The Ultimate and One-Year Views of Reserving Risk with Respect to Solvency and Risk Margins

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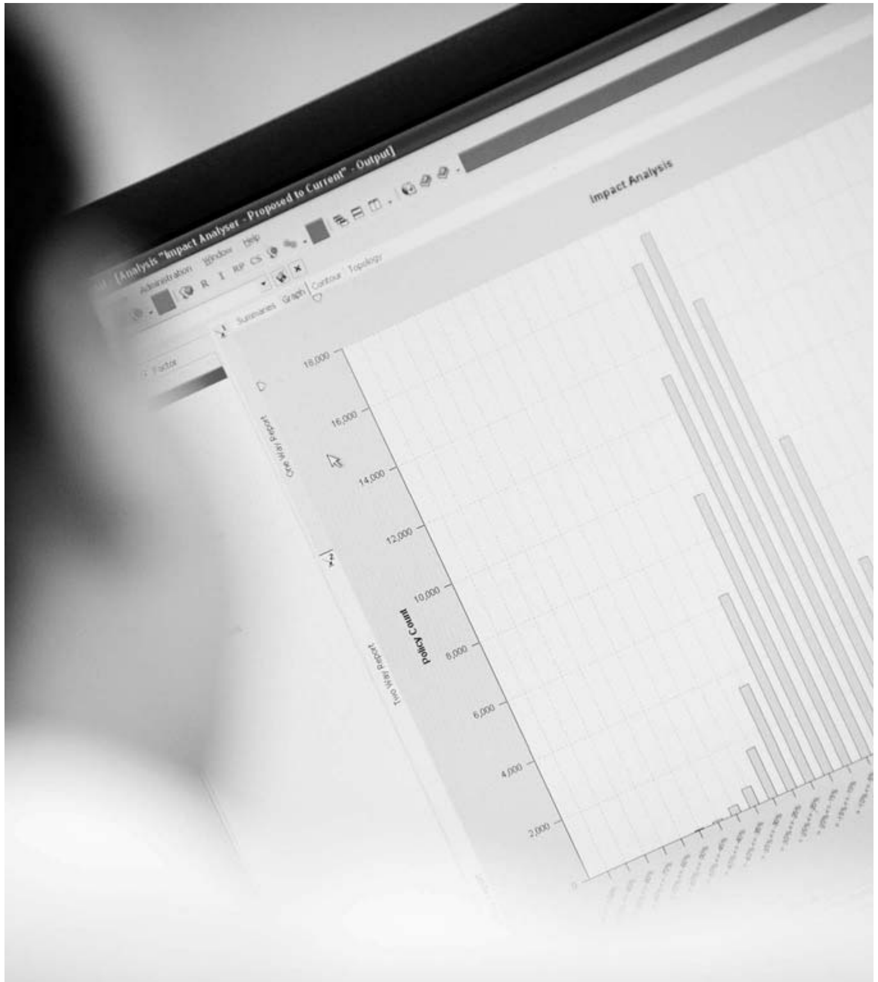
GIRO Conference, 08 October 2009
Edinburgh

Agenda

- An alternative view of reserve risk
  - The one-year view
- Implications for simulation based internal capital models
  - The overall SCR
  - Notional line of business SCRs
- Two proposed approaches for estimating the LoB risk margins
  - The ‘proportional proxy’
  - A ‘maximum entropy’ approach



The “ultimo” vs the one-year view of reserving risk

Solvency 2

- Solvency 2 is notionally projecting a balance sheet, and requires a distribution of "Net Assets" over a one year time horizon.
- Solvency 2 requires a view of the distribution of expected liabilities in one year
- For reserving risk, this requires a distribution of the profit/loss on reserves over one year
- This is different from the standard approach to reserving risk, which considers the distribution of the ultimate cost of claims (eg Mack 1993, England & Verrall 1999, 2002, 2006)

The one-year run-off result (undiscounted)
(the view of profit or loss on reserves after one year)

For a particular origin year, let:

The opening reserve estimate be R_0

The reserve estimate after one year be R_1

The payments in the year be C_1

The run-off result (claims development result) be CDR_1

Then

$$CDR_1 = R_0 - C_1 - R_1 = U_0 - U_1$$

Where the opening estimate of ultimate claims and the estimate of the ultimate after one year are U_0, U_1

The one-year run-off result
(the view of profit or loss on reserves after one year)

Merz & Wuthrich (2008) derived analytic formulae for the standard deviation of the claims development result after one year assuming:

- The opening reserves were set using the pure chain ladder model (no tail)
- Claims develop in the year according to the assumptions underlying Mack's model
- Reserves are set after one year using the pure chain ladder model (no tail)
- (The mathematics is quite challenging)

The M&W method is gaining popularity, but has limitations. What if:

- We need a tail factor to extrapolate into the future?
- Mack's model is not used – another model is used instead?
- We want another risk measure (say, VaR @ 99.5%)?
- We want a distribution of the CDR (not just a standard deviation)?

Merz & Wuthrich (2008)
Data Triangle

Accident Year	12m	24m	36m	48m	60m	72m	84m	96m	108m
0	2,202,584	3,210,449	3,468,122	3,545,070	3,621,627	3,644,636	3,669,012	3,674,511	3,678,633
1	2,350,650	3,553,023	3,783,846	3,840,067	3,865,187	3,878,744	3,898,281	3,902,425	
2	2,321,885	3,424,190	3,700,876	3,798,198	3,854,755	3,878,993	3,898,825		
3	2,171,487	3,165,274	3,395,841	3,466,453	3,515,703	3,548,422			
4	2,140,328	3,157,079	3,399,262	3,500,520	3,585,812				
5	2,290,664	3,338,197	3,550,332	3,641,036					
6	2,148,216	3,219,775	3,428,335						
7	2,143,728	3,158,581							
8	2,144,738								

Merz & Wuthrich (2008)
Prediction errors

	Analytic	
	Prediction Errors	
Accident Year	1 Year Ahead CDR	Mack Ultimate
0	0	0
1	567	567
2	1,488	1,566
3	3,923	4,157
4	9,723	10,536
5	28,443	30,319
6	20,954	35,967
7	28,119	45,090
8	53,320	69,552
Total	81,080	108,401

The one-year run-off result in a simulation model
(the view of profit or loss on reserves after one year)

For a particular origin year, let:

The opening reserve estimate be R_0

The expected reserve estimate after one year be $R_1^{(i)}$

The payments in the year be $C_1^{(i)}$

The run-off result (claims development result) be $CDR_1^{(i)}$

Then

$$CDR_1^{(i)} = R_0 - C_1^{(i)} - R_1^{(i)} = U_0 - U_1^{(i)}$$

Where the opening estimate of ultimate claims and the expected ultimate after one year are $U_0, U_1^{(i)}$

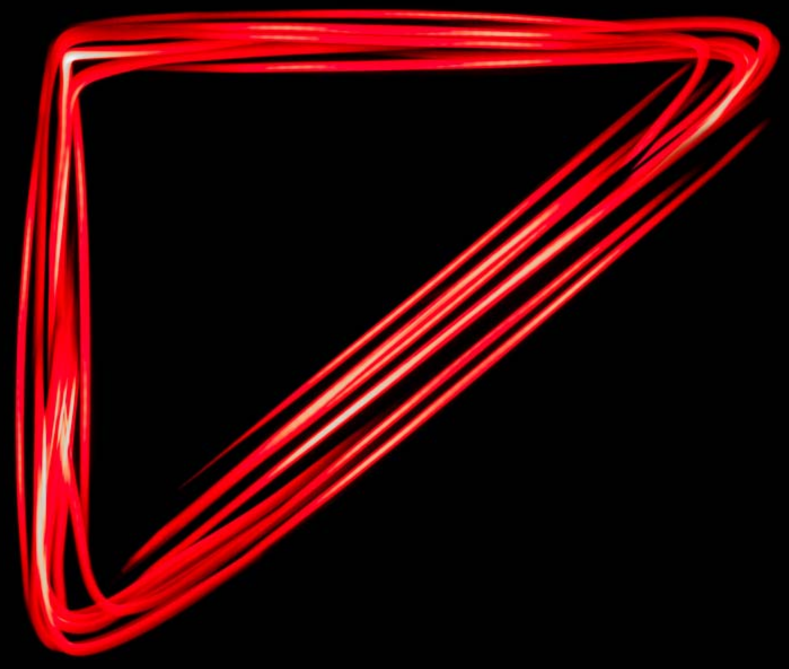
for each simulation *i*

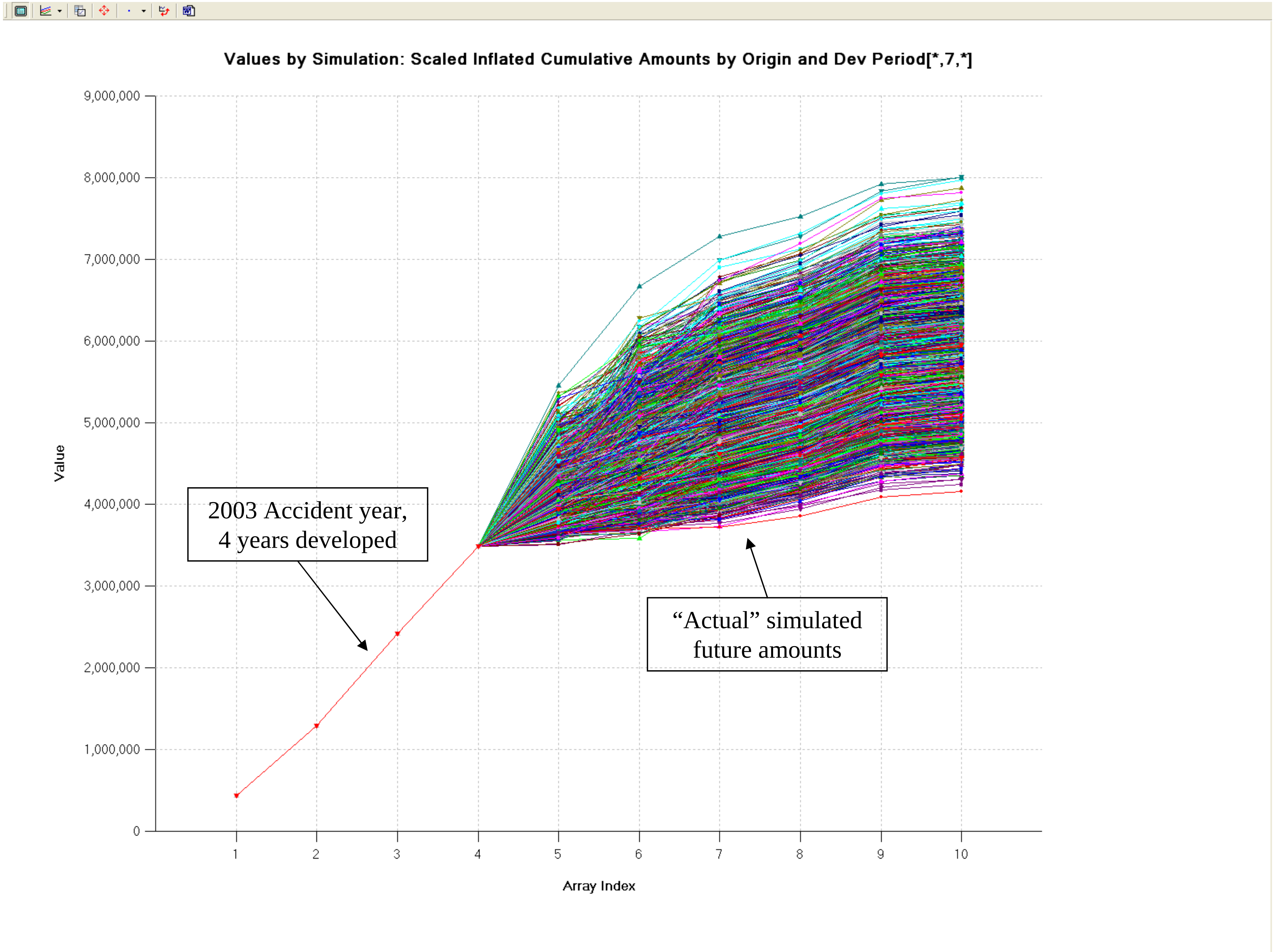
The one-year run-off result in a simulation model
Modus operandi

1. Given the opening reserve triangle, simulate all future claim payments to ultimate using a bootstrap or Bayesian MCMC technique.
2. Now forget that we have already simulated what the future holds.
3. Move one year ahead. Augment the opening reserve triangle by one diagonal, that is, by the simulated payments from step 1 in the next calendar year only. An actuary only sees what emerges in the year.
4. For each simulation, estimate the outstanding liabilities, conditional only on what has emerged to date. (The future is still "unknown").
5. A reserving methodology is required for each simulation – an "actuary-in-the-box" is required*. We call this re-reserving.
6. For a one-year model, this will underestimate the true volatility at the end of that year (even if the mean across all simulations is correct).

** The term "actuary-in-the-box" was coined by Esbjörn Ohlsson*

EMB ResQ Example




Values by Simulation: Scaled Inflated Cumulative Amounts by Origin and Dev Period[*,7,*]
9,000,000
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4,000,000
3,000,000
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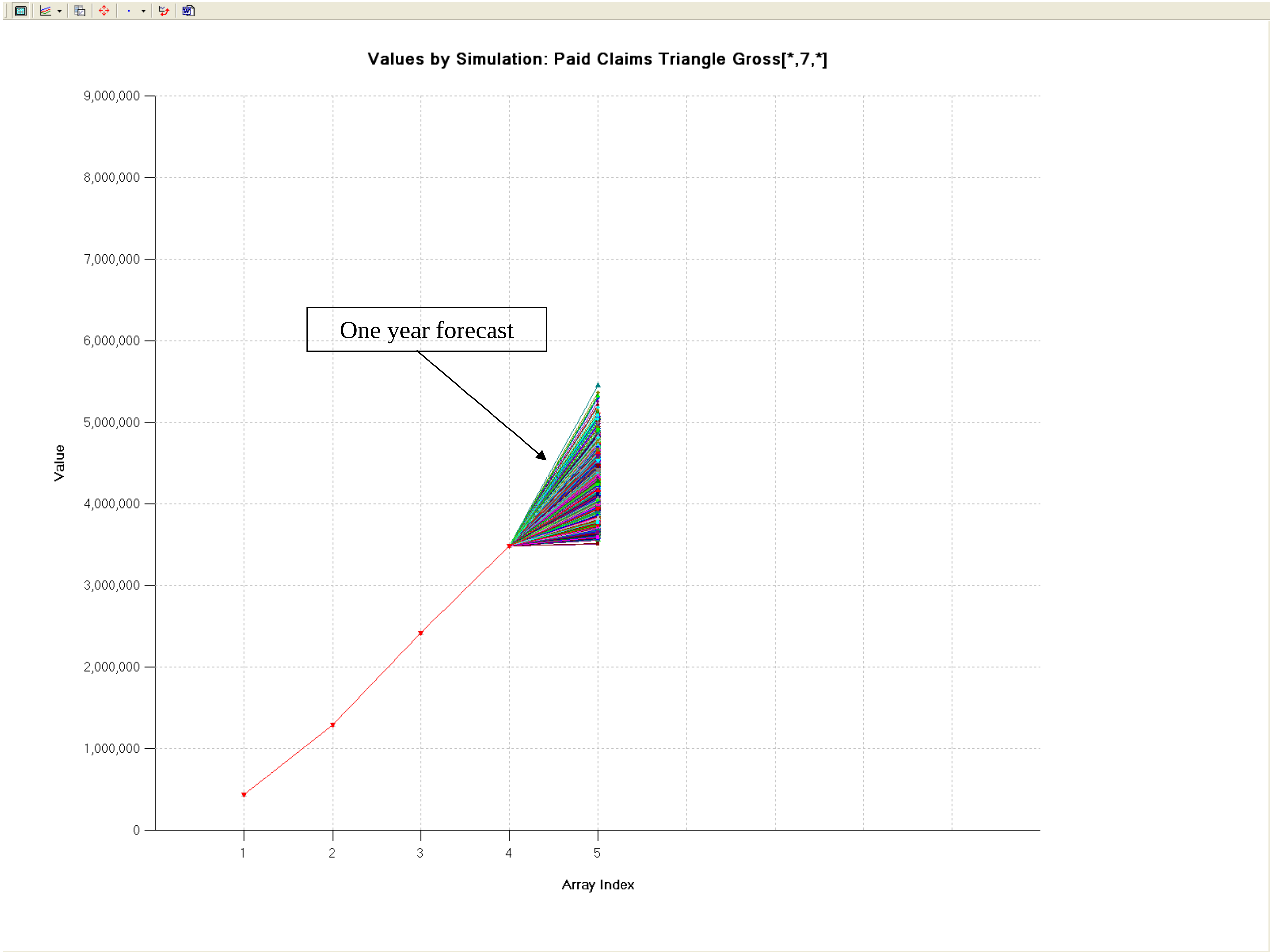
Array Index
2003 Accident year, 4 years developed
"Actual" simulated future amounts


Values by Simulation: Paid Claims Triangle Gross[*,7,*]
9,000,000
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One year forecast
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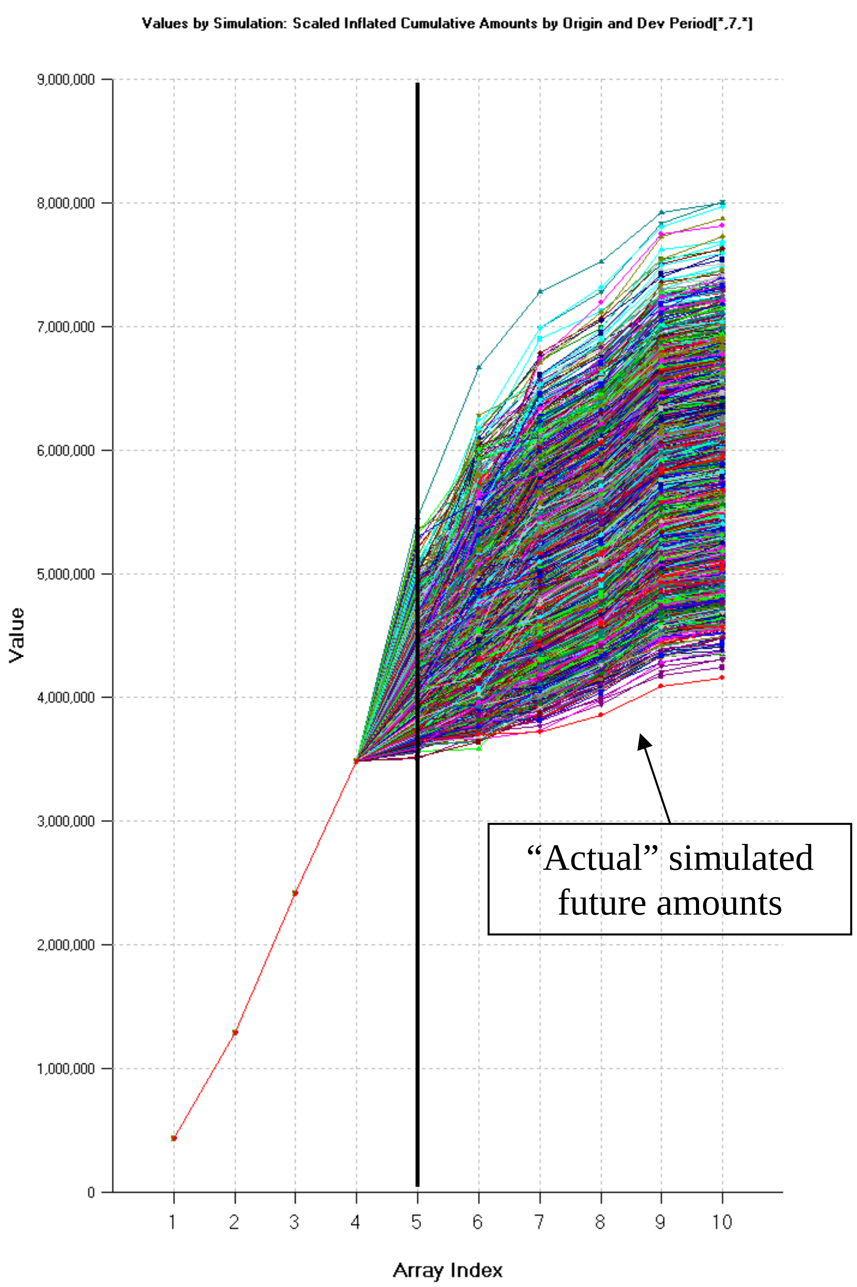
Array Index



Values by Simulation: Scaled Inflated Cumulative Amounts by Origin and Dev Period[*,7,*]
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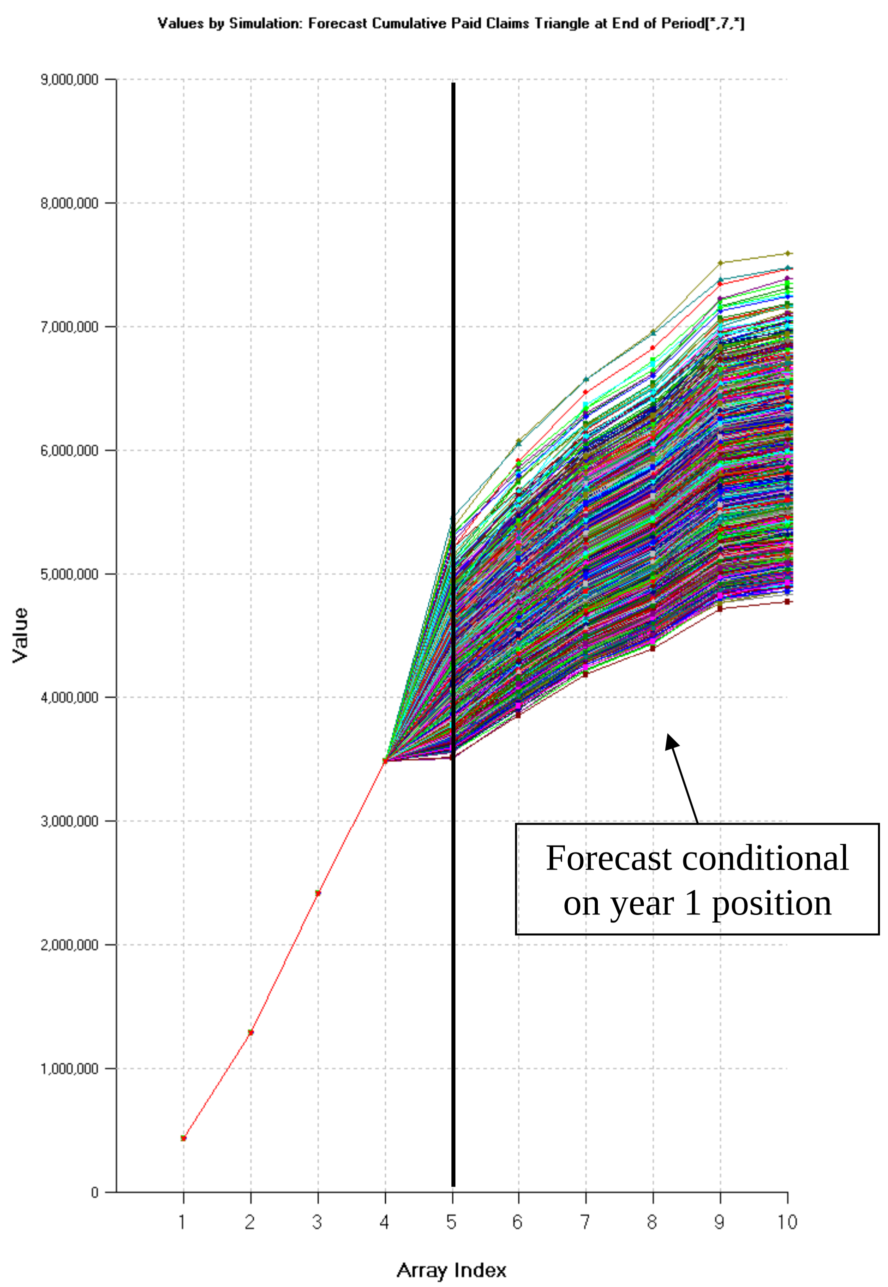
"Actual" simulated future amounts



Values by Simulation: Forecast Cumulative Paid Claims Triangle at End of Period[*,7,*]
9,000,000
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1,000,000
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Value
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Array Index
Forecast conditional on year 1 position

Merz & Wuthrich (2008)
Analytic vs Simulated

	Analytic Prediction Errors		Simulated Prediction Errors	
Accident Year	**1 Year Ahead CDR**	**Mack Ultimate**	**1 Year Ahead CDR**	**Mack Ultimate**
0	0	0	0	0
1	567	567	569	569
2	1,488	1,566	1,494	1,571
3	3,923	4,157	3,903	4,144
4	9,723	10,536	9,687	10,518
5	28,443	30,319	28,363	30,393
6	20,954	35,967	20,924	35,772
7	28,119	45,090	28,358	45,668
8	53,320	69,552	53,591	69,999
Total	81,080	108,401	81,159	108,442

Re-reserving in Simulation-based Capital Models

An advantage of investigating the claims development result (using re-reserving) in a simulation environment is that the procedure can be generalised:

- Not just the chain ladder model
- Can include curve fitting and extrapolation for tail estimation
- Can incorporate a Bornhuetter-Ferguson step
- Can be extended beyond the 1 year horizon to look at multi-year forecasts
- Can be used to help calibrate Solvency 2 internal models

The one-year run-off result in a simulation model

Further complications

So on an undiscounted basis we have:

$$CDR_1^{(i)} = R_0 - C_1^{(i)} - R_1^{(i)} = U_0 - U_1^{(i)}$$

If we use discounted reserves, then it gets harder, since we should also take account of allocated investment income (I) on the reserves held during the year:

$$CDR_t^{(i)} = R_{t-1}^{d(i)} + I_t^{(i)} - C_t^{(i)} - R_{t,d}^{d(i)}$$

If we use discounted reserves plus risk margins, then it gets harder still, since we need a risk margin (M) for each simulation conditional on that simulation and time period.

$$CDR_t^{(i)} = \left(R_{t-1}^{d(i)} + M_{t-1}^{(i)}\right) + I_t^{(i)} - C_t^{(i)} - \left(R_{t,d}^{d(i)} + M_t^{(i)}\right)$$

What is appropriate under Solvency 2, and how do we use the results?

Internal Capital Model Implications

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"The Solvency Capital Requirement corresponds to the economic capital a (re)insurance undertaking needs to hold in order to limit the probability of ruin to 0.5%, i.e. ruin would occur once every 200 years (see Article 101).

The Solvency Capital Requirement is calculated using Value-at-Risk techniques, either in accordance with the standard formula, or using an internal model: all potential losses, including adverse revaluation of assets and liabilities, over the next 12 months are to be assessed. The Solvency Capital Requirement reflects the true risk profile of the undertaking, taking account of all quantifiable risks, as well as the net impact of risk mitigation techniques."

DIRECTIVE OF THE EUROPEAN PARLIAMENT
Article 101

"The Solvency Capital Requirement shall be calibrated so as to ensure that all quantifiable risks to which an insurance or reinsurance undertaking is exposed are taken into account. With respect to existing business, it shall cover unexpected losses.

It shall *correspond* to the Value-at-Risk of the basic own funds of an insurance or reinsurance undertaking subject to a confidence level of 99.5% over a one-year period."

But how do we know that the SCR formula (with a capital amount calculated by risk type) corresponds to a 99.5% VaR applied to the basic own funds? In the absence of a distribution of the basic own funds, it is pure speculation!

DIRECTIVE OF THE EUROPEAN PARLIAMENT
Articles 87 and 74

Article 87

“Basic own funds shall consist of the following items:

(1) the excess of assets over liabilities, valued in accordance with Article 74 and Section 2 ;

(2) subordinated liabilities.”

Article 74

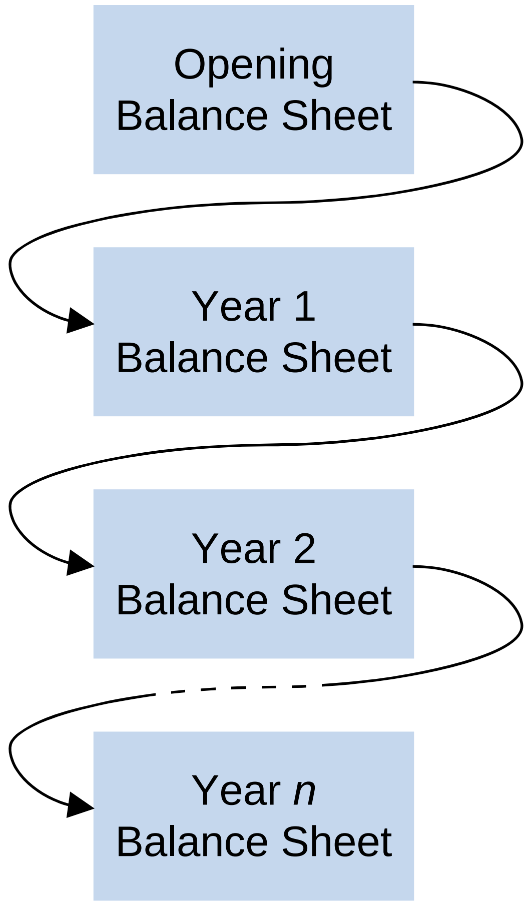
“Member States shall ensure that, unless otherwise stated, insurance and reinsurance undertakings value assets and liabilities as follows:

(a) assets shall be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction;

(b) liabilities shall be valued at the amount for which they could be transferred, or settled, between knowledgeable willing parties in an arm's length transaction.”

A Projected Balance Sheet View

- When projecting Balance Sheets for solvency, we have an opening balance sheet with expected outstanding liabilities
- We then project one year forwards, simulating the payments that emerge in the year
- We then require a closing balance sheet, with (simulated) expected outstanding liabilities conditional on the payments in the year
- The closing balance sheet after one year becomes the opening balance sheet in the second year, and so on



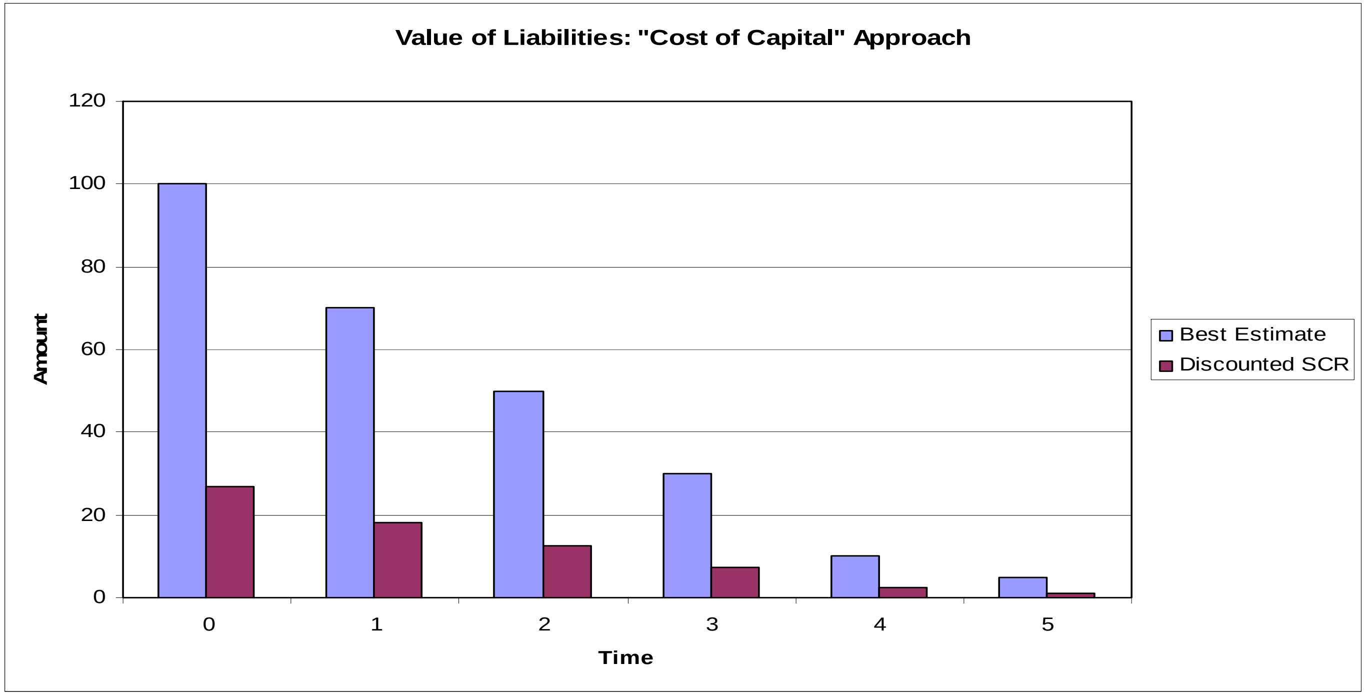
Solvency II Requirements
A slight problem

- The Solvency II requirements are worded as an overall company requirement based on a 1 year ahead balance sheet, and in a simulation based internal capital model, the SCR can be found naturally from a simulated balance sheet after 1 year
- However, to obtain risk margins by Solvency II line of business using the Cost-of-Capital approach, an ‘SCR’ by line of business is required, even though such a thing does not exist
- So we have to think in terms of overall capital requirements, AND artificial capital requirements by line of business
- We will try and consider both

Solvency 2 - QIS 4 Spreadsheets

- The QIS 4 formula based calculation of the overall "SCR Non-life" does not require risk margins as an input
  - The "Provisions for Claims Outstanding" (PCO) are required
  - These are the discounted expected values of outstanding claims, by line of business (and country)
  - A "standard deviation" is required for each line of business
    - It is not the standard deviation on an ultimate basis
- The SCR is compared to available capital from a balance sheet WITH risk margins in the liabilities
  - The risk margins are calculated separately, by Solvency II line of business
  - A 'line of business' SCR is required, which must be approximated
  - In the 'helper' spreadsheets, the 'proportional proxy' is used in the cost-of-capital risk margin calculations

“Cost of Capital” Approach: Core Components



Sum Discounted (LoB) Capital Requirements (incl. time 0 capital) = 68

Cost of Capital = 6% (*above risk free rate*)

Risk Margin = 68 * 6% = 4.08

The “problem” reduces to estimating the capital requirements at each time point

Risk Margin calculations
Estimating the capital requirements: A simple proxy

- Estimating the (LoB) capital required (in respect of reserves) at future time periods is not straightforward
- A proxy that has been suggested is to estimate the (LoB) capital required in the first year, then assume the capital required at further time periods is proportional to the outstanding liabilities at that time
- Let CR_0 be the opening capital required for reserving risk

  Let L_0 be the opening best estimate of outstanding liabilities

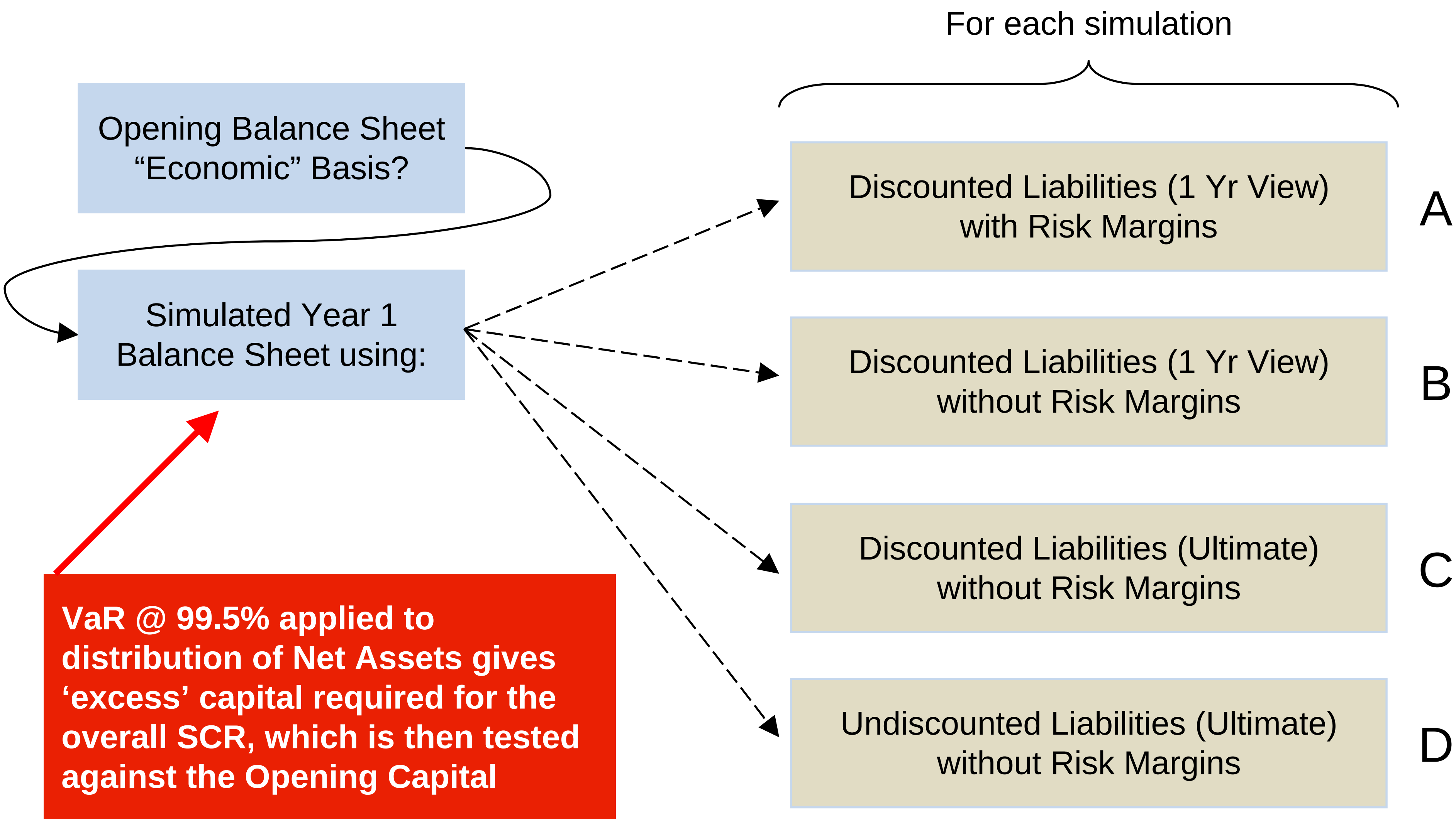
  Let L_t be the best estimate of outstanding liabilities at time t
- Then $CR_t = \frac{CR_0}{L_0} L_t$
- So the problem reduces further to estimating the opening (LoB) capital required under this simplification

Solvency II Questions
Simulation based internal capital models

Risk margins do not appear in the QIS 4 formula based SCR. So can we:

- Use a balance sheet excluding risk margins in the liabilities in a multi-year model for the opening position and at Year *t;*
  - Then calculate the excess capital required (using VaR @ 99.5% applied to the Yr 1 balance sheet) for the overall SCR calculation
- Then perform a "Cost-of-Capital" risk margin calculation, using an appropriate notional 'SCR' methodology by line of business;
- Then compare the overall SCR with a restated opening balance sheet with risk margins in the liabilities, for assessing capital adequacy?
- Or do we need an opening balance sheet with risk margins in the liabilities, and calculate risk margins for each simulation at each future time period for the Yr 1 balance sheet?
- Are there other options that simplify the modelling?
  - In particular, for the 'notional' SCR by line of business

Issue 1: Overall SCR
Simulated Year 1 balance sheet options



Issue 2: Line of business SCR
Simulated claims development result (CDR) options

For each simulation

Opening Reserves: Basis?

Year *n* Reserves:

Discounted Reserves (1 Yr View) with Risk Margins — A

Discounted Reserves (1 Yr View) without Risk Margins — B

Discounted Reserves (Ultimate) without Risk Margins — C

Undiscounted Reserves (Ultimate) without Risk Margins — D

VaR @ 99.5% applied to distribution of CDR gives the line of business capital required, which is then used for risk margin calculations

Issue 1:

Overall capital requirements

Simulated balance sheet definitions after 1 year

Option A – Discounted liabilities (1 yr view) with risk margins

Advantages

- Appears to obey the rules

Limitations

- Shareholder perspective: ensures profit is available for shareholders
- Does not adequately protect policyholders
- Extremely difficult to calculate risk margins on a simulation by simulation basis without simplifying assumptions
- Of limited practical use, since the business is not managed on that basis
- “One year” view of reserving risk calculated in a robotic way

Simulated balance sheet definitions after 1 year

Option B – Discounted liabilities (1 yr view) without risk margins

Advantages

- Straightforward to calculate in a simulation environment, using the "actuary-in-the-box" methodology
- Protects policyholders better, since the "total resources" are considered, which do not change if the risk margin method changes

Limitations

- At first sight, does not appear to match the Solvency II criteria
- "One year" view of reserving risk calculated in a robotic way

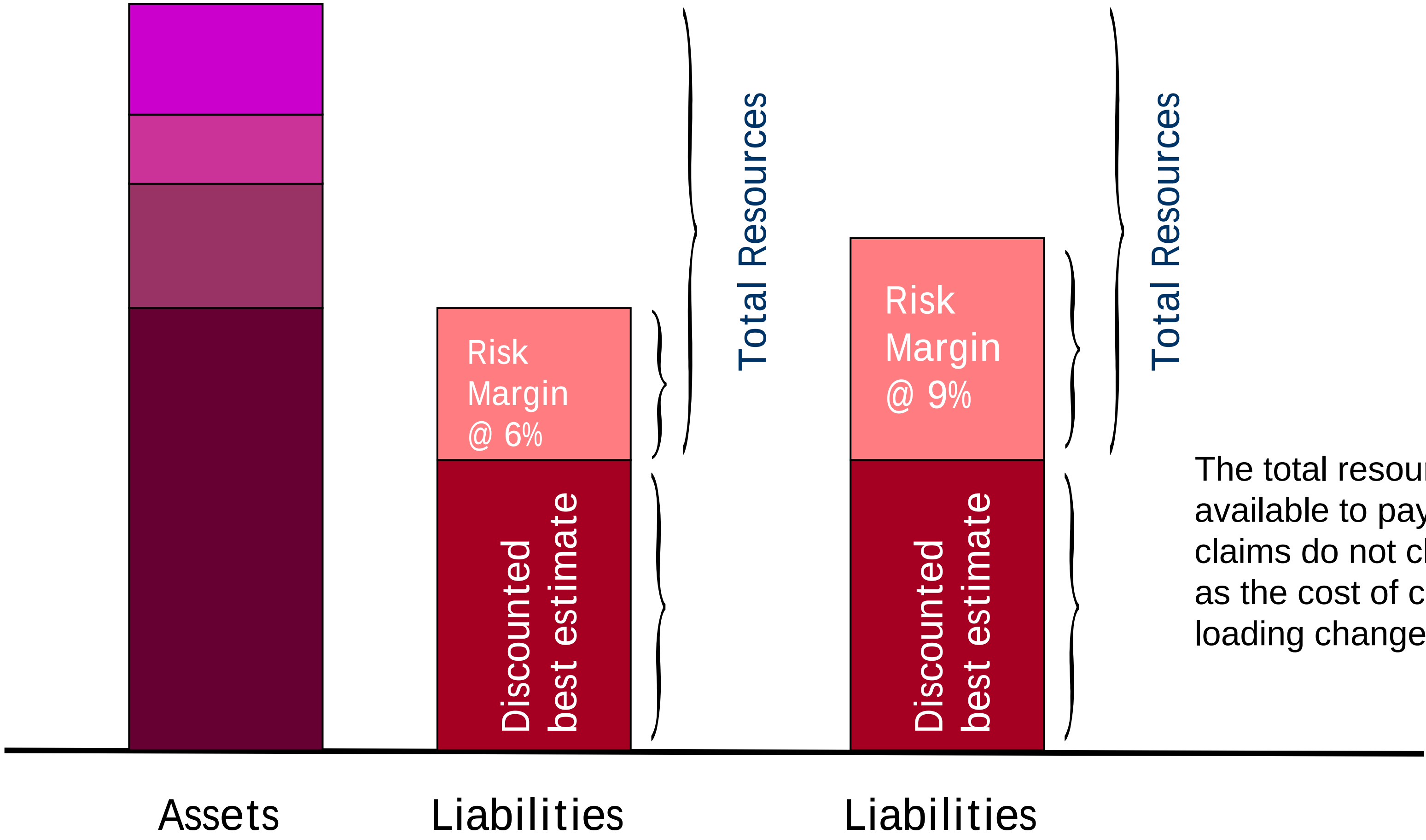
“Economic” Balance Sheet?

Suppose all other capital has been exhausted, except the Risk Margin, and another claim comes in. Does that claim get paid? That is, when does default occur?

It is the Total Resources that are important for protecting policyholders

- Avoids counter-intuitive results if the basis for the margin is strengthened
- Any argument about margins is then (almost) irrelevant, since it is just a partition of the Total Resources (which are fixed)

"Economic" Balance Sheet?



The total resources available to pay claims do not change as the cost of capital loading changes

Simulated balance sheet definitions after 1 year

Option C – Discounted liabilities on an ultimate basis without risk margins

Advantages

- Easy to calculate in a simulation environment, using standard reserving risk methods
- No need for a robotic "re-reserving" methodology and the additional assumptions required
  - We assume perfect foresight
- Protects policyholders, since the ultimate claims paying ability is considered

Limitations

- Does it satisfy the Solvency II rules?
  - May satisfy the Solvency II criteria if it can be shown that this approach is at least as strong
  - This will depend on the "Cost of Capital" percentage

Simulated balance sheet definitions after 1 year

Option D – Undiscounted liabilities on an ultimate basis without risk margins

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Advantages

- Even easier to calculate in a simulation environment, using standard reserving risk methods
- No need for a robotic “re-reserving” methodology and the additional assumptions required
  - We assume perfect foresight
- Protects policyholders, since the ultimate claims paying ability is considered

Limitations

- Does it satisfy the Solvency II rules?
  - May satisfy the Solvency II criteria if it can be shown that this approach is at least as strong
  - This will depend on the “Cost of Capital” percentage

Simulated balance sheet definitions after 1 year?
A convenient procedure

Opening Balance Sheet
without Risk Margins in the Liabilities

For each simulation

Simulated Year 1 Balance Sheet

Discounted Liabilities (1 Yr View) without Risk Margin

B

'Excess' capital calculated using VaR @ 99.5% applied to distribution of Net Assets

This is used to calculate the overall SCR which is then tested against the opening capital using a Balance Sheet WITH Risk Margins in the Liabilities

Under what assumptions can we use a balance sheet definition without risk margins in simulation based internal capital models for calculating the overall SCR?

(This would avoid unnecessary complications, and is analogous to the way QIS 4 seems to operate)

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Issue 2:

Line of business capital requirements for risk margin calculations

Issue 2: Line of business SCR

Simulated claims development result (CDR) options

- For the notional line of business SCR, in some ways the issues are slightly less complicated
  - We can ignore the assets
  - We use the distribution of the CDR as the 'risk profile', instead of a distribution of net assets
- We need to decide what items are included in the CDR (and which basis), and under what assumptions we can make simplifications

$$CDR_t^{(i)} = \left(R_{t-1}^{d\,(i)} + M_{t-1}^{(i)}\right) + I_t^{(i)} - C_t^{(i)} - \left(R_{t,d}^{d\,(i)} + M_t^{(i)}\right)$$

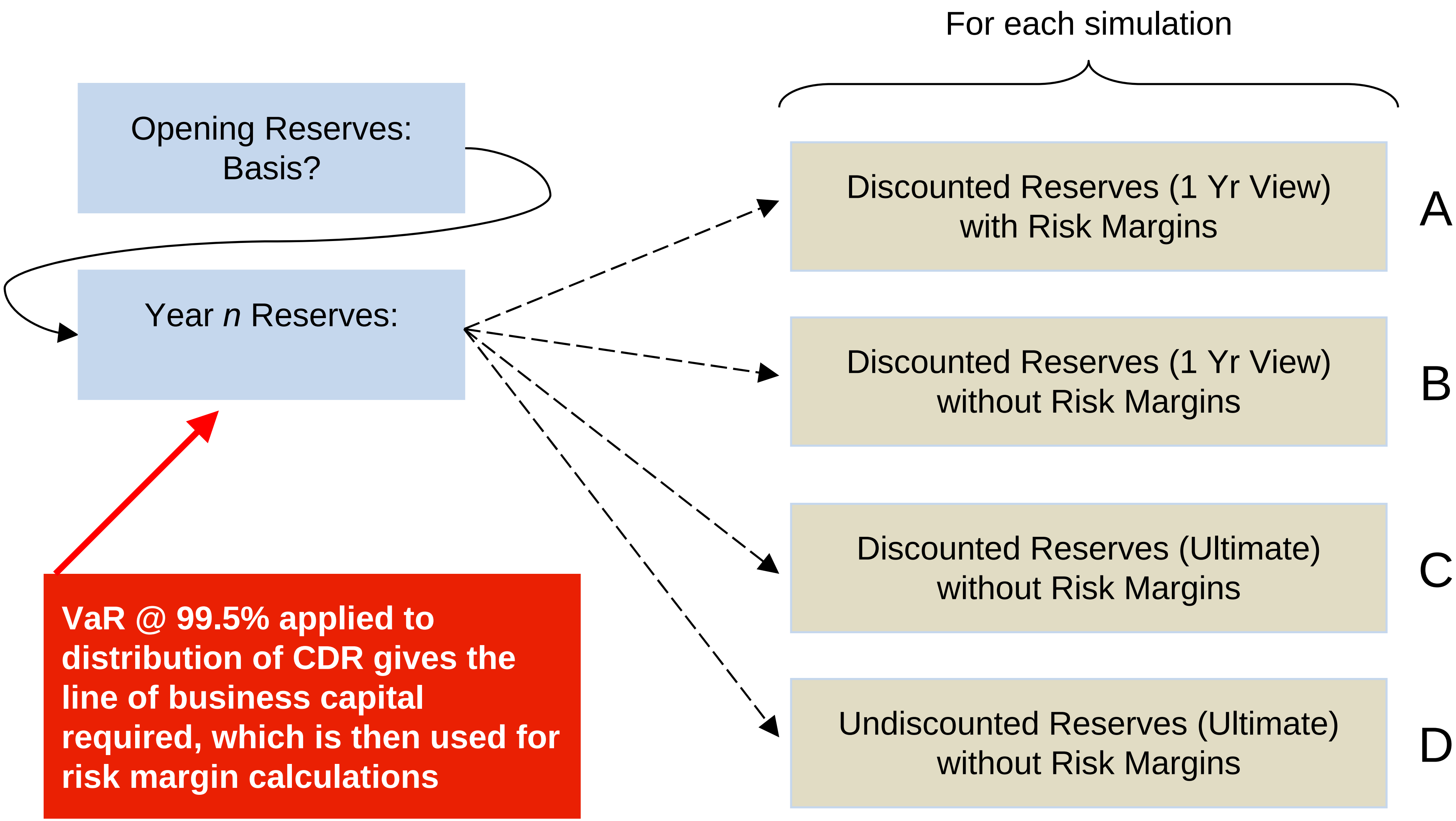
- If we can't make simplifications, we need an appropriate methodology
- The problem is that we need a notional line of business SCR *for each future year, for each simulation*

The notional LoB SCR for each future year

- It looks like the SCR depends on the risk margin, and the risk margin depends on the SCR
  - This paradox is resolved by starting at the end and working backwards
  - At the end of the run-off, the expected reserves are zero and the risk margin is zero
  - Moving one step back, the 99.5% VaR of the CDR is required for each simulation (conditional on information available up to that time), giving a distribution of the SCR
  - The risk margin can be obtained for each simulation (as the cost of capital)
  - The expected risk margin can also be calculated, which is required for the CDR at the previous step
- The problem is obtaining the 99.5% VaR of the CDR for each simulation, without performing simulation on simulation
- So, what are the options?

Issue 2: Line of business SCR

Simulated claims development result (CDR) options



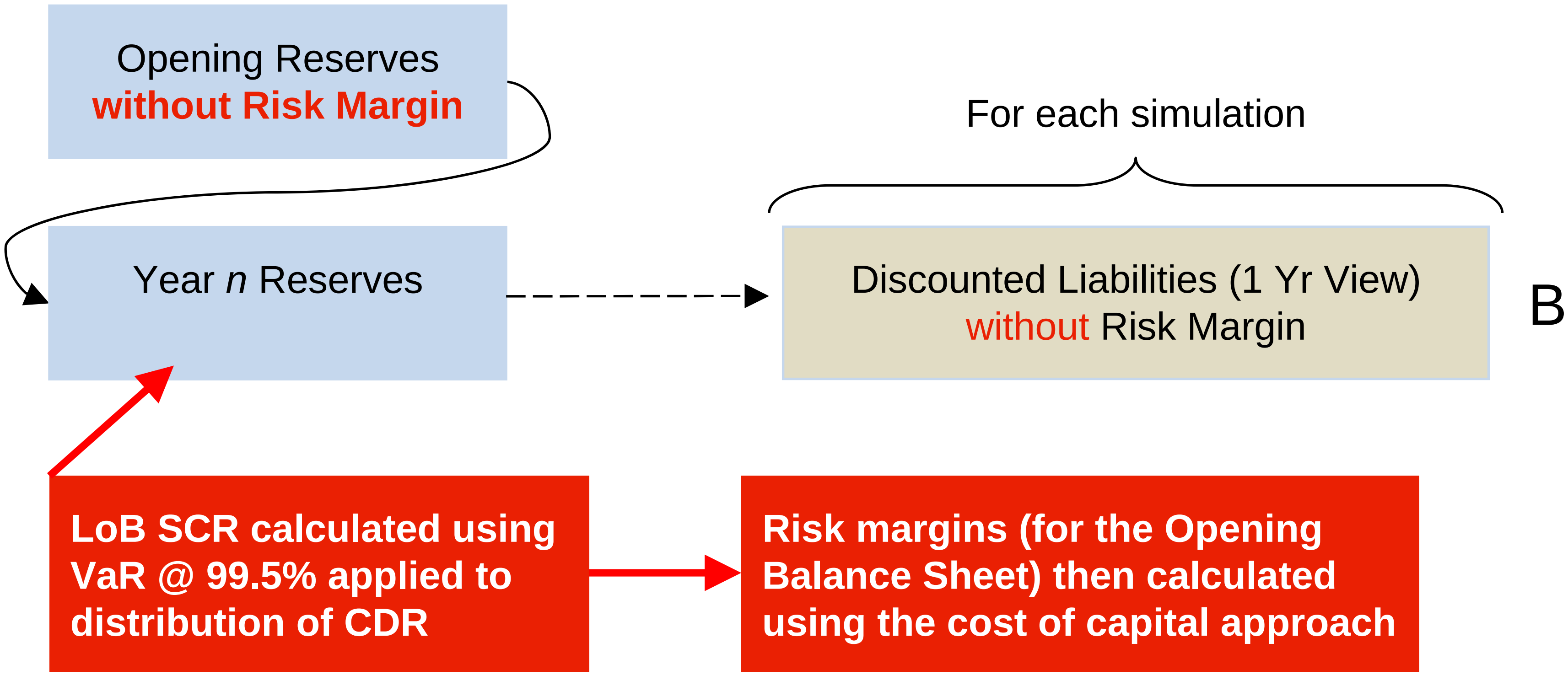
Issue 2: Line of business SCR

Simulated claims development result (CDR) options

- Option A
  - Seems technically correct
  - But very difficult to calculate in a simulation environment, without simplifying assumptions
- Option B
  - Easy to calculate in a simulation environment
  - But requires a re-reserving process for each simulation
- Option C
  - Easy to calculate in a simulation environment. Does not require a re-reserving process for each simulation.
  - Protects policyholders better
  - But does it satisfy the rules?
- Option D
  - Even easier to calculate in a simulation environment. Does not require a re-reserving process for each simulation.
  - Protects policyholders better
  - But does it satisfy the rules?

Risk margin calculations: An interesting result
Using the "proportional proxy" for the CoC approach

Ohlsson & Lauzeningks (2008/9) suggest that when using the "proportional proxy" for line of business capital requirements in the cost of capital approach, the risk margin itself drops out, so the (LoB) SCR can be calculated ignoring risk margins.



Risk margin calculations

- So, Option B seems to be a possibility under the ‘proportional proxy’ for a line of business risk margin calculation at the opening position
- But we still have the problem of a suitable definition of liabilities for estimating net assets for the overall SCR calculation
  - Do we need risk margins for each simulation in the liabilities at the Year 1 position, or can we use similar simplifications?
- If we do not use Option B for the line of business risk margin calculations, can we make progress with Option A (discounted reserves (1 year view) with risk margins)?
  - An example follows

Including risk margins on a simulation by simulation basis within internal capital models

EMB Igloo Example

Conclusions

- The one-year view of reserving risk and notional line of business SCRs require the ‘claims development result’
- Backwards recursion is required to avoid circularity of the line of business SCRs and risk margins
- Simplifications are required to avoid simulation on simulation
- Under the ‘proportional proxy’, the risk margins can be dropped
- Using the maximum entropy approach, progress can still be made without dropping risk margins

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