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## Uncertainty in short-term claims development.

GIRO 2008  
Gavin Hill  
Tom Wright



*Audit, Tax, Consulting, Corporate Finance*

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### Agenda

- Background
- A theoretical approach
- Practical implications

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## Background

- ICAs
  - Ultimate reserving uncertainty
  - Some focus on pattern of emergence
- Swiss solvency test (SST)
  - One year time horizon
  - Cost of capital / risk margin
  - 99% TVAR
- Solvency II
  - SCR based upon a one-year time horizon
  - Risk margin allows for emergence beyond one year
  - 99.5% VAR
- Requirements
  - Need to bridge the gap between models available and metrics required

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## A theoretical approach

- Objectives
- Notation
- Mixing distributions
- Some formulae
- Assumptions and simplifications
- Comparison to alternate method

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## Objectives

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### Long-term versus Short-term Uncertainty in Claim Development Result (CDR)

- Established actuarial methods for quantifying reserve uncertainty, eg:
  - Mack's method
  - Bootstrap method
- These give a probability distribution (or just a standard deviation) for the ultimate paid amount (each origin year, and all origin years combined).
- These methods take a long-term view: they aim to assess uncertainty in ultimate paid amount taking account of all possible future random variation.
- For regulatory capital, a short-term view is needed. Eg SST requires probabilistic assessment of the CDR over the next 1 year following the valuation date.
- There is less uncertainty over a finite time horizon (eg 1 year) than over the whole of the future
- So short-term standard deviation should be less than given by established actuarial methods (eg Mack, Bootstrap).

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### Short-Term Claims Development Result (CDR)

- The claims development result over a finite time horizon is defined as:  
$$\text{CDR} = (\text{best estimate ultimate now}) - (\text{best estimate at future point in time})$$
- This will appear in the profit and loss account at end of the period concerned.
- The first term is known at the present time: obtained by applying actuarial reserving methods to data available now.
- The second term will not be known until the future point in time: it can then be obtained by applying actuarial methods to data available at that point in time.
- Note that both terms are "best estimate ultimates": the short-term view (like the long term-view) is concerned with ultimate positions.
- However the short-term view is concerned with changes in the best estimate ultimate over a finite time horizon (eg 1 year).

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### Notation

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## Notation

Subscripts 0, 1, t relate to time

Valuation date is time zero ( $t = 0$ ), one year later is time  $t=1$

Quantities that are uncertain at time zero (that is, random variables at time zero) are shown in green.

$R_0$  = total payments made after time zero in respect of Prior Years

Expected values and variances at time t are denoted  $E_t()$  and  $Var_t()$ .

For example:  $E_0(R_0)$  and  $Var_0(R_0)$  are the best estimate reserve and its variance obtained by applying stochastic methods (eg Mack and/or bootstrap) to data available at time 0.

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## Notation

$R_0$  = total payments made after time zero in respect of Prior Years

$F_0(R_0)$  = cumulative distribution of  $R_0$  at time 0 (that is, the long-term predictive distribution eg from Mack and/or bootstrap).

$F_t(R_0)$  = cumulative distribution of  $R_0$  at time t

$F_t(R_0)$  differs from  $F_0(R_0)$  because of additional information at time t.

If  $D_t$  denotes information emerging between time zero and time t,  $F_t(R_0)$  can alternatively be expressed as  $F(R_0 | D_t)$

At time t,  $R_0$  will have a known component (amount paid between time 0 and time t) and an uncertain component (amounts paid after time t).

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## More Notation

$m_t$  and  $u_t$  are the mean and standard deviation of the distribution  $F_t(R_0)$

$m_t$  = best estimate of  $R_0$  at time t, that is  $m_t = E_t(R_0)$

$u_t$  = standard deviation of  $R_0$  at time t, that is  $u_t^2 = Var_t(R_0)$

Prior Year Claims Development Result is:  $CDR(0,t) = m_0 - m_t$

At time 0:  $m_0$  is known but  $m_t$  is a random variable (depends on  $D_t$ )

So  $Var_0(CDR(0,t)) = Var_0(m_t)$

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## Mixing distributions

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## Long-term predictive distribution at time 0 as mixture of possible long-term predictive distributions at fixed time $t$ ( $t > 0$ )

At time zero, there are many possibilities for  $F_t(\mathbf{R}_0)$  because this depends on information emerging between time 0 and time  $t$

$$F_t(\mathbf{R}_0) = F(\mathbf{R}_0 | D_t)$$

At time zero, the new information  $D_t$  that will emerge is uncertain

All possibilities for  $D_t$  approximated as a countable set  $\{D_t^{(i)} : i = 1, 2, \dots\}$

$P_0(D_t^{(i)})$  denotes the probability at time zero of  $D_t^{(i)}$

$$\text{Then } F_0(\mathbf{R}_0) = \sum_i F(\mathbf{R}_0 | D_t^{(i)}) \cdot P(D_t^{(i)})$$

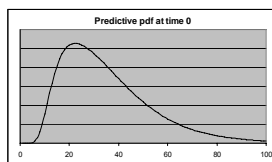
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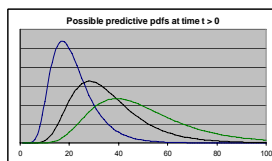
## Long-term predictive distribution at time zero as mixture

The horizontal axis is  $\mathbf{R}_0$  (ultimate prior year liabilities paid after time zero).

First graph shows long-term predictive distribution at time zero  $f_0(\mathbf{R}_0)$ : mean is  $m_0$ , std deviation is  $u_0$ .



Second graph shows three possibilities (at time 0) for the predictive distribution at time  $t$ .



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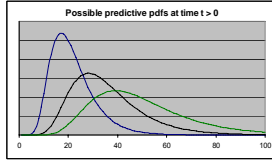
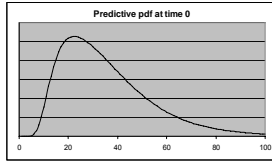
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## Long-term predictive distribution at time zero as mixture

Predictive distribution at time  $t$  depends on information emerging between time 0 and time  $t$ , so is unknown at time 0.

Suppose that at time 0, the three illustrated time- $t$  predictive distributions are considered to be the only possibilities, and all 3 are considered equally likely (probability  $1/3$  each).

Then the predictive distribution at time zero must be the probability weighted average (or "mixture") of the three possible time- $t$  predictive distributions (the weights being equal in this example).



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## Some Formulae

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## General Formula for Variance of Short-Term CDR

We have:  $F_0(\mathbf{R}_0) = \sum_i F(\mathbf{R}_0 | \mathbf{D}_i^{(0)}) \cdot P(\mathbf{D}_i^{(0)})$

Standard formulas of conditional probability:

$E_0(\mathbf{R}_0) = E_0(E(\mathbf{R}_0 | \mathbf{D}_i))$  and  $\text{Var}_0(\mathbf{R}_0) = \text{Var}_0(E(\mathbf{R}_0 | \mathbf{D}_i)) + E_0(\text{Var}(\mathbf{R}_0 | \mathbf{D}_i))$

Using  $m_t$  to denote best estimate of  $R_0$  at time  $t$ , and  $u_t$  to denote its standard deviation at time  $t$ , these become:

$m_0 = E_0(m_t)$  and  $u_0^2 = \text{Var}_0(m_t) + E_0(u_t^2)$

Prior Year claims development results is  $\text{CDR}(0,t) = m_0 - m_t$

At time zero,  $m_t$  is uncertain:  $\text{CDR}(0,t) = m_0 - m_t$

So:  $\text{Var}_0(\text{CDR}(0,t)) = \text{Var}_0(m_t) + u_0^2 - E_0(u_t^2)$

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### Evolution of long-term predictive uncertainty for prior years

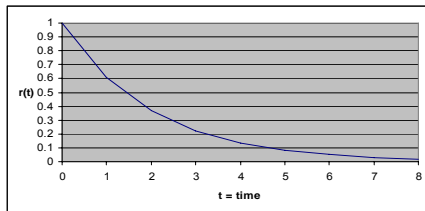
- $\text{Var}_0(\text{CDR}) = \text{Var}_0(m_t) = u_0^2 - E_0(u_t^2) = u_0^2 \cdot (1 - r_t)$  where  $r_t = E_0(u_t^2) / u_0^2$
- $u_0$  is known (by applying stochastic methods to data available at time zero)
- $u_t$  is not known for any future time  $t$  (because it will depend on data and other information emerging between time 0 and time  $t$ )
- $u_t$  can be estimated at time 0 by projecting observed claim development patterns forward to time  $t$
- In the normal course of events we should expect (for fixed prior years) that the long-term predictive uncertainty  $u_t$  will decrease as  $t$  increases, for three reasons:
  - future process uncertainty decreases because the expected outstanding liability decreases
  - parameter uncertainty decreases because the volume of data available to estimate model parameters (eg chain ladder development factors) increases
  - model uncertainty decreases because (a) there is less remaining time for future development patterns to deviate from past patterns, and (b) there is an increasing volume of data for model validation.

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### Evolution of long-term predictive uncertainty for prior years

- $\text{Var}_0(m_t) = u_0^2 \cdot \{1 - r(t)\}$  where  $r(t) = E_0(u_t^2) / u_0^2$
- $r(0) = 1$
- $r(t)$  decreases, tending to zero as  $t$  tends to infinity



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### To calculate distribution of short-term CDR

- **Method 1** – based on predictive standard error (eg Mack's method)
  - Calculate long-term predictive variance  $u_0^2$
  - Calculate expected reduction factor for long-term variance  $r(t) = E_0(u_t^2) / u_0^2$
  - Calculate short-term variance  $\text{Var}_0(\text{CDR}) = \text{Var}_0(m_t) = u_0^2 \cdot \{1 - r(t)\}$
  - Fit a distribution, eg Log-Normal with mean =  $m_0$ , variance =  $u_0^2 \cdot \{1 - r(t)\}$
- **Method 2** – based on complete predictive distribution (eg Bootstrap)
  - Calculate complete predictive distribution  $F_0(R_0)$  (mean =  $m_0$ , variance =  $u_0^2$ )
  - Calculate expected reduction factor for long-term variance  $r(t) = E_0(u_t^2) / u_0^2$
  - Calculate  $H_0(m_t) = F_0(m_0 + (m_t - m_0) / \sqrt{1 - r(t)})$
  - This distribution has mean =  $m_0$ , variance =  $u_0^2 \cdot \{1 - r(t)\}$
  - Example in Excel

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## Assumptions and simplifications

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## To calculate distribution of short-term CDR

- In either method, a key step is:
- Calculate expected reduction factor for long-term variance  $r(t) = E_0(u_t^2) / u_0^2$
- Simple pragmatic approach to achieve this:
  - a) Project triangle forward to time t using expected values
  - b) Apply the same methods as used when calculating  $u_0^2$  from the original triangle to the extended triangle (but being careful to allow for the absence of random process variation in the projected part of the triangle)
  - c) If step (b) is not practical because the method used to obtain  $u_0^2$  from the original triangle is too complex (eg involving much judgement), then use a simpler method (eg Mack) for both numerator and denominator of  $r(t)$

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## Estimation of $r(t) = E_0(u_t^2) / u_0^2$ for Mack's method

- Mack (1993 and 1999) gives formulas for the long-term predictive variance of chain-ladder estimates. This is the quantity denoted  $u_0^2$  here.
- To estimate what  $u_t^2$  would be by Mack's method at some future time t, Mack's formulas can be applied to the triangle projected to that future point in time.
- For example, if the triangle has annual development data, to estimate  $u_1^2$  one further diagonal of the triangle must be projected (using chain ladder projections) and Mack's formulas applied to the projected triangle.
- Estimates of the variance parameters (denoted  $\sigma_k^2$  in Mack 1999) should not be recalculated using the projected triangle because the absence of random variation in the projected diagonals would understate  $\sigma_k^2$ . Instead, these parameters should be maintained at their time-zero best estimates.

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### Comparisons to alternate method

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### Comparison with method of Merz and Wüthrich

- Michael Merz and Mario Wüthrich address the problem of calculating the variance of the short-term CDR in their paper "Modelling the Claims Development Result for Solvency Purposes" (presented at ASTIN 2008)
- In that paper, Merz & Wüthrich develop formulas, specific to Mack's model, for the variance of the one-year CDR.
- They use a completely different approach to the one described here, but the two methods give exactly the same variance for the one-year CDR.

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### Advantages of method described here compared to Merz-Wüthrich

- Merz & Wüthrich formula
  - requires development periods the same as origin periods (eg annual/annual)
  - looks forward one further diagonal of triangle
  - does not allow for a tail factor
- Method described here:
  - does not require development periods same as origin periods (eg for annual/quarterly triangle, one-year CDR is obtained by projecting 4 new diagonals to calculate  $E_0(u_t^2)$ )
  - works for any finite time horizon  $t$  (not just one year)
  - works where there is a tail factor (Mack's 1999 paper describes calculation of  $u^2$  when there is tail factor)
  - is not specific to Mack's method: the same principles can be applied whatever stochastic method is used for long-term predictions

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## Practical implications

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## Practical implications

- Continue to estimate ultimate reserve uncertainty, allowing for:
  - Parameter and process risk
  - Model risk
  - Systemic risk
- Estimate expected reduction factor for long-term variance  $r(t) = E_0(u_t^2) / u_0^2$
- Modify internal models to ensure reserve uncertainty emerges over time
  - Uncertainty emerging too soon will over-state SCR

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## Questions?

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