The Actuarial Profession
making financial sense of the future

Giro conference and exhibition 2011
Paul Figg & Neil Hyatt

UK Employers' Liability Insurance
Empirical Evaluation of Loss Development Factors

11-14 October 2011

Introduction
Purpose of workshop

- In this workshop we use real company and market data to examine loss development occurring in the UK Employers' Liability Market.
- Using this data to approach the following questions:
  - Is there an underlying distribution for loss development?
  - How predictable is loss development?
  - How suitable are widely used loss development models?
  - Are reported (ie incurred) claims figures reliable?

Introduction
Workshop structure

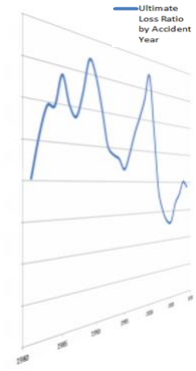
Part I	Overview of UK Employers' Liability Market
Part II	Characteristics of Loss Development
Part III	Theoretical Distribution
Part IV	Estimation Error
Part V	Standard Models
Part VI	Distribution Assumption
Part VII	Auto-Correlation
Part VIII	The "Calendar Year" Effect

Overview of Employers' Liability Market
Market results

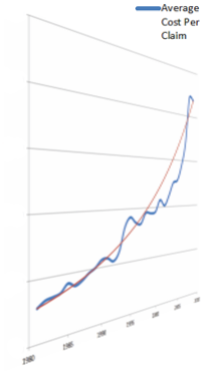
Part I

Overview of Employers' Liability Market

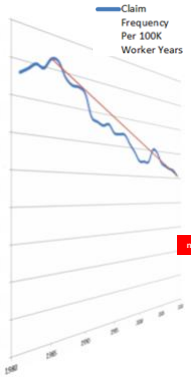
Part I - Overview of UK EL Market
Market Metrics



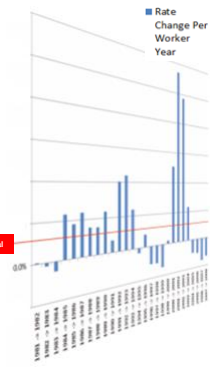
Accident Year Loss Ratios demonstrate clear cyclical behaviour



Claim Inflation has remained relatively stable over last 30 years



Claim frequency shows a declining trend



Primary rates demonstrate clear cyclical movements

Part I - Overview of UK EL Market
Summary

- EL Market exhibits stable claim characteristics:
  - Claim frequency and severity follow long-term trends.
  - No evidence of significant latent exposure (since 1980).
- Insurance Cycle is clearly visible (cyclical rate movements)
- Long-tail nature of this business means (gross) outstanding reserves are typically 350% to 450% of annual premium. Financial Year results are extremely sensitive to reserve redundancy/deficiencies.

⇨ EL seems a good candidate for studying development

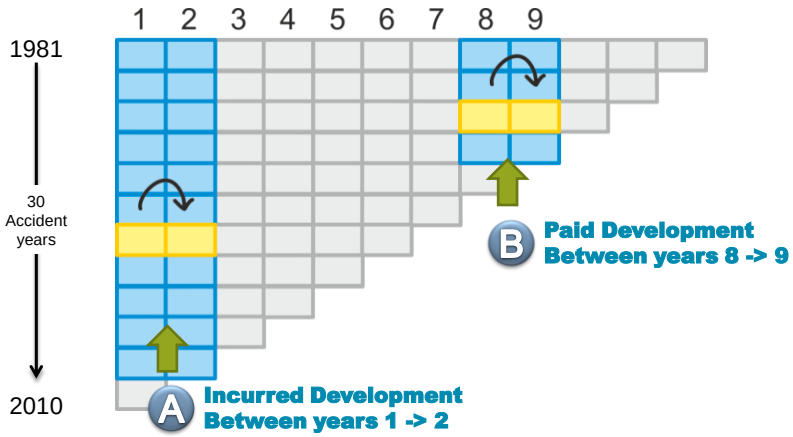
Characteristics of Loss Development
Empirical approach

Part II

Characteristics of Loss Development

Part II - Characteristics of Loss Development
Background

Purpose: use real company data to observe the distribution of two different points on a loss development triangle:



Part II - Characteristics of Loss Development
Background

- Source: observed development of 80 UK Companies (FSA Returns).
- Data dates back 30 accident years and comprises 7,000 data points.
- Gross of reinsurance.
- Results have been split by size of company: large, medium and small.

	small	medium	large
Annual Premium	< £6m	£6m -> £45m	£45m+

On-levelled premium, 33:33:33 split

- Loss Development studied in two (related) ways:
  - Traditional Loss Development Factors
  - Loss Movement

ie on-level £Sterling loss movement, expressed as % of on-level premium

Part II - Characteristics of Loss Development
Information Source

- Underlying source of information is the UK FSA Regulatory Returns. We used data from AM Best's Statement File UK Product and processed this to create our EL development data.
- More information on the Statement File UK Product is available from:

Bryan Martyn
Manager, Regional Sales
A.M. Best Europe – Information Services Ltd.
Tel: +44 (0) 20 7397 0292
Email: Bryan.Martyn@ambest.com

Part II - Characteristics of Loss Development
Data overview – average values

A Incurred Development Between years 1 -> 2

Data points: 800

	Movement	LDF
small	26.3%	1.54
medium	23.8%	1.51
large	28.8%	1.59
	26.3%	1.55

B Paid Development Between years 8 -> 9

Data points: 400

	Movement	LDF
small	2.3%	1.03
medium	2.7%	1.04
large	2.5%	1.03
	2.5%	1.03

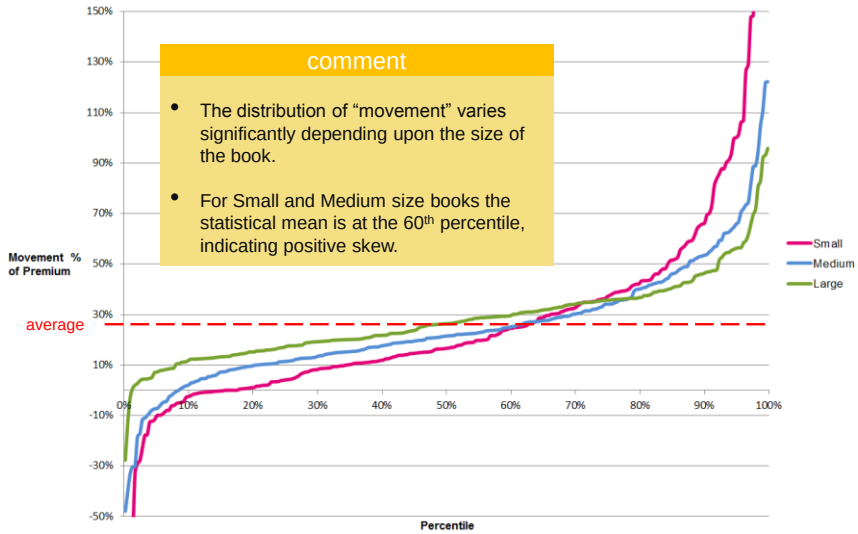
comment

Data suggests that different size companies have similar values.

Next we look at distribution shape

Part II - Characteristics of Loss Development
C.D.F of Movement

A Incurred Development Between years 1 -> 2



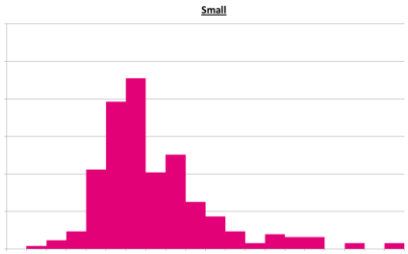
comment

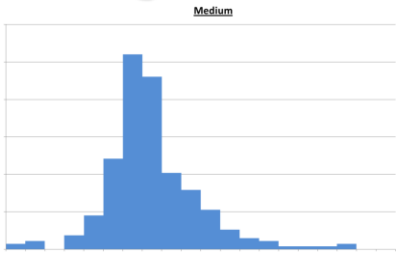
- The distribution of "movement" varies significantly depending upon the size of the book.
- For Small and Medium size books the statistical mean is at the 60th percentile, indicating positive skew.

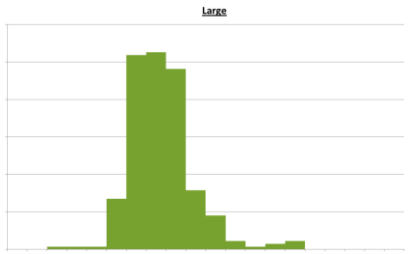
Part II - Characteristics of Loss Development

Histogram of Movement

A Incurred Development Between years 1 -> 2







comment

- Histograms demonstrate positive skew
- Large books have less skew, indicating the distribution tends towards Normality. Central Limit Theorem.

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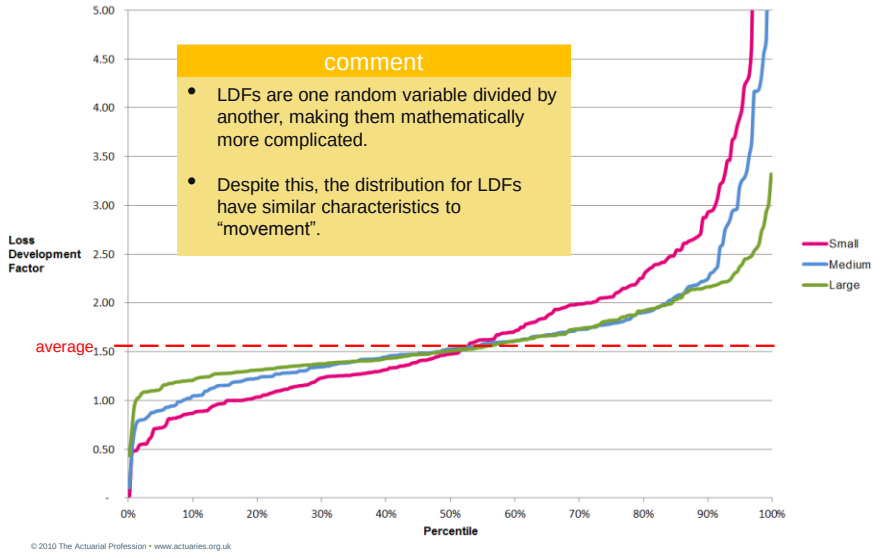
Part II - Characteristics of Loss Development

C.D.F. of LDF

A Incurred Development Between years 1 -> 2

comment

- LDFs are one random variable divided by another, making them mathematically more complicated.
- Despite this, the distribution for LDFs have similar characteristics to "movement".



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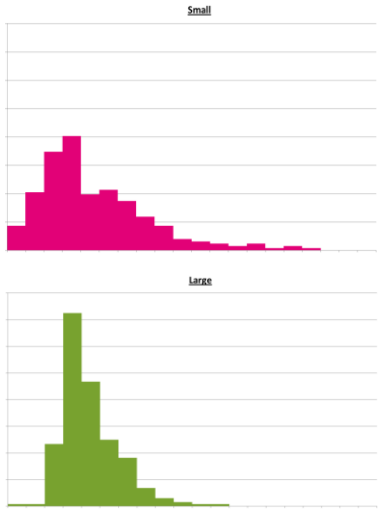
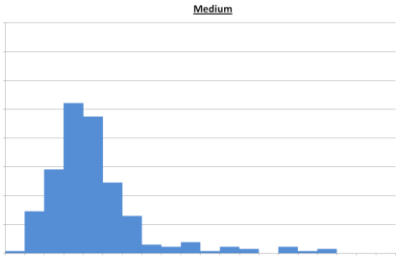
Part II - Characteristics of Loss Development

Histogram of LDF

A **Incurred Development Between years 1 -> 2**





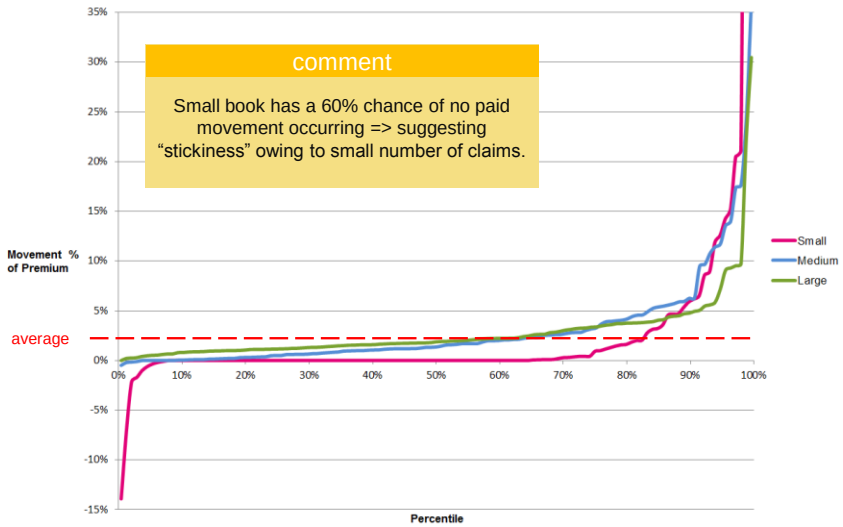
comment

- Histogram reveals LDFs are positive skewed.
- LDFs are clearly more skewed than "movement". This is not surprising as LDFs are a ratio.

Part II - Characteristics of Loss Development

C.D.F of Movement

B **Paid Development Between years 8 -> 9**



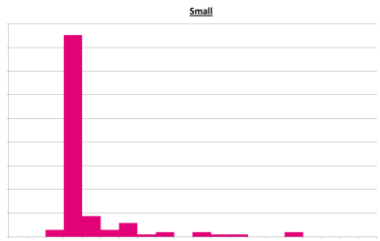
comment

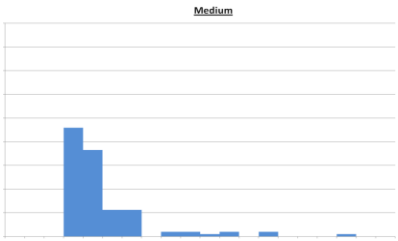
Small book has a 60% chance of no paid movement occurring => suggesting "stickiness" owing to small number of claims.

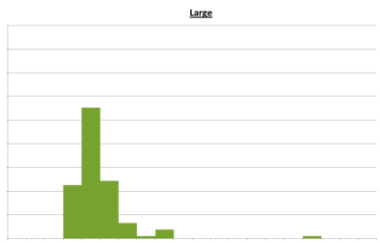
Part II - Characteristics of Loss Development

Histogram of Movement

B Paid Development Between years 8 -> 9







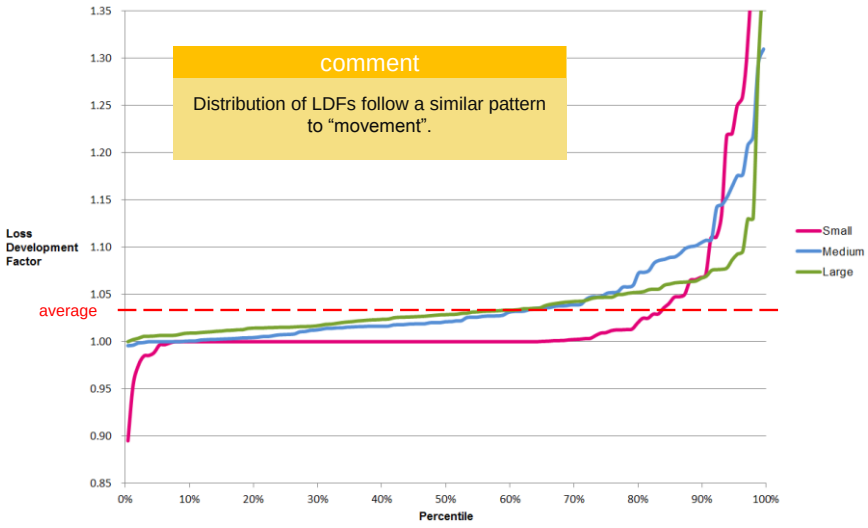
comment

"Movement" is more positively skewed than the previous Incurred data.

Part II - Characteristics of Loss Development

C.D.F. of LDF

B Paid Development Between years 8 -> 9

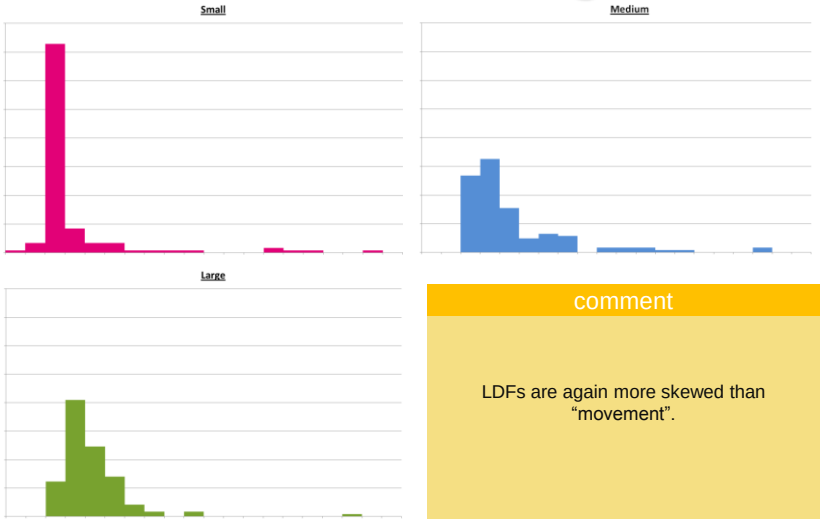


comment

Distribution of LDFs follow a similar pattern to "movement".

Part II - Characteristics of Loss Development
Histogram of LDF

B **Paid Development Between years 8 -> 9**



comment

LDFs are again more skewed than "movement".

Part II - Characteristics of Loss Development
Summary

- Loss development appears to follow a distribution, but the distribution varies depending upon the size of the book.
- Evidence indicates that "mean" of development isn't effected by size of book.
- We picked two development periods / items at either end of the spectrum. Other periods look similar and demonstrate similar, if less extreme, behaviour.
- It is clear that positive skew (3rd moment) is an important feature of loss development. A distribution for loss development isn't defined by the first two moments alone (ie mean and variance).
- Tendency for small books to have nil movement is also apparent.

Loss Development
Theoretical Model

Part III

Theoretical Distribution

Part III - Theoretical Distribution
Is there an underlying process?

- Purpose: propose a possible distribution / process that explains how the distribution of development changes as the size of book increases.
- Focusing on “movement” is mathematically preferable over LDFs.
- In the previous section we saw evidence that:
  - The variance of the “movement” is lower for larger books of business.
  - The distribution of “movement” is positively skewed but appeared to tend towards a normality as the size of the book grew.
  - The “movement” mean appears to be the same regardless of the size of book.
  - For small books, and at later development periods (where fewer individual claims experience movements): there is a tendency for nil “movement”.

Part III - Theoretical Distribution
Proposed distribution / process

Inputs:
- μ = mean "movement"
- σ = sd "movement"
- α = skew of "movement"
- N = number of claims
- P = probability individual claim has nil movement

Parameters describing individual claim movement; assuming claim does move during period.

Proportional to on-level premium?

$$\text{r.v. Movement} = \sum_{i=1}^{N} M_i$$

where:

$$M_i = \begin{cases} 0 & \text{with probability } P \\ \text{SkewNormal}(\mu,\sigma,\alpha) & \text{with probability } 1 - P \end{cases}$$

Comment

- This formulation is an example of the approach we feel would be necessary to properly explain and model loss development.
- Requires a market-wide dataset to be parameterised.

Loss Development
Theoretical Model

Part IV

Estimation Error

Part IV – Estimation Error
Introduction

We asked two simple questions of the data :

Question 1

How many years of data do you need for a 75% probability that your sample estimate of loss movement lies within 5%, (either side), of the "true" value?

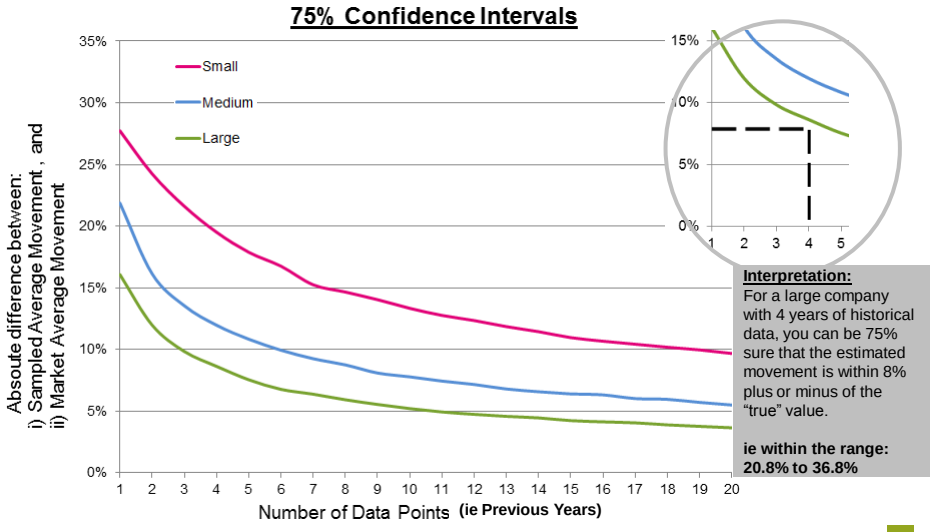
Question 2

How does a downside estimate for LDF - derived using a LogNormal fitted to a sample – compare with the observed one-in-a-two-hundred year estimate?

Part IV – Estimation Error
Q1–Estimating The Mean

Question 1

A Incurred Development Between years 1 -> 2



Interpretation:
For a large company with 4 years of historical data, you can be 75% sure that the estimated movement is within 8% plus or minus of the "true" value.

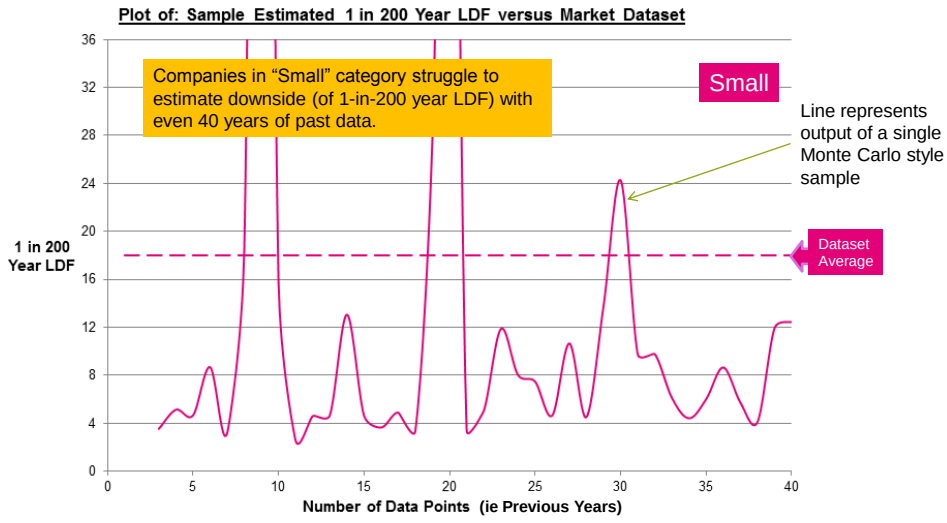
ie within the range: 20.8% to 36.8%

Part IV – Estimation Error

Q2–Estimating Downside Risk

Question2

A Incurred Development Between years 1 -> 2

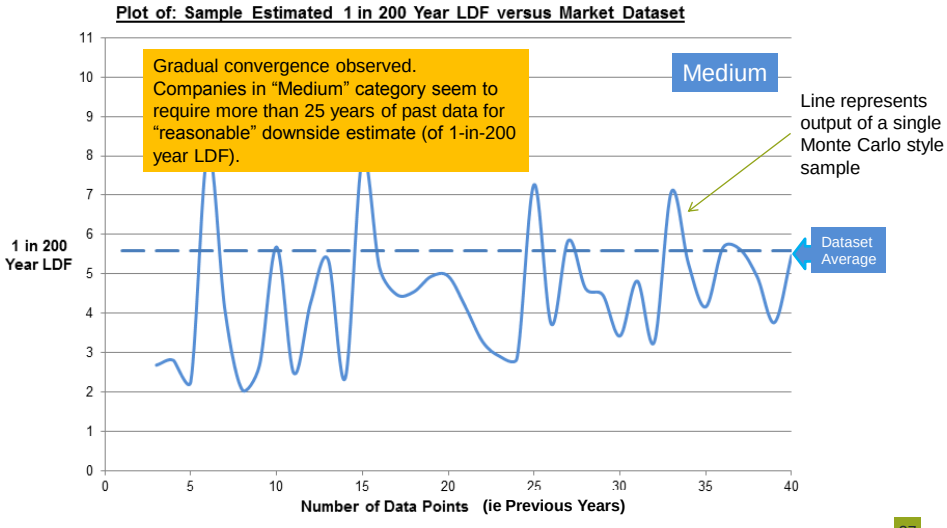


Part IV – Estimation Error

Q2–Estimating Downside Risk

Question2

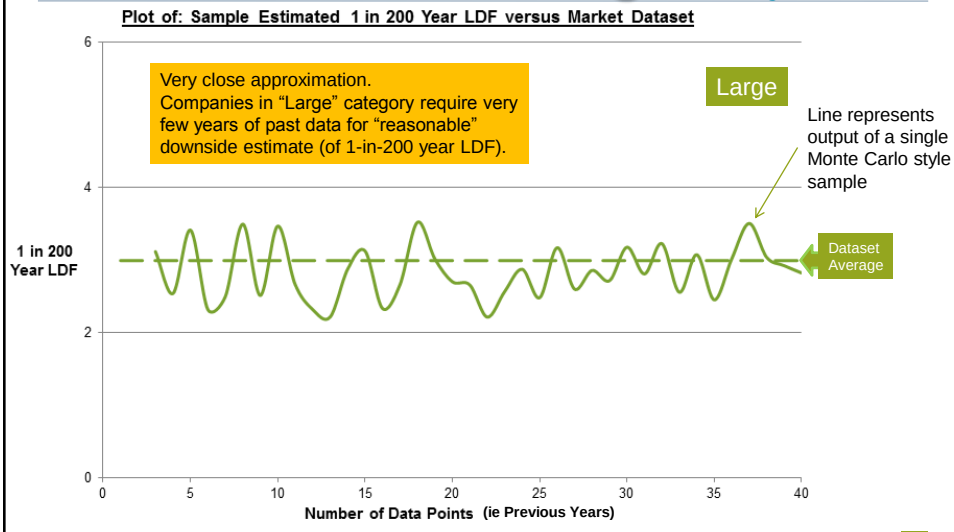
A Incurred Development Between years 1 -> 2



Part IV – Estimation Error
Q2–Estimating Downside Risk

Question2

A Incurred Development Between years 1 -> 2



Part IV – Estimation Error
Summary

- Previous (four) slides are intended to illustrate estimation error that companies of different size face. More work could be done on this topic.
- The slides show that companies with small EL books of business face significant challenges using their internal data to estimate loss development.
- Elevated levels of random variation within small books means that firms struggle identifying the true underlying characteristics – particularly downside.
- It could be argued that smaller companies would appear to be better off using external data than relying on their own internal experience.

Loss Development
Theoretical Model

Part V

Standard Models

Part V – Standard Models
Testing Underlying Assumptions

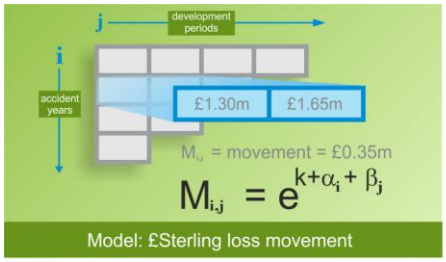
For the next part of this presentation we use market and individual company data to assess key assumptions underlying three widely used models for loss development.

Models tested:

1. Over dispersed poisson
2. Mack
3. Simple linear regression

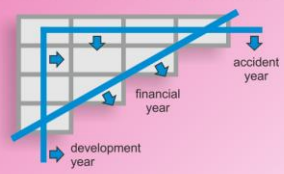
definitions follow ...

Part V – Standard Models
Definition: Over Dispersed Poisson



Model: £Sterling loss movement

Errors / Deviations from expected are i.i.d.



Implicit Assumptions

$C_{i,j}$ = cumulative Paid or Incurred at triangle point: i,j

$M_{i,j} = C_{i,j} - C_{i,j-1}$

k = constant

α_i = explains claim movement attributable to accident year i

β_j = explains claim movement attributable to development year j

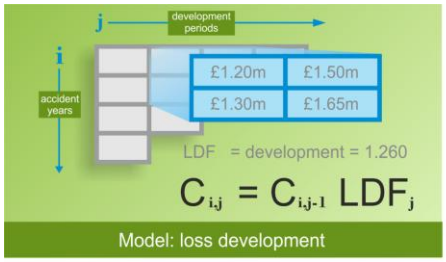
$E[M_{i,j}] = e^{k+\alpha_i+\beta_j}$

$Var[M_{i,j}] = \phi\, e^{k+\alpha_i+\beta_j}$

ϕ = constant

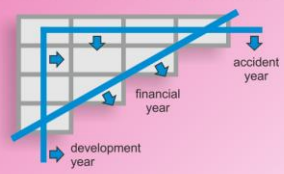
Definitions

Part V – Standard Models
Definition: Mack



Model: loss development

Errors / Deviations from expected are i.i.d.



Implicit Assumptions

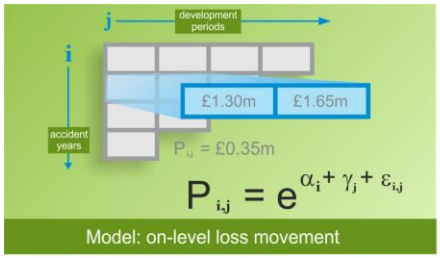
$C_{i,j}$ = cumulative Paid or Incurred at triangle point: i,j

$E[C_{i,j}] = C_{i,j-1}\, E[LDF_j]$

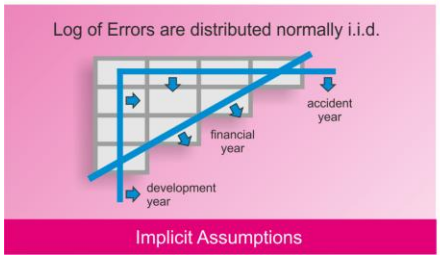
$Var[C_{i,j}] = C_{i,j-1}\, Var[LDF_j]$

Definitions

Part V – Standard Models
Definition: Linear Regression



Model: on-level loss movement



Implicit Assumptions

$P_{i,j}$ = incremental at triangle point: i,j

α_i = explains claim movement attributable to accident year i

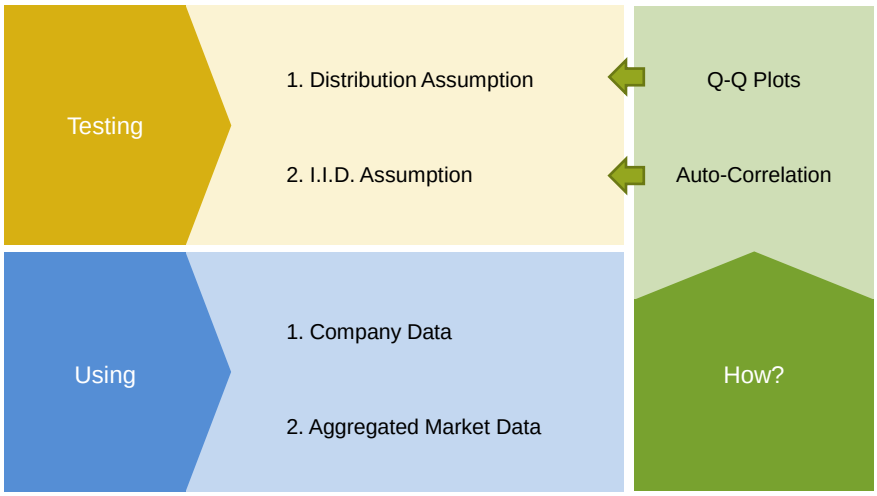
γ_j = explains claim movement attributable to development year j

$\varepsilon_{i,j}$ = error term at triangle point: i,j

$\text{Var}[\varepsilon_{i,j}] = \sigma^2$

Definitions

Part V – Standard Models
Testing What?



Part VI – Distribution Assumption
Standard Models

Part VI

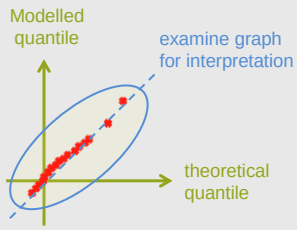
Distribution Assumption

Part VI – Distribution Assumption
Q-Q Plots

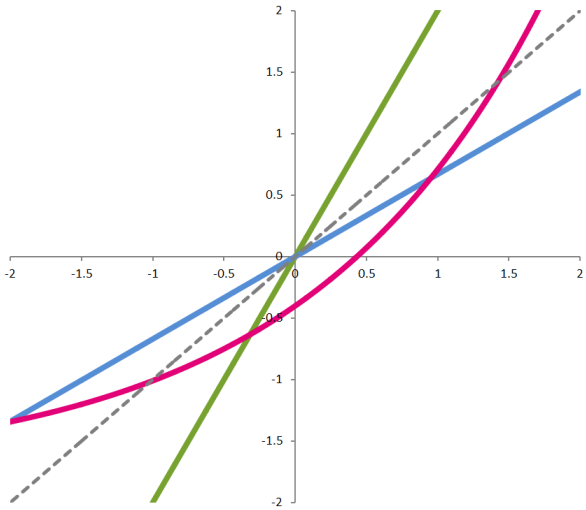
Methodology:

Step 1	Fit each model to data
Step 2	Calculate the Normalised Residual of each observed point, and determine the Quantiles.
Step 3	Plot these against Quantiles of Normal(0,1)

$$\text{Normalised Residual} = \frac{\text{Data Point} - \text{Fitted Mean}}{\text{Fitted SD}}$$



Part VI – Distribution Assumption
Q-Q Plots - Interpretation



True σ > modelled

- Fitted model **understates** variability in observed data set.

True σ < modelled

- Fitted model **overstates** variability in observed data set.

right skew

- Observed data set is more right skew than fitted model allows.

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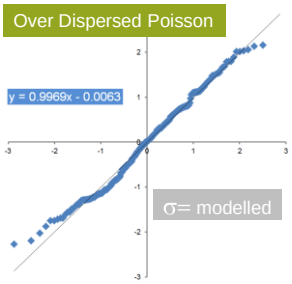
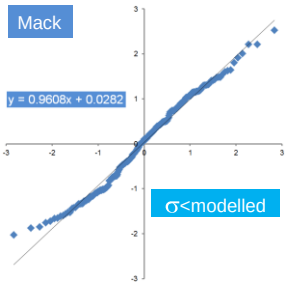
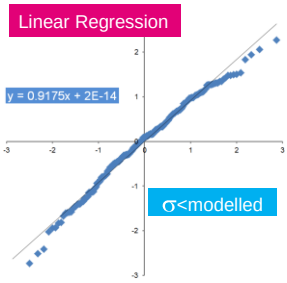
Part VI – Distribution Assumption
Q-Q Plots: Market Aggregated (paid, inflation adjusted)

Over Dispersed Poisson	Mack	Linear Regression
		
y = 0.9969x - 0.0063	y = 0.9608x + 0.0282	y = 0.9175x + 2E-14
σ= modelled	σ<modelled	σ<modelled
Models SD reasonably well	SD overestimated (assuming Normal)	SD overestimated

Comment	Effect of Central Limit Theorem seen at whole market level (ie little skew)

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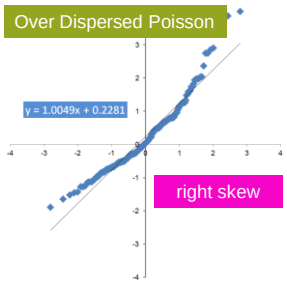
Part VI – Distribution Assumption

Q-Q Plots: A Large Company (paid, inflation adjusted)

on-level annual premium of company ≈ £130m

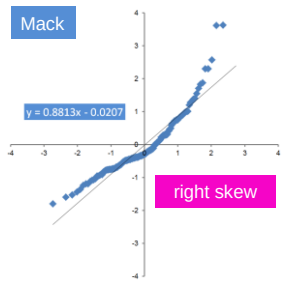
Over Dispersed Poisson



right skew

Right skew – 3rd moment is clearly visible, even for this company

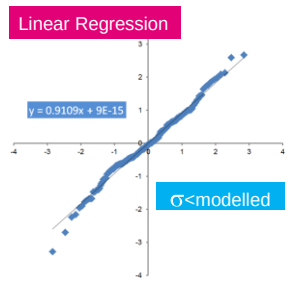
Mack



right skew

Right skew – 3rd moment is clearly visible, even for this company

Linear Regression



σ<modelled

Linear Regression Log Normal assumption reasonable, though SD overstated.

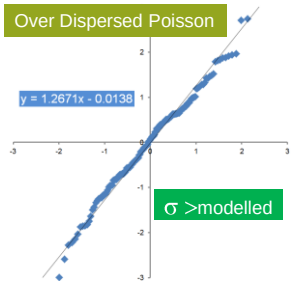
Comment

Skew is clearly visible even for one of the largest companies.
Skew effect is even more pronounced for smaller companies.

Part VI – Distribution Assumption

Q-Q Plots: Market Aggregated (incurred)

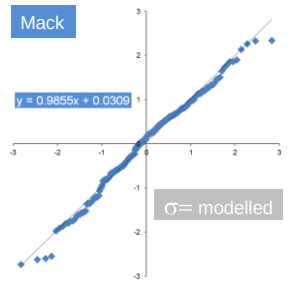
Over Dispersed Poisson



σ >modelled

Suggests Normality, but SD understated

Mack



σ= modelled

Models SD reasonably well

Linear Regression

NOT POSSIBLE

Model unable to cope with negative movements

Fundamental problem with this family of models

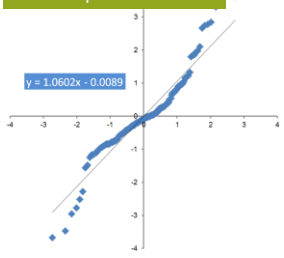
Comment

Market data appears Normally distributed.

Part VI – Distribution Assumption
Q-Q Plots: A Large Company (incurred)

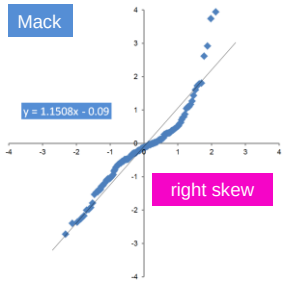
on-level annual premium of company ≈ £130m

Over Dispersed Poisson



Thicker tailed than Normal distribution

Mack



right skew

Right skew – 3rd moment is clearly visible, even for this company.

Linear Regression

NOT POSSIBLE

Model unable to cope with negative movements

Fundamental problem with this family of models

Comment

Skew is clearly visible even for one of the largest companies.
Skew effect is even more pronounced for smaller companies.

Part VI – Distribution Assumption
Summary

- Market (level) data
  - When fitting the models at the market level both paid and incurred claims appear Normally distributed.
  - However, the models appear to potentially overestimate the variance of the paid claims, whilst underestimating the variance of the incurred
- Company (level) data
  - Examining paid and incurred at a company level indicates that either the distributions are right skew, or that the tails are much thicker than would be implied by a normal distribution.
  - Consequently care should be taken to ensure that the variability of the reserves are not understated.

Part VII – Auto Correlation
Standard Models

Part VII

Auto-Correlation

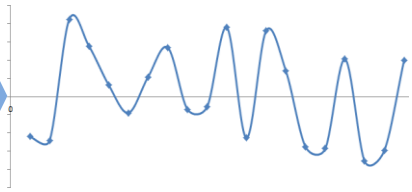
Part VII – Auto Correlation
What is Auto Correlation?

Auto Correlation is a statistical test used to measure the level of dependence that one term in a sequential series has on surrounding terms.

Random / I.I.D. Data

Knowing the value of any particular point tells you nothing about values of surrounding points.

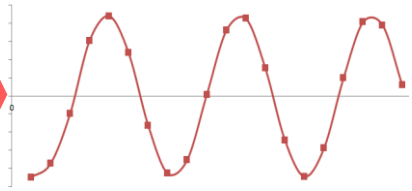
Ie nil correlation



Correlated Data

Knowing the value of any particular point tells you a great deal about value of surrounding points.

Ie strong correlation



Why?

- Industry models assume that paid and incurred claim figures are I.I.D. We want to test this assumption.
- We are interested in: Is there auto correlation amongst paid and incurred data?

Part VII – Auto Correlation
How Auto Correlation is Measured?

In Maths

	1	2	3	4	5	6	7
X(t)	-0.90	-0.75	-0.20	0.61	0.88	0.47	-0.33

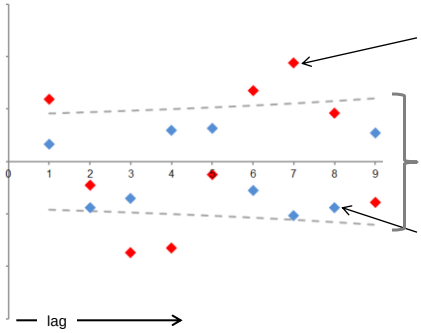
lag of 3

consider relationship

Auto Correlation is calculated by considering all data points separated by the same lag.

$$\alpha \sum (X_t - \mu)(X_{t+lag} - \mu)$$

Interpretation



Correlated Data

Data points clearly outside confidence interval indicate strong auto correlation

Confidence Interval
Data points inside appear to be uncorrelated (ie random)

Random / IID Data

Graph indicates that no auto correlation exists (all points within confidence interval)

Part VII – Auto Correlation
Confidence interval

If x(t) ~ IID(0,σ)
then the auto correlation is asymptotically distributed as a normal random variable with mean 0 and variance n^{-1} where n is the sample size.

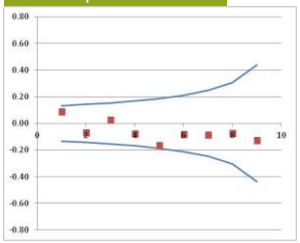
This result means that if x(t) is an iid process the 95% confidence interval is given by:

$$\left[\frac{-1.96}{\sqrt{n}}, \frac{1.96}{\sqrt{n}}\right]$$

Auto Correlation tests are performed on the normalised residuals.

Part VII – Auto Correlation
Market Incurred Claims: Accident year

Over Dispersed Poisson



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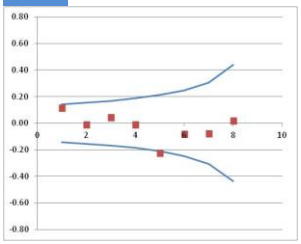


Linear Regression

NOT POSSIBLE

Model unable to cope with negative movements

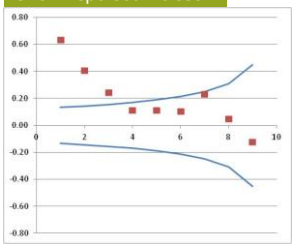
Comment

All the correlations are within the 95% confidence interval. IID assumption appears valid for the incurred claims.

This is also observed at an individual company level.

Part VII – Auto Correlation
Market Incurred Claims: Calendar year

Over Dispersed Poisson



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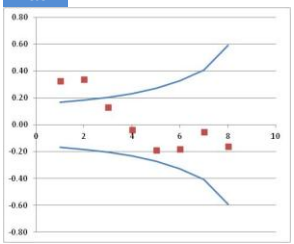


Linear Regression

NOT POSSIBLE

Model unable to cope with negative movements

Comment

Calendar year incurred are very highly correlated. Does this imply that incurred loss figures are cyclical?

This is observed at the market level – but for individual companies: this feature becomes increasingly hard to identify as the company size reduces.

Part VII – Auto Correlation
Summary

- Market data
  - Whilst incurred claims don't appear to exhibit auto correlation within an individual accident year, they are correlated when considered from a calendar year perspective – does this, in part, explain the Reserving Cycle?
- Company Data
  - We observed that for progressively smaller firms, these trends became increasingly harder to identify amongst the greater statistical noise. This poses a significant challenge for smaller firms as the implication is that many firms are unable to properly identify emerging trends.

Part VIII – The Calendar Year Effect
A Closer Look

Part VIII

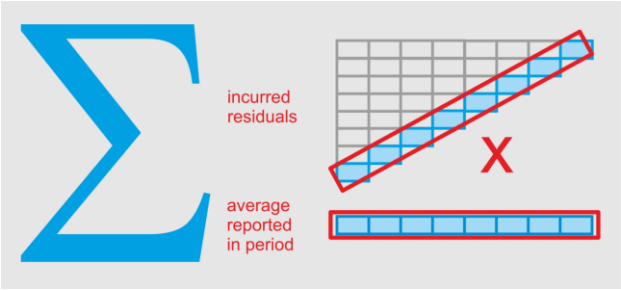
The "Calendar Year" Effect

Part VIII – The Calendar Year Effect
Observations

- The correlation of incurred (reported) claims on a Calendar year basis is a significant observation and it may go someway towards explaining the reserving cycle.
- We decided to investigate the effect in more detail.
- Auto correlation is a blunt instrument and overlooks two important aspects:
  - All development periods are treated equally, whereas the first couple of development years largely determine the £sterling result.
  - It doesn't address the question of whether there is a relationship with the insurance cycle.
- To address these points we created a LDF / money weighted statistic.
- We've used the fitted Over Dispersed Poisson model for the normalised residuals because it includes an opinion on the adequacy of reported claims in the most recent accident year.

Part VIII – The Calendar Year Effect
Test Statistic

Test statistic used to overcome problems discussed on the previous slide:



Test statistic is defined as the sum of the product of:

i) Normalised residuals

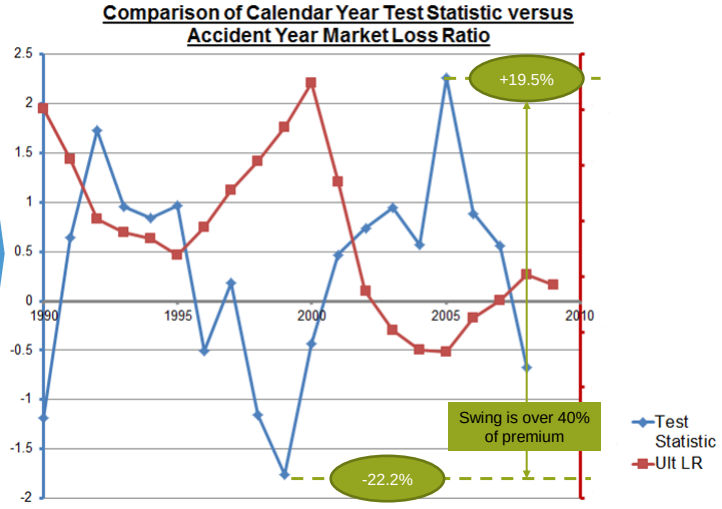
i) Fitted non-cumulative reporting (incurred) pattern

Statistic is then Normalised (ie scaling variance back to: 1)

Part VIII – The Calendar Year Effect
Results

Test Statistic demonstrates cyclical behaviour.

Clear link to the insurance cycle .



Part VIII – The Calendar Year Effect
Summary

- Test statistic demonstrates that the Calendar Year effect is itself cyclical:
  - Largest increases to reported (incurred) claim figures occur when the best business is being written (ie hardest point in underwriting cycle).
  - Slowest build-up of reported (incurred) claim figures occur when the worst best business is being written (ie softest point in cycle).
- This is quite a surprising result. On a Financial Year basis reported loss ratios are 20% (as a percentage of premium) below where they should be at the softest point in the market cycle. Inadequate reported losses would have a geared effect on reserves held and probably goes someway to explaining the reserving cycle.
- Firms increasingly come under earnings pressure as the soft market intensifies. Does this pressure lead to what we've observed?
- It would be intriguing to know whether a similar effect would be observed with other liability classes of business? If so, whether these cycles coincide?

Questions or comments?

Expressions of individual views by members of The Actuarial Profession and its staff are encouraged.

The views expressed in this presentation are those of the presenter.