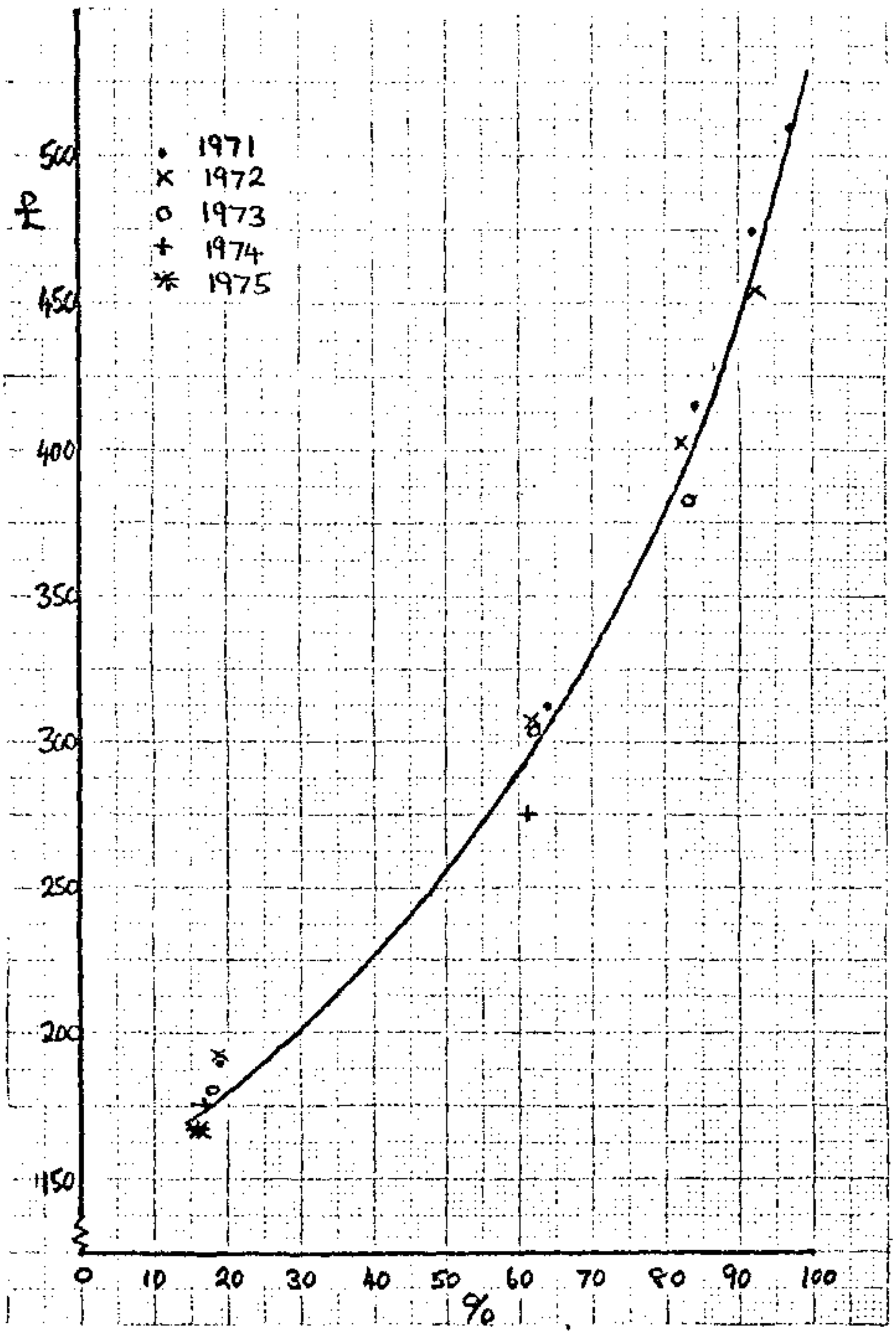


% of Expected Total Number of Positive Cost Claims Settled to date

Figure 3 (b)

Payments deflated by the Index of Average Earnings lagged by 6 months

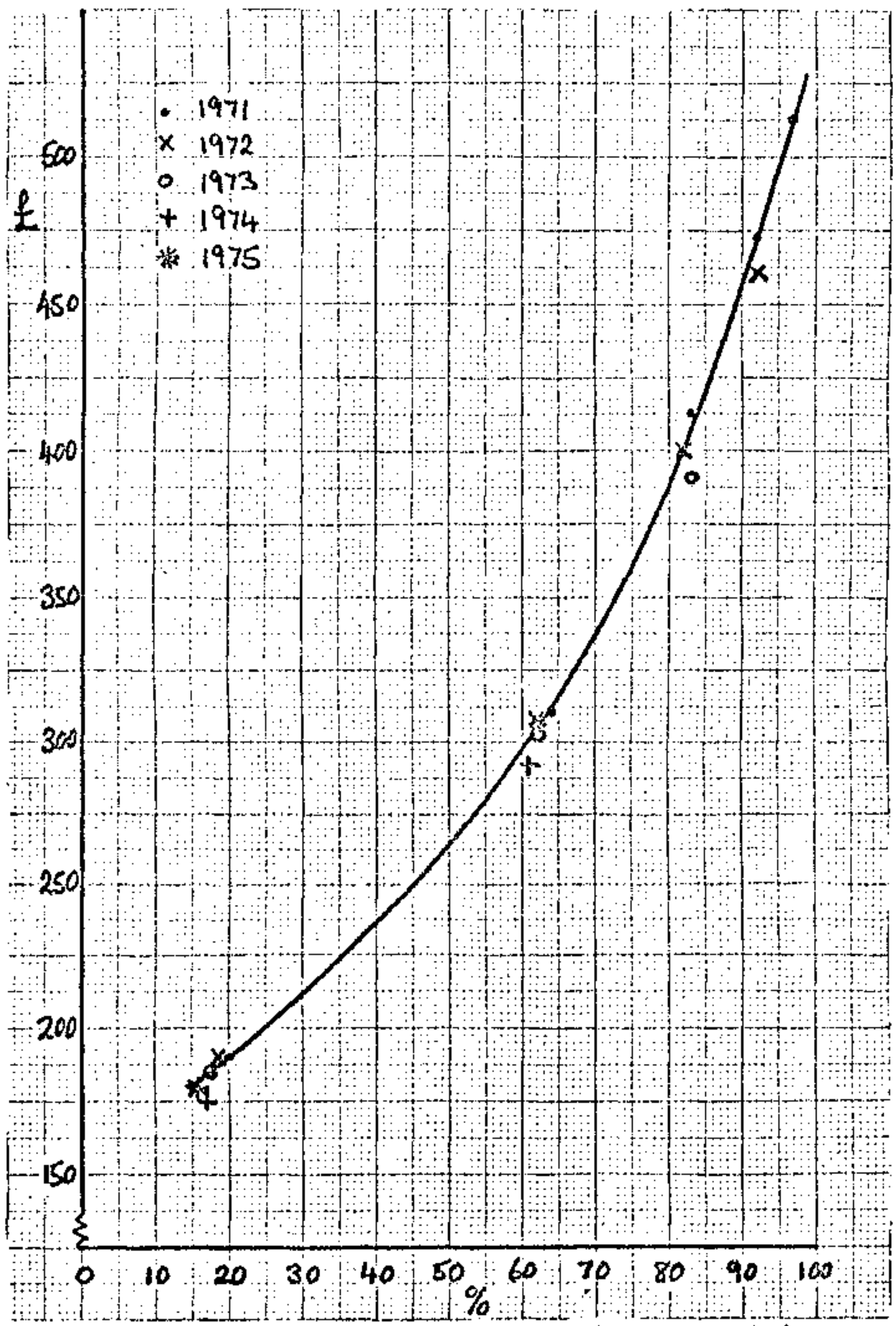
Average Amount of Positive Cost Claims Settled to date



% of Expected Total Number of Positive Cost Claims Settled to date

Figure 3 (c)

Payments deflated by the Index of Average Earnings lagged by 12 months



Average Amount of Positive Cost Claims Settled to date

% of Expected Total Number of Positive Cost Claims Settled to date

Reserves on Short-Term Reinsurance Business

D.H.Craighead.

1.1. This paper is concerned with the determination of reserves for Reinsurance business - the particular problems faced and to what extent suitable reserves can actually be determined with any degree of certainty.

1.2. The following are the factors (in order of importance) which affect the observed incurred loss ratio, at any point of time during development of a particular class and type of business under consideration.

(a) The ultimate loss ratio that will develop, dependant on conditions applying to the portfolio of business for the period of time during which it was at risk (partly purely stochastic; partly conditions of weather, economic tides and other similar factors) together with the aggregation of the premiums for which individual risks were written (basically, the market level of premium rates applying).

(b) "Lag" factors in reporting inherent in the type of business and the structure of the reinsurance world (hence tending to be relatively constant but perhaps varying slowly with time as technical methods and patterns of reinsurance alter).

(c) Further delay factors in reporting and errors in data not corrected until later (seen particularly acutely in the reporting of outstanding claims but also applying to premiums and paid claims).

This last factor causes temporary fluctuations and gives added "roughness" to figures but has no permanent effect on the results.

1.3. The aim in setting up reserves is to:

(a) determine the profit of the company at an accounting point for purposes of distributing profits to shareholders.

cont ,,,,

(b) determine the profit of the class and type of business for the purposes of determining whether changes are required in the terms at which the business is written.

In this latter requirement, time is of the essence and a rather rough forecast to the ultimate loss ratio developing is of more importance than a more accurate assessment given, say, two years later.

Hence we should not eschew the production of approximations which later will be found have to be corrected considerably, so long as those approximations are in the "right parish". The Actuary who refuses to take some chances in this connection is of little help to the underwriter.

1.4. In trying to obtain a picture of the ultimate loss ratio that will develop, we much chance first a picture of the lag factor involved and this is likely to vary with:

(a) whether the loss is property damage or liability, i.e. "short-tail" or "long-tail".

(b) how the reinsurance is written, i.e.

Facultative
Facultative R/I
Contributing Treaty
Excess of Loss Treaty
(generally called "type of business").

(c) To a lesser extent; class of business

Property
Aviation etc.

1.5. This paper is concerned largely with an examination of the lag factor, in an effort to see whether it can lead to any meaningful results or, at least, give some insight into the reasons for variations in development patterns and into the way in which provision should be made for IBNR.

cont ,,,,

1.6. The degree of lag appears to show the following features

(a) Direct Business

Normally notified fairly quickly after the loss, particularly as time bars of various types normally operate, but a lag can arise in liability and may easily be as much as several years. A suitable curve giving the distribution function of the log would probably be skew rather than normal in type. A curve that might fulfil these conditions is of the form

$$re^{-kr^2} \quad \text{or even} \quad re^{-kr^a}$$

(which has no particular appeal other than it seems to be of the correct shape).
For insurances, all starting at a given date, say 1st January, the total of notified claims at any point of time is then

$$\int_{r=0}^{t} re^{-kr^2}$$

$$\text{which is} \quad \frac{1 - e^{-kt^2}}{2k}$$

For insurances with "spread" dates of inception throughout the year, the integral becomes more complex and is more difficult to follow.

(b) Facultative R/I

Similar to direct business of a single risk, but with an added delay (referring to both premiums and claims).

(c) Proportional Treaties

Similar to a portfolio of direct business with dates of inception spread throughout the period of attachment, but with an added delay referring to both premiums and claims.

cont ...

Further sub-grouping needs to be considered:

(i) Non-Marine business (particularly from the U.S.A.) often involves a "cut-off" by the use of portfolio transfers. This may involve a premium transfer after 12 months and a loss portfolio transfer after 24 months, thus shortening the period of reporting.

(ii) Portfolio transfers are very rarely found in Marine business and there is thus a long "tail" in reporting. Furthermore, Hull facultative business exhibits a notorious delay on the claim side, in that a ship will travel around the world for several years, sustaining various minor damages (all separate claims) before entering dry dock for general repair.

(iii) Proportional Treaties also involve enormous delays in reporting block claim outstanding figures. A peculiar instance of this feature is that many London Underwriters adjust their claim outstanding figures fully only at the end of the third year. They then report those adjustments to their Reinsurers, who find them entering their statistics in the third quarter of the fourth year, giving a "hump".

(iv) Excess of loss business involve premium adjustments after the end of the period of attachment, and sometimes the ripples affect premiums for several years.

(v) Catastrophe Excess of Loss involve the gathering of claims statistics for several years and, with recycling in the market, can result in an almost unending stream of minor correction to claim advices.

(vi) Stop Loss on a burning cost basis will also mean adjustments to premiums for several years.

2.1. As a preliminary, it is necessary to examine standard approaches used in direct business and see to what extent, if any, they are applicable. Here this section of the paper is concerned with the differences between Reinsurance business and direct business; the differences are considerable.

cont ....

2.2. All figures relating to loss ratios in reinsurance business are extremely rough. Underwriters accept proportionately much bigger lines than for direct business and protect themselves through reinsurance outwards, often on a "whole account excess of loss basis (divided only into Marine, Aviation and Non-Marine) so that "net account" figures cannot be used for analysis. Hence any mathematical basis, in terms of distribution functions, are used mainly for convenience but are based more on conjecture than on fact.

2.3. The number of claims is not known and any attempt to work in that direction will lead up a blind alley.

(a) In regard to proportional treaties, the reassured does not advise the number of claims involved in block settlements. They vary from very small to larger. Only claims above a stated limit are advised separately. Any pressure to obtain a "number of claims" is much more likely to lead to a loss of the business than to success.

(b) Much reinsurance business today is of the excess of loss type, often catastrophe excess of loss or even stop loss, where the separate claims are not independent and the apparent "number of claims" is a function of how the reinsurance happens to be placed. For catastrophe X/L "number of claims" statistically is meaningless.

2.4. The "base" figure of premiums, to be used in loss ratio calculations, is itself too indeterminate for too long a period of time. Examine for example, the following two tables, which are reasonably typical and arise from world-wide reinsurance accounts.

Example 1 Premium Income by development year.

In £000 - all currencies converted to £

Company (a):

Year	"All Other"	Non-Marine Business
1	359	486
2	125	281
3	18	81
4	9	-4
5	13	7
6	6	1
7	2	2
8	-	-1
9	3	2
10	-1	
11	3	

Company (b):

Year	Short tail -	Non-Marine
½	1027	2846
1	1819	
1½	1201	2078
2	877	
2½	288	352
3	64	
3½	143	150
4	7	
4½	-352	- 352
5	-	

2.5. On the other hand, most reinsurance companies keep entirely separate statistics of premiums, claims paid and claims outstanding by underwriting year: hence cohorts of development can be clearly traced. Also most maintain 3-year accounting, not only for M.A.T. classes, but for all classes of business, thus reducing somewhat problems in the first two years where the greatest degree of uncertainty exists. (On the other hand, the actual methods of collecting and maintaining statistics are legion, making comparisons difficult).

2.6. It is impossible even to guess at the average rate of inflation involved in the claim run-off of a world-wide reinsurance portfolio. Statistics must be corrected in some way (failing which the curve of cumulative incurred claims does not converge but becomes open-ended unless stopped at an arbitrary point of time).

I have used, as a correcting factor, the rate of interest earned on the insurance funds, both in order to obtain convergence of the development series and because it parallels the actual profit and loss experience of the underwriting account. Figures used in tables in this paper have been corrected in this way.

cont ....

2.7. Reinsurance covers so many diverse fields that practically no homogeneity exists in data. The groupings must cover large sweeps of roughly consistent classes. Any attempt to break down groupings further (e.g. by minor class or location of risk) will simply result in data that is too sparse to be meaningful in other than a few cases.

2.8. Both the notification and the assessment of claims outstanding is notoriously haphazard and deficient. On a first investigation one has grave doubts whether the figures are meaningful at all. Yet in fact the figures appear to be more consistent than one might expect. As will be shown below, there are very definite advantages in using the figures. Consistency in reporting as between one underwriting year and another is more important (and more readily possible) than speed or accuracy in reporting.

2.9. Since we have no starting point in the number of claims, but do have figures separately for each underwriting year, should we try to parallel methods used in direct insurance by setting up a "development" triangle of loss ratios, based either on claims paid or on the increase in claims paid and claims outstanding, and on loss ratios calculated on the total premium income eventually reported (which is usually reasonably consistent after the end of the third year)? The following tables are drawn on this basis.

Example 2(a)

Example of an "All-Other" Claim Settlement Triangle

Based on Claims Paid in each separate year(as a percentage of total premiums paid after 3 years)

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	O/S %
1963	16.67	29.37	16.21	8.76	7.21	3.43	1.92	1.39	0.86	6.82	6.79	0.46	0.33	10.86
1964	11.14	35.45	19.14	14.35	3.66	5.29	0.62	2.67	3.27	1.58	1.02	0.87		11.36
1965	6.54	13.50	8.48	17.74	11.03	5.80	5.70	2.61	2.40	2.05	1.50			10.08
1966	7.14	23.27	19.45	17.24	13.04	5.39	6.48	2.16	1.44	2.03				10.13
1967	8.75	20.62	20.26	7.07	6.05	4.21	2.11	2.54	2.00					11.21
1968	5.29	22.15	12.63	9.10	12.10	4.78	3.64	1.08						14.64
1969	4.24	20.36	11.27	8.06	5.54	7.43	3.51							21.95

Normalised to a loss ratio of 100%:

1963	15.01	26.44	14.59	7.89	6.49	3.09	1.73	1.25	.77	6.14	6.11	.41	.30	9.78
1964	10.09	32.11	17.34	13.00	3.31	4.79	.56	2.42	2.96	1.43	.92	.79		10.29
1965	7.48	15.44	9.70	20.29	12.62	6.63	6.52	2.99	2.75	2.34	1.72			11.53
1966	6.63	21.59	18.05	16.00	12.10	5.00	6.01	2.00	1.34	1.88				9.40
1967	10.32	24.31	23.89	8.34	7.13	4.96	2.49	2.99	2.36					13.22
1968	6.24	26.12	14.89	10.73	14.27	5.64	3.58	1.27						17.26
1969	5.15	24.72	13.68	9.79	6.73	9.02	4.26							26.65
Average	8.70	24.39	16.02	12.29	8.95	5.59	3.59	2.15	2.04	2.95	2.92	.60	.30	

Example 2 (b)

Based on Claims Incurred Increases/Decreases (as a percentage of total premiums paid) - same as for 2 (a)
(claims outstanding not available until end of third year)

Year	3	4	5	6	7	8	9	10	11	12	13
1963	87.51	3.51	2.69	8.35	-3.35	2.46	-4.26	7.75	3.51	.38	.66
1964	87.68	8.18	16.61	-11.71	.60	7.77	-2.68	-1.21	.16	1.60	
1965	68.45	23.57	1.19	7.02	1.54	2.65	3.32	1.75	2.81		
1966	79.46	12.83	14.79	- 1.06	4.02	-2.00	-1.78	1.51			
1967	74.61	5.78	5.41	- 1.74	-4.24	2.31	2.68				
1968	73.74	18.01	3.30	- 5.10	-1.19	-3.24					
1969	68.43	9.25	- 1.26	7.64	4.14						

Normalised to a loss ratio of 100%:

1963	80.13	3.21	2.46	7.65	-3.07	2.25	-3.90	7.10	3.21	.34	.60
1964	81.94	7.49	15.52	-10.94	.56	7.26	-2.50	-1.13	.15	1.40	
1965	60.95	20.99	1.06	6.25	1.37	2.36	2.96	1.56	2.50		
1966	73.74	11.91	13.72	-.98	3.73	-1.86	-1.65	1.40			
1967	87.16	6.75	6.32	-2.03	-4.95	2.70	3.13				
1968	87.35	21.32	3.91	-6.04	-1.41	-3.96					
1969	77.59	10.49	-1.43	8.66	4.69						
Average	78.41	11.74	5.94	3.67	.13	1.46	- .39	2.23	1.95	.87	.60

Apart from difficulty arising from waiting until the end of the third year in order to obtain the figure of the premium income involved, which means waiting until the first two critical years are already passed, I am very doubtful whether these development triangles give average percentages that are of value.

2.10. One then turns to the type of development triangle much more in use in the reinsurance market, embodying the following features:

(a) Loss ratios are based, at each point in time, on the actual cumulative premium income figure reported up to that point in time.

(b) Cumulative figures are used.

(c) It is a comparison of the "line across" development ratios with those of other underwriting years relating to roughly the same overall portfolio of business that is more important than any "column average" ratios.

(d) The incurred loss ratios used include both claims paid and claims outstanding. The following two examples give a comparison of including/omitting claims outstanding notified . In spite of the degree of unreliability in claims o/s figures, their inclusion seems to give ratios no rougher nor less consistent than that of claims paid alone, while having the distinct advantage of giving an earlier meaningful picture of results ultimately expected.

Example 3

Loss Ratio % - "All other" Account

(Based on claims paid only)

Cumulative basis:

Year of Development / Year	1	2	3	4	5	6	7	8	9	10	11	12	13	o/s %
1	22.5	47.4	62.4	71.0	78.1	81.3	83.2	84.5	83.7	90.5	97.7	97.3	97.6	10.9
2	17.4	47.3	64.8	79.0	82.5	88.3	88.8	91.5	94.7	96.5	98.2	99.1		11.4
3	15.5	35.7	49.1	66.8	76.7	81.9	87.5	90.1	92.3	94.3	95.6			10.1
4	12.0	34.3	53.1	68.5	80.7	85.9	92.3	94.3	95.8	97.6				10.1
5	14.8	32.3	49.9	57.3	63.0	67.1	69.1	71.7	73.6					11.2
6	7.4	29.0	40.1	48.6	60.8	65.1	68.7	69.9						14.5
7	5.9	26.6	36.6	44.7	49.8	56.9	60.4							22.0

Example 4

Incurred Loss Ratio % - "All other" account
(Based on claims paid plus claims outstanding)

Cumulative basis:

Year	Year of Development										
	3	4	5	6	7	8	9	10	11	12	13
1	87.8	91.0	93.6	101.7	98.3	100.6	96.4	104.2	108.1	107.8	108.5
2	86.4	94.6	110.8	99.9	100.5	104.8	105.5	104.5	105.4	110.4	
3	71.9	95.3	94.7	101.0	102.3	105.0	101.4	103.1	105.7		
4	84.6	94.2	107.8	106.4	110.3	108.2	106.4	107.8			
5	75.0	81.2	86.1	84.3	79.9	82.2	84.8				
6	73.7	90.6	94.4	88.6	87.5	84.3					
7	69.8	79.0	77.0	78.2	82.4						

3.1. This section of the paper deals with a suggested method of approach in regard to which some work has been completed, but a great deal more remains to be done.

It proceeds from the assumption that the apparent incurred loss ratio (as defined in par. 2.10) for any given underwriting year x at any elapsed point of time t (i.e. year of development) can be expressed in the form

$$\frac{l_{x:t}}{L_x} = 1 - e^{-\left(\frac{t}{k}\right)^a}$$

where L_x is the ultimate loss ratio for underwriting year x. a and k are parameters (see par. 3.3. below).

3.2. The use of this function is largely pragmatic.

Advantages

(a) It seems to be of the correct shape.

(b) The parameters a and k are meaningful in terms of the type of business involved (see below).

(c) It may possibly lead to the ability to forecast L_x at an early stage in development with sufficient rough accuracy to help underwriters materially.

(d) The IBNR factor becomes $\dfrac{e^{-\left(\frac{t}{k}\right)^a}}{1 - e^{-\left(\frac{t}{k}\right)^a}}$ at point in time t.

Disadvantages

(a) It is awkward to handle, particularly in regard to curve fitting.

(b) The value of L_x must be estimated or guessed before any part of the curve can be plotted.

3.3. From tests of the usage of this function that have been made to date (as mentioned below), it would appear practicable to determine the values of a and k for specific portfolios of business, over a number of underwriting years, to justify their use as a first approximation to the value they will take for a new underwriting year.

From this point onwards, the value of L_x can be determined from values of $l_{x:t}$ to date. Since the values of $l_{x:t}$ will fluctuate about their "real" values and will, unfortunately, be rather rough, a further degree of approximation is involved. If however, values of $l_{x:t}$ are taken at quarterly intervals (which in itself involves the capturing continually of data related to outstanding claim notification) then several converging approximations to L_x can be built up. It is probably wise to avoid values of $l_{x:\frac{1}{4}}$ $l_{x:\frac{1}{2}}$ and to give more weight to values of $l_{x:t}$ as t increases.

3.4. First tests were carried out on government statistics in Canada relating to a large volume of direct motor business where reinsurance is placed in the Lloyds market and statistics are set out on an underwriting year basis and include outstanding claims.

Taking an average of four underwriting years which have developed sufficiently for (1967/8/9/70) the fit was very good for the average of the four years (with a = 2 and k = .825) The value of a gives the shape of the curve, while k shows the rapidity with which it develops and hence of the "lag" factor mentioned in section 1 of the paper.

The fit is quite good on individual years and a = 2 appears to give a satisfactory shape for the type of business involved.

The degree of change in k then becomes the crucial factor. If $l_{x:1}$, for example, is sufficiently stable (in terms of short term fluctuations resulting from the volume of data), or at least if values $l_{x:\frac{3}{4}}$, $l_{x:1}$ and $l_{x:1\frac{1}{4}}$ are sufficiently stable when taken in conjunction one with another, then estimation of L_x within 5% would be very useful.

For k = .825 IBNR at t =3 is about 1.35%

Example 4 shows some of the results, in respect of estimating L_x from $l_{x:t}$ at values of t (i) $\frac{1}{2}, \frac{3}{4}, 1$

(ii) $\frac{3}{4}, 1, 1\frac{1}{4}$

Assuming $k = .825$

By this method, L_x has been estimated at the end of year 1 to within 4.1%.

Example 4

Canadian Motor Statistics

Incurred Loss Ratio

Year	Value of $l_{x,t}$			
t	1967	1968	1969	1970
$\frac{1}{4}$	7.8	10.6	1.9	
$\frac{1}{2}$	13.7	17.0	16.1	16.1
$\frac{3}{4}$	23.9	27.3	27.0	27.0
1	34.5	42.2	44.0	39.1
$1\frac{1}{4}$	53.8	62.0	57.3	57.3
$1\frac{1}{2}$	63.5	70.6	66.8	66.8
$1\frac{3}{4}$	68.8	78.6	72.5	72.5
2	71.3	82.1	78.1	76.0
$2\frac{1}{4}$	71.3	82.2	78.2	78.2
$2\frac{1}{2}$	71.7	83.2	78.4	83.5
$2\frac{3}{4}$	72.3	84.0	79.6	79.6
3	72.8	85.8	80.7	80.0
$3\frac{1}{4}$	73.1	85.8	80.7	80.0
$3\frac{1}{2}$	73.6	85.8	80.7	80.0
$3\frac{3}{4}$	73.7	85.8	80.7	80.0
4	73.7	85.8	80.7	80.0
Lx	73.7	85.8	80.7	80.0

Using lag factor throughout of k = .825

(1) Matching on loss ratios at points of time $\frac{1}{2}$, $\frac{3}{4}$, 1 we get

Lx = 70.2, 84.3, 84.8, 80.1 % respectively.

(2) Matching on loss ratio at points of time $\frac{3}{4}$, 1, $1\frac{1}{4}$ we get

Lx = 72.8, 85.3, 83.3, 80.1 % respectively.

3.5. Contributing Treaties:

Using $a = 2$ in each case

(a) A long-tail Non Marine account traced (at quarterly intervals) for 4 years
A rough fit with $k = 2.274$

(b) Quite a good fit on a Marine account traced for 4 years except for $t = 1$
($k = 1.672$)

See example 5

The relatively higher value of k represents a greater lag factor.

3.6. Facultative Business

Curve appears to fit better with a value of a of $1\frac{1}{2}$

Relationship between IBNR and Lag Factor
(IBNR calculated as a percentage of known claims paid and outstanding at the point of time t concerned)

<u>Where $a = 2$</u>

If IBNR at point $t = 3$ is	Lag factor $k =$
.5%	.920
1.0%	.986
2.0%	1.070
5.0%	1.215
10.0%	1.369
20.0%	1.584

If Lag Factor is :	IBNR is (when expressed as a percentage of claims paid and outstanding):			
k	t = 1	2	3	4
.9	117%	9.3%	.004%	-
1.0	154	15.6	1.1	-
1.25	265	38.5	5.9	.6
1.5	400	69.9	15.6	3.0
2.0	751	154	48.1	15.6
2.5	1200	265	95	38.5

Since, in long tail accounts, we are dealing with lag factors in excess of 2, it is obvious that the IBNR calculation at $t < 3$ is largely a matter of conjecture, based as it is on very rough figures of claims paid and outstanding. Even at t = 3 the calculations must be clouded with a considerable degree of uncertainty.

t	Contributing Treaties Values of $l_{x:t}$			
	Non-Marine - "All other"		Marine - "All other"	
	Calculated	Actual	Calculated	Actual
$\frac{1}{2}$	1.8%	.8%	4.1 %	11.5%
1	7.0	13.6	15.4	30.6
$1\frac{1}{2}$	14.7	19.7	31.1	31.8
2	24.1	30.2	48.0	45.2
$2\frac{1}{2}$	34.0	30.0	63.2	56.0
3	43.6	37.2	75.1	66.5
$3\frac{1}{4}$	52.0	55.2	83.4	70.0
4	59.0	54.2	88.5	88.0
$4\frac{1}{2}$	64.4	64.4	91.4	91.9
5	68.3	79.0	92.8	92.8
l_x	= 75.0	?	= 93.9 %	?

CALCULATION OF EARNED PREMIUM
EFFECT OF NON-UNIFORM EXPOSURE

Author: J. M. Taylor

1. Problem

The traditional '24ths-Method' of calculating unexpired premium reserve (UPR) assumes an even spread of risk intensity over time, and also an even spread of policy dates over each month.

But what if the claim risk over time for any particular policy unit is not uniform? What if one calendar quarter of each year has a higher propensity to risk than the remainder of the year? This can arise from non-uniform exposure to risk, seasonal variations in claim frequency or cost per claim, or any combination of these variations. In practice this is quite likely to be found (inter alia) in risks such as yachts and motor boats, certain property risks, motor cycles, etc.

In these situations, does the normal 24ths-Method make proper provision for UPR, or should a method be used which makes more specific allowance for the risk distribution?

This paper therefore compares the year-end provision for UPR on the standard 24ths-Method with the correct values required when risk distribution is taken into account, under different situations of risk distribution and premium growth.

Using the same model, the paper also looks at the UPR provisions by each method at individual quarter-ends.

A comparable situation arises in reserving for UPR under inflationary claim cost conditions, since later policy months need to absorb a higher proportion of a given risk premium than the earlier months. The paper therefore looks at the year-end provision using the 24ths-Method and the 'correct' weighted method.

No allowance for inadequate premium reserve (URR) is considered.

2. Annual Provision for UPR

To demonstrate the differences in UPR under the two reserving methods, a simple discrete model was constructed, with one quarter's risk 'peaking' above the uniform level of the other 9 months. Different assumptions of premium growth between successive calendar years were made.

(a) Model

i) Period of Risk:

All policies are assumed to be annual, covering 12 months' risk.

continued ...

ii) Claim Risk:

Risk is assumed to be uniform in 9 calendar months of each year. The remaining 3 months are assumed to peak at higher risk values, due to any combination of the causes mentioned in Section 1.

The proportionate risk assumed in each calendar month is

$$\left.\begin{array}{l} .1 \\ .125 \\ .1 \end{array}\right\} \text{ 3 peak months}$$

$$.075 \quad \text{9 uniform months}$$

The model is varied so that the peak appears successively as Quarter 1, 2, 3 and 4 of each calendar year.

iii) Premiums:

Premiums are assumed to be uniformly written by numbers and amount over the calendar year.

Premium growth at an annual rate g is assumed, such that if

$P_{Y,t}$ = premium written, year Y month t

then

$$P_{Y+1,t} = (1+g)\, P_{Y,t}$$

Growth rates of

g = 0, 20, 40% are assumed.

iv) Initial Expenses:

UPR is calculated net of 20% initial expenses.

(b) Calculations

To demonstrate the calculation of UPR at the year end for premiums written during the year, consider the model where the peak risk is in months 1, 2 and 3; the annual amount of premiums written is £100,000 with 0% growth.

The 24ths-Method reserves $^1/_{24}$ of the risk premium for each half-month not expended at the year end.

The weighted method reserves risk premium in relation to the months not expended at the year end, according to the risk values for each such calendar month.

The UPR factors for each calendar month (before deduction of 20% for expenses) are then calculated as follows:

continued....

Calendar Month	Risk	UPR Factors (100%) 24ths	Weighted
1	.1	1	$\frac{1}{2} \times .1 = .05$
2	.125	3	$.1 + \frac{1}{2} \times .125 = .1625$
3	.1	5	$.1 + .125 + \frac{1}{2} \times .1 = .275$
4	.075	7	$.1 + .125 + .1 + \frac{1}{2} \times .075 = .3625$
⋮	⋮	⋮	⋮
12	.075	23	$.1 + .125 + .1 + .075 + \ldots + \frac{1}{2} \times .075 = .9625$

Then, if

P_t = Premium written in month t

U_t = UPR factor, month t

the UPR reserved at the year end is given by

$$\sum_{t=1}^{12} .8\, P_t U_t$$

In the model stated, with annual premium of £100,000 and 0% growth between years, the calculation of earned premium under the two methods looks as follows:

	24ths-Method	Weighted Method
	£	£
Premiums, written	100,000	100,000
+ UPR, b/fwd	40,000	43,000
- UPR, c/fwd	40,000	43,000
Premium, earned	100,000	100,000

We thus see that although in a zero growth situation the earned premium is the same under either method, this is only achieved by a balance of UPR with substantially different values, the 'correct' UPR being some 7.5% greater than that actually reserved by the standard 24ths-Method.

The full results for the different combinations of the model are shown in the next section.

(c) Results

i) UPR Values:

The relation of the UPR values calculated at the year end under the two methods, when the peak risk appears at different quarters of the year, is as follows:

(Expressed as Weighted/24ths %)

Peak risk in

	Quarter 1	2	3	4	
Weighted/ 24ths value	107.5	102.5	97.5	92.5	%

continued...

Thus, on the model distribution, the correct value required for UPR varies by up to ± 7.5% from the value normally reserved under the 24ths-Method.

ii) Earned Premium

Using the model for the full range of risk and growth assumptions, the relation of earned premium is as follows:

(Weighted 24ths earned premium, %)

Premium Growth	Quarter	Peak risk in 1	2	3	4	
0		100	100	100	100	
20%		99.5	99.8	100.2	100.5	
40%		99.0	99.7	100.3	101.0	%

Thus we see that although the UPR on the two methods varies by up to ± 7.5%, the earned premium error in a complete account year varies by less than ± 1%, even under conditions of 40% premium growth, since the redistributed risk components roughly cancel out over a complete year's risk cycle.

There is, however, a basic difference hidden between the two methods in calculation. Whilst in a continuing revenue situation the 24ths-Method achieves an earned premium approximating to the correct value, on a run-off basis the 24ths-Method would eventually uncover a UPR value which was substantially different from the correct value. The 24ths-Method may therefore disguise the need for an inadequate premium reserve (URR), in addition to the UPR.

(d) Non-Uniform Premium Distribution

In the above model we have only considered the situation where premium was uniformly written over each calendar year, and we saw that in this situation the earned premium using the 24ths-Method approximates to the correct required value.

But what if written premiums, as is often the case, are not uniformly distributed by policy date? Does this cause the 24ths-Method to give significantly wrong values for earned premium in a continuing revenue situation?

To test this, the model was varied to allow premium to be written with the same peaked distribution as the risk (although not necessarily in phase). The full 4 x 4 replication of this model, allowing the peak premium distributions and peak risk to vary independently, might have been expected to produce greater variation

continued...

in the earned premium values calculated under the two methods than for a uniform premium distribution. In fact, there was virtually no difference from the original variations.

Hence the 24ths-Method appears to be fairly robust in a continuing revenue situation, even when premiums are subject to high growth rate and uneven distribution, and risk has a pronounced peak in each year.

3. Quarterly Earned Premium

From the above it may appear that in a continuing situation the method of UPR calculation makes little difference to the end product of earned premium in the period. But, as we shall see, this is true only in an annual account period.

The adequacy of the 24ths-Method becomes more exposed if we consider earned premium calculated for quarterly account periods (e.g. for internal allocation of premium to cover risk is more critical, since the exposure period does not cover a complete annual cycle of risk.

To demonstrate this, the same model is adopted as before, and the earned premium calculated for cumulative calendar quarters.

Taking the particular case of £100,000 premium written uniformly over the year with 0% growth, and peak risk in Quarter 1, the calculation of earned premium at the end of Quarter 1 looks as follows:

	24ths-Method £	Weighted Method £
Premium, written	25,000	25,000
+ UPR, b/fwd	40,000	43,000
- UPR, c/fwd	40,000	37,000
Premium, earned	25,000	31,000

This case shows that earned premium for Quarter 1 should be 24% greater than that shown by the 24ths-Method - a serious difference if management decisions stem from use of this figure.

The full relation of earned premium, <u>at 0% growth</u>, comparing Weighted/24ths % for the cumulative calendar quarters, looks as follows:

(Weighted/24ths earned premium, %)

Account, to end of	Peak risk in Quarter 1	2	3	4	
Quarter 1	124.0	92.0	92.0	92.0	
2	108.0	108.0	92.0	92.0	
3	102.7	102.7	102.7	92.0	
4	100.0	100.0	100.0	100.0	%

continued...

This table shows clearly the distorting effect of the 24ths-Method in the intermediate quarters' earned premium, when risk is unevenly spaced over the year, even when there is no premium growth present. In these cases, the use of earned premium on the 24ths-Method distorts the true position and should be corrected accordingly to make the results meaningful.

4. Allowance for Inflation

Under the influence of inflation, a claim incurred towards the end of a policy year risk period will, on average, cost more than one incurred earlier in the policy year. Everything else being equal, it would therefore be reasonable to allocate for UPR on an increasing basis, reserving proportionately more of each risk premium to the later part of the exposure period than to the earlier part.

This raises similar queries about the adequacy of the normal 24ths-Method in this situation, and is investigated below, using a simple model.

(a) Model

i) Period of risk:

All policies assumed to be annual, covering 12 months' risk.

ii) Exposure:

Risk of claim is constant, but claim costs increase with inflation.

iii) Inflation:

Inflation is assumed at rates of 0, 10, 20 and 30% p.a.
The rate is assumed to be constant over time.

iv) Premiums:

Premiums are written uniformly over each calendar month.
Premium growth is assumed at annual rates g, such that
if

$P_{Y,t}$ = premium written, year Y, month t

then

$$P_{Y+1,t} = (1+g)P_{Y,t}$$

Growth rates of g = 0, 10, 20 and 30% p.a. are assumed.

v) Initial Expenses

UPR is calculated net of 20% initial expenses.

vi) Inadequate Premium:

Risk premiums are assumed to adequately cover the full cost of claims incurred during the risk period. No allowance is therefore made for URR.

(b) Calculations

If inflation continues at a rate i per month, then the risk premium will be expended over each policy month in the ratio

$$1 : (1+i) : (1+i) : \ldots\ldots : (1+i)$$

continued...

Thus, UPR at the end of the account (calendar) year is calculated by the following factors:

Calendar month	UPR Factor (100%)	
1	$\frac{1}{2}(1+i)^{11} \Big/ \sum_{1}^{12}(1+i)^{t-1}$	$= U_1$
2	$\left[\frac{1}{2}(1+i)^{10}+(1+i)^{11}\right] \Big/ \sum_{1}^{12}(1+i)^{t-1}$	$= U_2$
⋮	⋮	⋮
12	$\left[\frac{1}{2}(1)+(1+i)+\dots+(1+i)^{11}\right] \Big/ \sum_{1}^{12}(1+i)^{t-1}$	$= U_{12}$

and UPR at the year-end is given by $\sum_{t=1}^{12} .8\, P_t U_t$

for premium in month t = P_t

In the model stated, with annual premium of £100,000, and 0% premium growth, the detailed calculations of earned premium under the Standard-24ths and Weighted Methods look as follows:

	24ths-Method	Weighted Method Inflation at (% p.a.) 0	10	20	30
Premium, written	100,000	100,000	100,000	100,000	100,000
+ UPR b/fwd	40,000	40,000	40,632	41,208	41,736
- UPR c/fwd	40,000	40,000	40,632	41,208	41,736
Premium, earned	100,000	100,000	100,000	100,000	100,000

This shows again that, in a 0% growth situation, the correct earned premium result is achieved by a balance of UPR values which, under inflation, should be greater than those produced by the normal 24ths-Method.

(c) Results

The relation of Weighted/Standard earned premium values (as a %) for the full range of premium growth and inflation conditions is as follows:

Weighted/Standard earned premium (%)

Premium Growth (% p.a.)	Inflation at (% p.a.) 0	10	20	30
0	100.0	100.0	100.0	100.0
10	100.0	99.9	99.9	99.8
20	100.0	99.9	99.8	99.7
30	100.0	99.8	99.7	99.6 %

In practice, therefore, there initially appears to be little justification for changing to a weighted method which allows for inflation, since the 24ths-Method produces approximately the correct values for earned premium even under high inflation/premium growth conditions.

Again, however, it should be noted that the 24ths-Method understates the UPR values under these conditions, which has implications on UPR and the run-off of unexpired risk.

5. Use in Practice

In the author's own office, it was found that the Yacht & Motor Boat account habitually showed a more favourable underwriting position at the end of the first and second quarters of each calendar year than the ultimate out-turn at the year end.

Upon investigation, it was realised that exposure for each risk was greatest between the months April - October when vessels were in commission, and relatively lower in the winter months when vessels tended to be laid up. In these conditions, the use of the standard-24ths method was allocating too much earned premium to the first and second quarters and not reserving sufficient for the later periods.

It was therefore decided to adopt, for internal analysis purposes at least, a weighted method of calculating UPR which allocated premium more precisely against risk. Investigation indicated that for practical purposes the relative risk for each month would be fairly allocated as follows:

Jan - Mar	.06	each month
Apl - Oct	.10	" "
Nov - Dec	.06	" "

Using the weighted method, it was evident that the earned premium calculation had been at the root of the problem, since the claim ratios in each quarter were now much more comparable, within the variation imposed by claims estimates. In particular the earned premium in the first quarter of 1976 was now only 77% of the value produced by the standard calculation, which therefore had the effect of increasing the corresponding first quarter claims ratio by 30%.

An interesting point, however, was that not only were the earned premium values at the year end comparable under the two methods of calculation, but also the actual UPR values were not dissimilar, due to the combined distribution of exposure and policy dates. Thus the continued use of the standard-24ths method is not appropriate for the formal year-end account in this particular case, provided that the distributions remain roughly as at present.