Using the ODP Bootstrap Model: A Practitioner's Guide

Using the ODP Bootstrap Model: A Practitioner's Guide

Author:
Mark R. Shapland, FCAS, FSA, MAAA

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Monograph Outline

- Introduction
- Notation
- Basic ODP Model / GLM Framework
- Generalizing the GLM Framework
- Practical Data Issues / Algorithm Enhancements
- Model Diagnostics
- Using Multiple Models
- Aggregation Issues / Model Uses
- Testing & Future Research

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ODP Bootstrap Overview

- Non-parametric bootstrap (Section 4) involved sampling of age-to-age ratios.
- For semi-parametric bootstrap, we will use parameters to calculate residuals and sample the residuals.
- The residuals create new samples of the triangle.
- Then for each new triangle we can make a projection.
- And for each projection we can add random noise.
- Let's start with a simple example to review the algorithm… then review the theory.

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ODP Bootstrap Overview

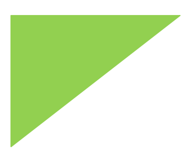
For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Cumulative Data

	1	2	3	4	5	6
1	95	150	180	200	210	215
2	110	160	175	205	210	
3	105	165	190	210		
4	120	155	180			
5	130	170				
6	125					

1) Actual Cumulative Data



Cumulative Data

	1	2	3	4	5	6
1	95	150	180	200	210	215
2	110	160	175	205	210	
3	105	165	190	210		
4	120	155	180			
5	130	170				
6	125					
Factors:	1.429	1.151	1.128	1.037	1.024	

2) Avg. Age-to-Age Factors

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ODP Bootstrap Overview

For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Fitted Cumulative Data

	1	2	3	4	5	6
1	109.16	155.94	179.45	202.50	210.00	215.00
2	109.16	155.94	179.45	202.50	210.00	
3	113.20	161.71	186.10	210.00		
4	109.49	156.41	180.00			
5	119.00	170.00				
6	125.00					
Factors:	1.429	1.151	1.128	1.037	1.024	

3) "Fit" Cumulative Data

Fitted Incremental Data

	1	2	3	4	5	6
1	109.16	46.78	23.51	23.05	7.50	5.00
2	109.16	46.78	23.51	23.05	7.50	
3	113.20	48.51	24.39	23.90		
4	109.49	46.92	23.59			
5	119.00	51.00				
6	125.00					

4) "Fitted" Incremental Data

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ODP Bootstrap Overview

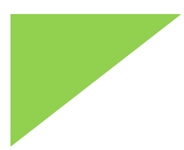
For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

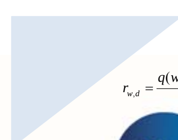
5) Actual Incremental Data



Unscaled Pearson Residuals

	1	2	3	4	5	6
1	-1.35	1.20	1.34	-0.64	0.91	0.00
2	0.08	0.47	-1.76	1.45	-0.91	
3	-0.77	1.65	0.12	-0.80		
4	1.00	-1.74	0.29			
5	1.01	-1.54				
6	0.00					

6) Unscaled Residuals



$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m^z_{w,d}}}$$

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ODP Bootstrap Overview

For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

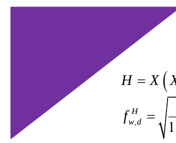
Hat Matrix Factors

	1	2	3	4	5	6
1	1.65	1.27	1.23	1.29	1.44	0.00
2	1.65	1.27	1.23	1.29	1.44	
3	1.68	1.28	1.23	1.31		
4	1.80	1.30	1.24			
5	2.06	1.35				
6	0.00					

Standardized Pearson Residuals

	1	2	3	4	5	6
1	-2.24	1.53	1.64	-0.82	1.31	0.00
2	0.13	0.60	-2.15	1.87	-1.31	
3	-1.30	2.12	0.15	-1.04		
4	1.80	-2.26	0.36			
5	2.07	-2.07				
6	0.00					

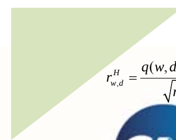
7) Hat Matrix Factors



$$H = X\left(X^T W X\right)^{-1} X^T W$$

$$f^H_{w,d} = \sqrt{\frac{1}{1-H_{i,i}}}$$

8) Standardized Residuals



$$r^H_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m^z_{w,d}}} \times f^H_{w,d}$$

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ODP Bootstrap Overview

For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

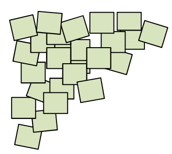
Random Residuals

	1	2	3	4	5	6
1	-1.31	-2.07	0.36	-2.07	-2.07	1.64
2	-0.82	-0.82	-2.26	1.64	0.36	
3	2.07	1.87	1.87	-2.15		
4	-1.31	-0.82	1.80			
5	-0.82	1.80				
6	0.60					

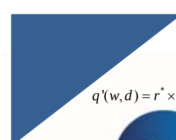
Sample Incremental Triangle

	1	2	3	4	5	6
1	95.42	32.59	25.26	13.09	1.82	8.66
2	100.56	41.16	12.55	30.92	8.49	
3	135.27	61.57	33.64	13.38		
4	95.73	41.29	32.34			
5	110.03	63.88				
6	131.70					

9) Sample Random Residuals



10) Sample Incremental Data



$$q'(w,d) = r^* \times \sqrt{m^z_{w,d}} + m_{w,d}$$

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ODP Bootstrap Overview

For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Sample Cumulative Triangle

	1	2	3	4	5	6
1	95.42	128.01	153.27	166.36	168.18	176.84
2	100.56	141.72	154.27	185.19	193.68	
3	135.27	196.84	230.49	243.87		
4	95.73	137.02	169.37			
5	110.03	173.91				
6	131.70					
Factors:		1.448	1.172	1.107	1.029	1.052

11) Sample Age-to-Age Factors

Projected Cumulative Data

	1	2	3	4	5	6
1	95.42	128.01	153.27	166.36	168.18	176.84
2	100.56	141.72	154.27	185.19	193.68	**203.65**
3	135.27	196.84	230.49	243.87	**251.02**	**263.95**
4	95.73	137.02	169.37	**187.43**	**192.93**	**202.87**
5	110.03	173.91	**203.81**	**225.55**	**232.16**	**244.13**
6	131.70	**190.67**	**223.46**	**247.30**	**254.55**	**267.66**

12) Project Ultimate Values

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ODP Bootstrap Overview

For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Projected Incremental Data

	1	2	3	4	5	6	Point Estimate
1							
2						9.98	9.98
3					7.15	12.93	20.08
4				18.07	5.49	9.94	33.50
5			29.91	21.74	6.61	11.96	70.22
6		58.98	32.79	23.84	7.25	13.11	135.97
							269.74

13) Project Incremental Values

Incremental Data w/ Process Variance

	1	2	3	4	5	6	Possible Outcome
1							
2						15.20	15.20
3					9.85	9.71	19.57
4				24.60	9.35	13.52	47.47
5			29.81	14.12	5.97	13.53	63.43
6		44.97	38.24	24.46	11.79	9.10	128.56
							274.23

14) Add Process Variance

$$\phi = \frac{\sum r_{w,d}^2}{N - p}$$

- Repeat steps 9 – 14, a large number of times!

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ODP Bootstrap Overview

- Start with a triangle of cumulative data:

		d: 1	2	3	...	n-1	n
w	1	c(1,1)	c(1,2)	c(1,3)	...	c(1,n-1)	c(1,n)
	2	c(2,1)	c(2,2)	c(2,3)	...	c(2,n-1)	
	3	c(3,1)	c(3,2)	c(3,3)	...		
	...	...	...				
	n-1	c(n-1,1)	c(n-1,2)				
	n	c(n,1)					

- For GLM, we will use the incremental data:

		d: 1	2	3	...	n-1	n
w	1	q(1,1)	q(1,2)	q(1,3)	...	q(1,n-1)	q(1,n)
	2	q(2,1)	q(2,2)	q(2,3)	...	q(2,n-1)	
	3	q(3,1)	q(3,2)	q(3,3)	...		
	...	...	...				
	n-1	q(n-1,1)	q(n-1,2)				
	n	q(n,1)					

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ODP Bootstrap Overview

- The GLM formulation is as follows:

$$E[q(w,d)] = m_{w,d}$$

$$Var[q(w,d)] = \phi E[q(w,d)] = \phi m_{w,d}^{z}$$

$$\ln[m_{w,d}] = \eta_{w,d}$$

$\eta_{w,d} = c + \alpha_w + \beta_d$, where: $w = 1, 2, \ldots, n$; $d = 1, 2, \ldots, n$; and $\alpha_1 = \beta_1 = 0$

$z = 0(Normal), 1(Poisson),$

$2(Gamma),\ or\ 3(Inverse\,Gaussian)$

$\phi = Scale\,Parameter$

- Alternatively:

$\eta_{w,d} = \alpha_w + \beta_d$, where: $w = 1, 2, \ldots, n$ and $d = 2, 3, \ldots, n$

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ODP Bootstrap Overview

- Let's consider a simple example:

	1	2	3
1	$q(1,1)$	$q(1,2)$	$q(1,3)$
2	$q(2,1)$	$q(2,2)$	
3	$q(3,1)$		

- Transforming to a log scale:

	1	2	3
1	$\ln[q(1,1)]$	$\ln[q(1,2)]$	$\ln[q(1,3)]$
2	$\ln[q(2,1)]$	$\ln[q(2,2)]$	
3	$\ln[q(3,1)]$		

- Specify a system of equations with vectors α_w and β_d:

$$\begin{aligned}
\ln[q(1,1)] &= 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 0\beta_2 + 0\beta_3 \\
\ln[q(2,1)] &= 0\alpha_1 + 1\alpha_2 + 0\alpha_3 + 0\beta_2 + 0\beta_3 \\
\ln[q(3,1)] &= 0\alpha_1 + 0\alpha_2 + 1\alpha_3 + 0\beta_2 + 0\beta_3 \\
\ln[q(1,2)] &= 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 1\beta_2 + 0\beta_3 \\
\ln[q(2,2)] &= 0\alpha_1 + 1\alpha_2 + 0\alpha_3 + 1\beta_2 + 0\beta_3 \\
\ln[q(1,3)] &= 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 1\beta_2 + 1\beta_3
\end{aligned}$$

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ODP Bootstrap Overview

- Converting to matrix notation we have:

$$Y = X \times A$$

- Where:

$$Y = \begin{bmatrix} \ln[q(1,1)] & 0 & 0 & 0 & 0 & 0 \\ 0 & \ln[q(2,1)] & 0 & 0 & 0 & 0 \\ 0 & 0 & \ln[q(3,1)] & 0 & 0 & 0 \\ 0 & 0 & 0 & \ln[q(1,2)] & 0 & 0 \\ 0 & 0 & 0 & 0 & \ln[q(2,2)] & 0 \\ 0 & 0 & 0 & 0 & 0 & \ln[q(1,3)] \end{bmatrix},$$

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

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ODP Bootstrap Overview

- Solving for the parameters in A that minimize the difference between Y and W, where:

$$W = \begin{bmatrix} \ln[m_{1,1}] & 0 & 0 & 0 & 0 & 0 \\ 0 & \ln[m_{2,1}] & 0 & 0 & 0 & 0 \\ 0 & 0 & \ln[m_{3,1}] & 0 & 0 & 0 \\ 0 & 0 & 0 & \ln[m_{1,2}] & 0 & 0 \\ 0 & 0 & 0 & 0 & \ln[m_{2,2}] & 0 \\ 0 & 0 & 0 & 0 & 0 & \ln[m_{1,3}] \end{bmatrix}$$

- X = Design Matrix, and W = Weight Matrix
- Then we have:

$$\ln[m_{1,1}] = \eta_{1,1} = \alpha_1$$
$$\ln[m_{2,1}] = \eta_{2,1} = \alpha_2$$
$$\ln[m_{3,1}] = \eta_{3,1} = \alpha_3$$
$$\ln[m_{1,2}] = \eta_{1,2} = \alpha_1 + \beta_2$$
$$\ln[m_{2,2}] = \eta_{2,2} = \alpha_2 + \beta_2$$
$$\ln[m_{1,3}] = \eta_{1,3} = \alpha_1 + \beta_2 + \beta_3$$

and

	1	2	3
1	$\ln[m_{1,1}]$	$\ln[m_{1,2}]$	$\ln[m_{1,3}]$
2	$\ln[m_{2,1}]$	$\ln[m_{2,2}]$	
3	$\ln[m_{3,1}]$		

Finally, exponentiating we get:

	1	2	3
1	$m_{1,1}$	$m_{1,2}$	$m_{1,3}$
2	$m_{2,1}$	$m_{2,2}$	
3	$m_{3,1}$		

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ODP Bootstrap Overview

- Using this "GLM Framework" and setting z=1 (Poisson), the solution exactly replicates the "fitted" values using volume-weighted average age-to-age ratios!
- This is generally referred to as the Over-Dispersed Poisson (ODP) Bootstrap model.
- Instead of solving the GLM, we can simplify by using the volume-weighted average ratios.
- We refer to this as the "ODP Bootstrap"
- The "ODP Bootstrap" also improves issues with negative incremental values.

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ODP Bootstrap Overview

- Using a model fit to the data, bootstrapping involves sampling the residuals with replacement, using:

$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}}$$

- From the sampled residuals and fitted incremental values, we can derive a sample triangle using:

$$q'(w,d) = r^{*} \times \sqrt{m_{w,d}^{z}} + m_{w,d}$$

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ODP Bootstrap Overview

- However, in order to correct for a bias in (and standardize) the residuals, the GLM framework requires a hat matrix adjustment factor:

$$H = X\left(X^{T}WX\right)^{-1}X^{T}W$$

$$f_{w,d}^{H} = \sqrt{\frac{1}{1-H_{i,i}}}$$

- Standardized Residuals:

$$r_{w,d}^{H} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}} \times f_{w,d}^{H}$$

- Continuing the bootstrap process…

Can approximate with Degrees of Freedom Adjustment Factor:

$$f^{DoF} = \sqrt{\frac{N}{N-p}}$$

Where: *N* = Number of Data Cells [e.g., *n* x (*n*+1) ÷ 2]
p = Number of Parameters [e.g., 2 x *n* -1]

Or Scaled Residuals:

$$r_{w,d}^{S} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}} \times f^{DoF}$$

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ODP Bootstrap Overview

- Each sample incremental triangle can be converted to cumulative values
- Sample age-to-age factors can be calculated (parameter risk)
- A point estimate can be calculated
- We can add process variance to the future incremental values (from the point estimate) using a Poisson (or Gamma) distribution assuming each incremental cell is the mean and the variance is the cell value times the scale parameter (i.e., to over-disperse the variance):

$$\phi = \frac{\sum r_{w,d}^2}{N-p}$$

- Repeat a significant number of iterations.

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"Fitted" Incremental Triangle

- Work Backwards from observations on diagonal to create estimated cumulative triangle
  - $\hat{c}(w,n\text{-}1) = c(w,n)/F(n\text{-}1)$
  - $\hat{c}(w,n\text{-}2) = \hat{c}(w,n\text{-}1)/F(n\text{-}2)$
  - Fill in all cumulative entries on triangle
- Compute estimated or "fitted" incremental triangle

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Exercises

- Compute fitted incremental triangle from "Exercise" data
  - Use weighted average loss development factors
  - Compute fitted cumulative triangle
  - Compute fitted incremental triangle

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Standardized Residuals

- In "Diagnostics" section, we used standardized residual:

$$z = \frac{x_i - \mu}{\sigma}$$

- More general Pearson Residual used with GLM models:

$$r = \frac{x - \mu}{\sqrt{Var(\mu)}}$$

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Exercise

- Compute Unscaled Pearson Residuals from the "Exercise" incremental data
  – Assume Var(x) = E(x) = fitted incremental value

$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}}}$$

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Pearson Residuals

- If data assumed Normally distributed, Pearson Residual = standardized residual
- If data assumed Poisson, then:

$$Var(\mu) = \mu, \text{ so } r = \frac{x_i - \mu}{\sqrt{\mu}}$$

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Over-Dispersed Poisson

- Often Poisson distributed: Var > Mean
- One of the "Over-Dispersed Poisson" models uses the constant ϕ to inflate Variance:

$$Var(\mu) = \phi\mu, \text{ and Scaled Pearson Residual is } \frac{x-\mu}{\sqrt{\phi\mu}}$$

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Scale Parameter

- Using the Chi-Squared statistic:
  - N = sample size, p = # parameters

$$\phi = X^2 = \sum_N \frac{[x_i - E(x_i)]^2}{(N-p)E(x_i)}$$

- Scaled Residual is:

$$r_i = \frac{x_i - E(x_i)}{\sqrt{\phi E(x_i)}}$$

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Poisson Triangles

- The Poisson has been a useful Parametric assumption in modeling loss development triangles
- The "semi" Parametric bootstrap does not require a distribution assumption but
  - It uses a Pearson Residual
  - The Standardized (or Scaled) Pearson Residual follows the Poisson assumption

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Triangle of Residuals

- Using actual and "fitted" incremental triangles, compute Unscaled Pearson Residuals:

$$e_U(w,d) = [q(w,d) - \hat{q}(w,d)] / \sqrt{\hat{q}(w,d)}$$

- Calculate Degrees of Freedom Adjustment:

$$f^{DoF} = \sqrt{\frac{N}{N-p}}$$

- Calculate Scaled Pearson Residuals:

$$e_s(w,d) = e_u(w,d) \times f^{DoF}$$

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Scale Parameter

- Use Unscaled Pearson Residuals and Degrees of Freedom to calculate the Scale Parameter:

$$\phi = \frac{\sum e_u(w,d)^2}{N-p}$$

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Exercise

- Using "Exercise" data and results of prior exercises
  1. Compute triangle of square of Unscaled Pearson Residuals
  2. Compute the triangle's degrees of freedom
  3. Using 1. and 2. compute the Scale Parameter for "Exercise" data
  4. Compute the triangle's Degrees of Freedom Adjustment Factor
  5. Compute triangle of Scaled Pearson Residuals

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Bootstrap Residuals

- For each cell in the triangle, randomly select a Scaled Pearson Residual (with replacement)
- Transform residual into an incremental value for the triangle

$$q_s(w,d) = \hat{q}(w,d) + [e_s(w,d) \times \sqrt{\hat{q}(w,d)}]$$

- Calculate cumulative sample triangle
- Compute age-to-age factors

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Exercise

- Create a table of Scaled Pearson Residuals, using results of previous exercise
- Simulate a bootstrap triangle of residuals
- Create a triangle of incremental values from bootstrapped residuals
- Compute a cumulative triangle
- Compute weighted average age-to-age factors

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Process Variance

- Use Age-to-Age factors to compute ultimate for sample data
- Calculate incremental values for completed triangle
- Use the Gamma distribution to simulate random incremental values with:
  - Mean = sample incremental
  - Variance = sample incremental x Scale Parameter

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Distribution of Estimates

- Add incremental values after process variance to get ultimate and unpaid estimates
- Sum the unpaid amounts to get total unpaid
- Repeat many times...

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Distribution of Estimates

- A "trick" is needed to easily compute a distribution of unpaid amounts
- Use the Data Table (menu) function of Excel
- ={Table(,input cell=a constant)}
- Or use VBA to write a macro

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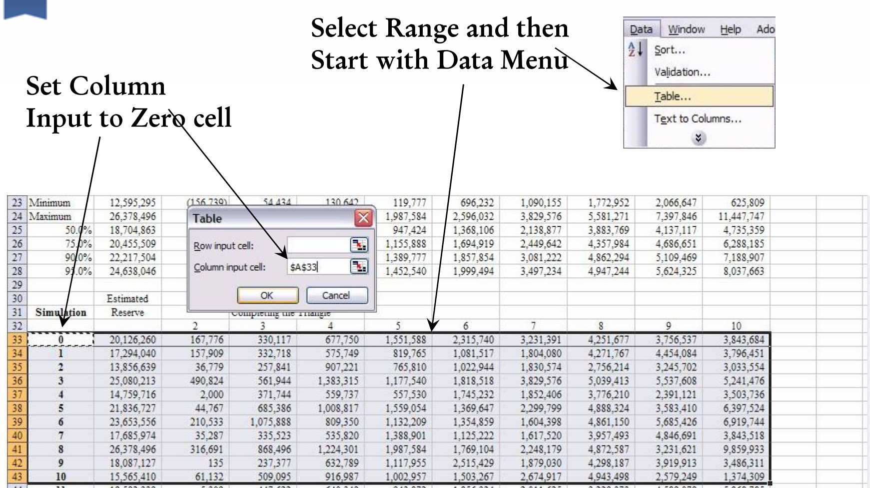
Data Table Function

Select Range and then
Start with Data Menu

Set Column
Input to Zero cell



23	Minimum	12,595,295	(156,739)	54,434	130,642	119,777	696,232	1,090,155	1,772,952	2,066,647	625,809
24	Maximum	26,378,496				1,987,584	2,596,032	3,829,576	5,581,271	7,397,846	11,447,747
25	50.0%	18,704,863				947,424	1,368,106	2,138,877	3,883,769	4,137,117	4,735,359
26	75.0%	20,455,509				1,155,888	1,694,919	2,449,642	4,357,984	4,686,651	6,288,185
27	90.0%	22,217,504				1,389,777	1,857,854	3,081,222	4,862,294	5,109,469	7,188,907
28	95.0%	24,638,046				1,452,540	1,999,494	3,497,234	4,947,244	5,624,325	8,037,663
29											
30		Estimated									
31	Simulation	Reserve									
32			2	3	4	5	6	7	8	9	10
33	0	20,126,260	167,776	330,117	677,750	1,551,588	2,315,740	3,231,391	4,251,677	3,756,537	3,843,684
34	1	17,294,040	157,909	332,718	575,749	819,765	1,081,517	1,804,080	4,271,767	4,454,084	3,796,451
35	2	13,856,639	36,779	257,841	907,221	765,810	1,022,944	1,830,574	2,756,214	3,245,702	3,033,554
36	3	25,080,213	490,824	561,944	1,383,315	1,177,540	1,818,518	3,829,576	5,039,413	5,537,608	5,241,476
37	4	14,759,716	2,000	371,744	559,737	557,530	1,745,232	1,852,406	3,776,210	2,391,121	3,503,736
38	5	21,836,727	44,767	685,386	1,008,817	1,559,054	1,369,647	2,299,799	4,888,324	3,583,410	6,397,524
39	6	23,653,556	210,533	1,075,888	809,350	1,132,209	1,354,859	1,604,398	4,861,150	5,685,426	6,919,744
40	7	17,685,974	35,287	335,523	535,820	1,388,901	1,125,222	1,617,520	3,957,493	4,846,691	3,843,518
41	8	26,378,496	316,691	868,496	1,224,301	1,987,584	1,769,104	2,248,179	4,872,587	3,231,621	9,859,933
42	9	18,087,127	135	237,377	632,789	1,117,955	2,515,429	1,879,030	4,298,187	3,919,913	3,486,311
43	10	15,565,410	61,132	509,095	916,987	1,002,957	1,503,267	2,674,917	4,943,498	2,579,249	1,374,309
44	11	19,592,339	5,309	447,622	640,349	943,872	1,856,324	2,011,625	3,229,373	4,589,079	5,868,785

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Exercise

- Review bootstrap calculation in Bootstrap spreadsheet
- Where is the calculation of
  - The fitted cumulative triangle?
  - The fitted incremental triangle?
  - The residuals?
- What formula is used to resample residuals?
- Where is estimation of bootstrapped unpaid?
- What diagnostics are used?
- Paste value a new triangle into the Inputs sheet and run a new model for 100 iterations

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Parametric Bootstrap

- Sampling of residuals is limited to past observations
- For a 10 x 10 triangle, 53 residuals are sampled so extremes could be considered 1 in 53 events, but we usually want 1 in 100 or 1 in 200 events.
- Solution: Parameterize residuals and simulate from a distribution

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Monograph Outline

- Introduction
- Notation
- Basic ODP Model / GLM Framework
- Generalizing the GLM Framework
- Practical Data Issues / Algorithm Enhancements
- Model Diagnostics
- Using Multiple Models
- Aggregation Issues / Model Uses
- Testing & Future Research

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ODP Bootstrap

For the "ODP Bootstrap":

- Using Incurred data (instead of Paid) results in a distribution of IBNR
- We can adjust by converting Incurred ultimate values to a "random" paid development pattern
- We could also adjust the "calculate point estimate step" by switching to a Bornhuetter-Ferguson, Cape Cod or other methodology.

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GLM Bootstrap Overview

For the "GLM Bootstrap":

- We can abandon the need to calculate age-to-age factors.
- Here's a simple example using a 6 x 6 triangle:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.69
α_2	4.69
α_3	4.73
α_4	4.70
α_5	4.78
α_6	4.83
β_2	-0.85
β_3	-0.69
β_4	-0.02
β_5	-1.12
β_6	-0.41

Fitted Values

	1	2	3	4	5	6
1	109.16	46.78	23.51	23.05	7.50	5.00
2	109.16	46.78	23.51	23.05	7.50	
3	113.20	48.51	24.39	23.90		
4	109.49	46.92	23.59			
5	119.00	51.00				
6	125.00					

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GLM Bootstrap Overview

For the "GLM Bootstrap":

- We can abandon the need to calculate age-to-age factors.
- And use only one α_w, or "level", parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.74
β_2	-0.87
β_3	-0.70
β_4	-0.02
β_5	-1.13
β_6	-0.41

Fitted Values

	1	2	3	4	5	6
1	114.17	48.00	23.75	23.33	7.50	5.00
2	114.17	48.00	23.75	23.33	7.50	
3	114.17	48.00	23.75	23.33		
4	114.17	48.00	23.75			
5	114.17	48.00				
6	114.17					

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GLM Bootstrap Overview

For the "GLM Bootstrap":

- We can abandon the need to calculate age-to-age factors.
- Or, use only one β_d, or "development", parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.65
α_2	4.65
α_3	4.68
α_4	4.61
α_5	4.71
α_6	4.83
β_2	-0.65

Fitted Values

	1	2	3	4	5	6
1	104.61	54.76	28.66	15.00	7.85	4.11
2	104.17	54.53	28.54	14.94	7.82	
3	108.20	56.64	29.65	15.52		
4	100.14	52.42	27.44			
5	111.59	58.41				
6	125.00					

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GLM Bootstrap Overview

For the "GLM Bootstrap":

- We can abandon the need to calculate age-to-age factors.
- Or, use only one α_w and one β_d parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.69
β_2	-0.66

Fitted Values

	1	2	3	4	5	6
1	108.53	55.93	28.83	14.86	7.66	3.95
2	108.53	55.93	28.83	14.86	7.66	
3	108.53	55.93	28.83	14.86		
4	108.53	55.93	28.83			
5	108.53	55.93				
6	108.53					

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GLM Bootstrap Overview

For the "GLM Bootstrap":

- We can abandon the need to calculate age-to-age factors.
- Or, add a Calendar period parameter using:

$$\eta_{w,d} = \alpha_w + \beta_d + \gamma_k$$

Where: $w = 1, 2, \dots, n$

$d = 2, 3, \dots, n$

$k = 2, 3, \dots, n$

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GLM Bootstrap Overview

For the "GLM Bootstrap":

- We can abandon the need to calculate age-to-age factors.
- And use only one α_w and one β_d and one γ_k parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.63
β_2	-0.67
γ_2	0.02

Fitted Values

	1	2	3	4	5	6
1	102.20	53.31	27.81	14.51	7.57	3.95
2	104.65	54.59	28.48	14.85	7.75	
3	107.16	55.90	29.16	15.21		
4	109.74	57.24	29.86			
5	112.37	58.62				
6	115.07					

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GLM Bootstrap Overview

For the "GLM Bootstrap":

- Selection of model parameters is very flexible
- The rest of the theory still applies (e.g., hat matrix)
- Sample triangles used to fit new parameters for each iteration
- Sample parameters used to project incremental values to ultimate
- Can still add process variance to future mean incremental values

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Questions on Bootstrap Model?

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